Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.2-d-sec-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [879]. This is test number [118].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (879)	% 0.00 (0)
Mathematica	% 98.86 (869)	% 1.14 (10)
Maple	% 83.62 (735)	% 16.38 (144)
Maxima	% 35.15 (309)	% 64.85 (570)
Fricas	% 44.71 (393)	% 55.29 (486)
Sympy	% 5.57 (49)	% 94.43 (830)
Giac	% 29.81 (262)	% 70.19 (617)
Mupad	% 36.75 (323)	% 63.25 (556)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

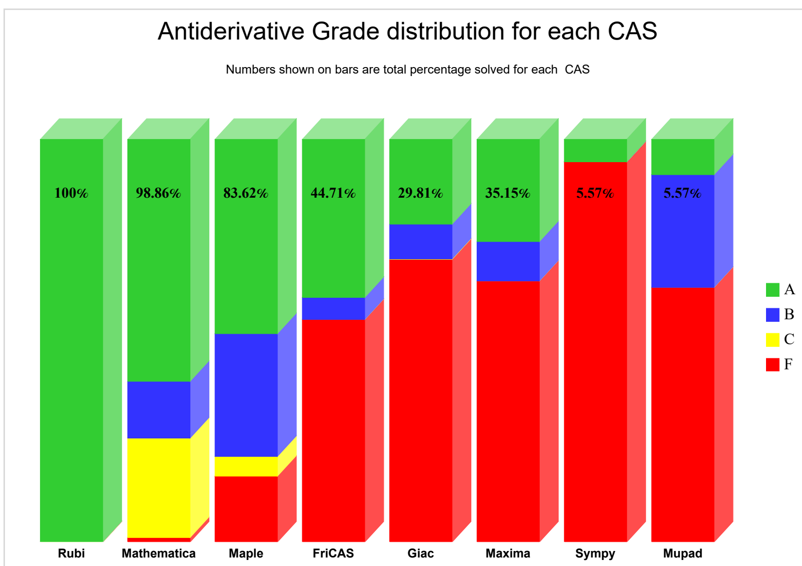
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

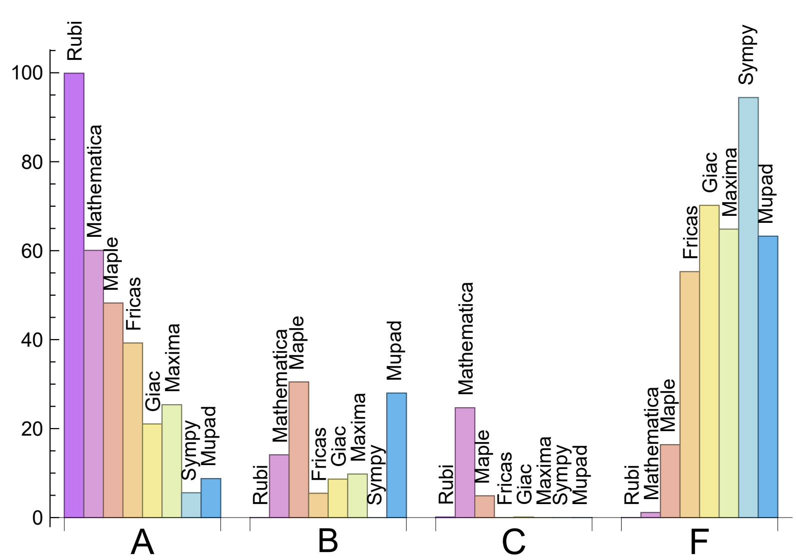
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.89	0.00	0.11	0.00
Mathematica	60.07	14.11	24.69	1.14
Maple	48.24	30.49	4.89	16.38
Maxima	25.37	9.78	0.00	64.85
Fricas	39.25	5.46	0.00	55.29
Sympy	5.57	0.00	0.00	94.43
Giac	21.05	8.65	0.11	70.19
Mupad	8.76	27.99	0.00	63.25

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	100.00 %	0.00 %	0.00 %
Maple	144	100.00 %	0.00 %	0.00 %
Maxima	570	76.49 %	16.14 %	7.37 %
Fricas	486	81.48 %	18.31 %	0.21 %
Sympy	830	66.39 %	33.61 %	0.00 %
Giac	617	91.57 %	7.46 %	0.97 %
Mupad	556	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

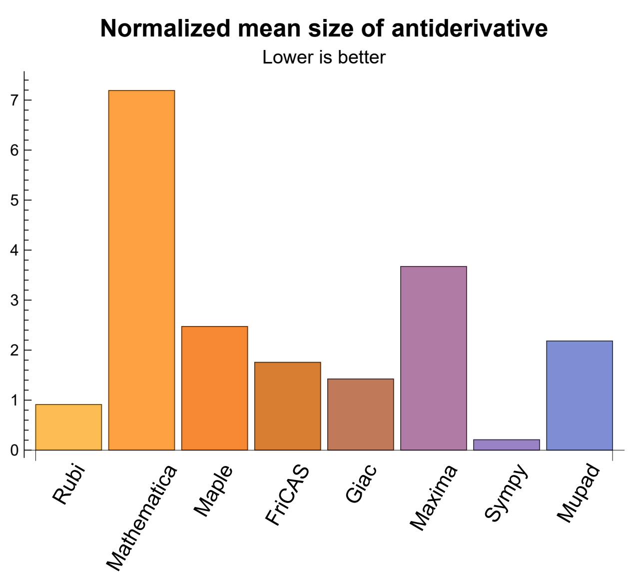
1.3 Performance

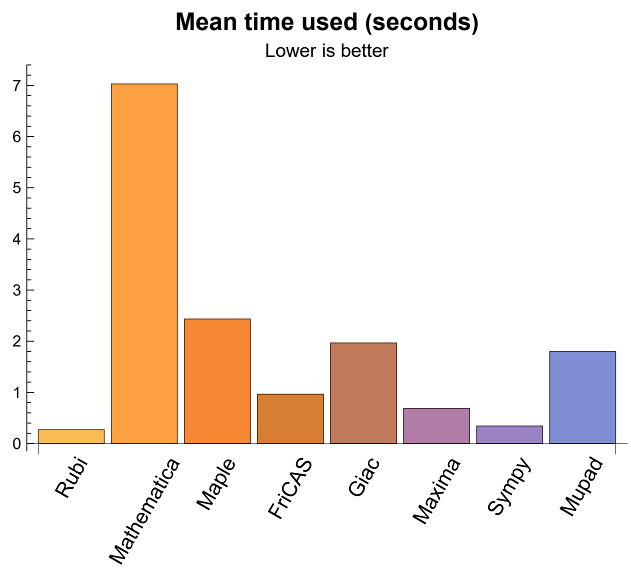
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	152.93	0.91	128.00	1.00
Mathematica	7.03	1428.94	7.19	155.00	1.08
Maple	2.43	516.90	2.47	219.00	1.79
Maxima	0.69	502.75	3.67	97.00	1.16
Fricas	0.96	222.01	1.76	121.00	1.30
Sympy	0.34	3.80	0.21	0.00	0.00
Giac	1.97	160.93	1.42	115.00	1.40
Mupad	1.80	376.02	2.18	88.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {286}

Mathematica {11, 21, 111, 112, 123, 131, 139, 146, 147, 151, 152, 157, 158, 163, 164, 186, 219, 227, 228, 235, 236, 237, 245, 249, 255, 256, 257, 264, 265, 266, 267, 269, 270, 271, 280, 281, 282, 283, 284, 285, 287, 289, 290, 294, 297, 298, 301, 302, 305, 306, 307, 310, 311, 314, 315, 318, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 388, 391, 392, 395, 396, 397, 408, 415, 416, 417, 425, 426, 427, 428, 433, 434, 436, 437, 440, 441, 442, 497, 531, 535, 536, 537, 538, 539, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 635, 642, 643, 662, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 837, 838, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
```

```
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

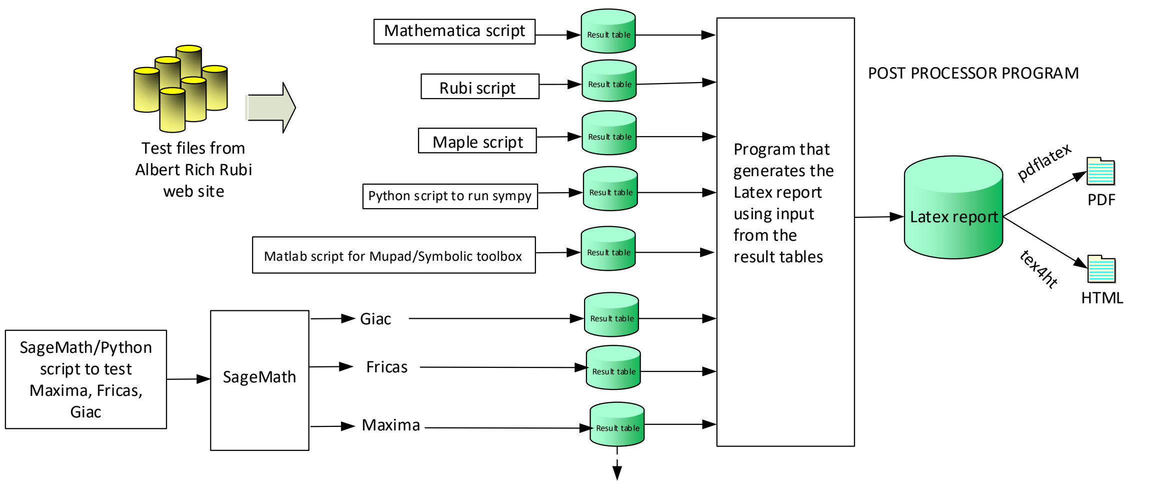
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

B grade: { }

C grade: { 286 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 46, 47, 49, 50, 51, 55, 56, 57, 59, 60, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 140, 165, 166, 167, 168, 169, 170, 171, 175, 189, 204, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 286, 291, 295, 296, 299, 300, 303, 304, 308, 309, 312, 313, 316, 317, 341, 342, 343, 344, 363, 385, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 440, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 537, 538, 539, 540, 544, 545, 546, 547, 554, 555, 556, 558, 559, 562, 563, 564, 565, 566, 570, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 621, 627, 629, 630, 631, 632, 633, 636, 637, 638, 639, 644, 645, 646, 649, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 875, 876, 877 }

B grade: { 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 42, 43, 44, 45, 48, 52, 53, 54, 58, 61, 62, 70, 71, 77, 78, 80, 88, 146, 147, 151, 152, 157, 158, 163, 164, 253, 260, 280, 281, 282, 283, 284, 297, 298, 301, 302, 305, 306, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 431, 438, 480, 529, 530, 549, 551, 557, 568, 571, 576, 616, 617, 619, 620, 622, 623, 624, 625, 626, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 822, 878, 879 }

C grade: { 96, 97, 98, 111, 112, 113, 114, 115, 117, 118, 123, 131, 133, 138, 139, 141, 142, 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 233, 234, 244, 272, 273, 274, 275, 276, 277, 278, 279, 285, 287, 289, 290, 294, 307, 320, 321, 322, 325, 326, 327, 328, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 418, 419, 441, 442, 505, 535, 536, 541, 542, 543, 548, 550, 552, 553, 560, 561, 567, 569, 575, 577, 578, 628, 634, 635, 640, 641, 642, 643, 647, 648, 654, 655, 661, 662, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F grade: { 288, 292, 293, 323, 324, 332, 336, 340, 444, 445 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 112, 116, 118, 121, 123, 142, 167, 168, 169, 170, 171, 175, 176, 177, 178, 182, 183, 184, 185, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 248, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 262, 263, 264, 269, 270, 271, 351, 352, 354, 355, 359, 360, 361, 363, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 386, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 404, 405, 406, 407, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 497, 498, 500, 501, 503, 504, 508, 509, 510, 516, 517, 518, 519, 523, 524, 525, 526, 527, 528, 529, 530, 534, 558, 559, 581, 582, 583, 584, 585, 589, 590, 591, 592, 596, 597, 598, 599, 605, 606, 607, 608, 611, 612, 650, 678, 679, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 798, 799, 806, 813, 819, 820, 858 }

B grade: { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 113, 114, 115, 117, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 165, 166, 172, 173, 174, 179, 180, 181, 186, 187, 188, 189, 194, 204, 210, 218, 219, 220, 226, 227, 228, 235, 236, 243, 244, 245, 246, 247, 252, 258, 259, 265, 266, 267, 268, 353, 356, 357, 358, 362, 364, 365, 366, 371, 372, 373, 380, 385, 387, 388, 396, 402, 403, 408, 409, 439, 440, 487, 488, 494, 495, 496, 499, 502, 505, 506, 507, 511, 512, 513, 514, 515, 520, 521, 522, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 604, 609, 610, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 670, 671, 796, 797, 800, 801, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

C grade: { 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 242, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 875, 876, 877, 878, 879 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 221, 222, 229, 238, 247, 248, 269, 400, 401, 407, 414, 422, 423, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 523, 524, 525, 526, 527, 528, 529, 530, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 30, 41, 42, 44, 48, 94, 95, 96, 97, 98, 103, 104, 111, 112, 117, 118, 218, 219, 220, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 237, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 254, 255, 258, 259, 261, 262, 265, 266, 267, 268, 270, 271, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 417, 418, 419, 420, 421, 424, 425, 426, 429, 430, 432, 433, 436, 437, 439, 440 }

C grade: { }

F grade: { 90, 91, 92, 93, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 236, 253, 256, 257, 260, 263, 264, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 416, 427, 428, 431, 434, 435, 438, 441, 442, 443, 444, 445, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 137, 140, 141, 142, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 270, 271, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 496, 500, 501, 503, 504, 505, 508, 513, 523, 524, 525, 526, 527, 528, 529, 530, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 4, 5, 14, 117, 118, 129, 130, 131, 136, 138, 139, 219, 220, 227, 228, 243, 244, 252, 259, 265, 266, 267, 268, 269, 449, 450, 460, 488, 489, 497, 498, 499, 502, 506, 507, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522 }

C grade: { }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.6 Sympy

A grade: { 4, 5, 6, 449, 450, 451, 686, 691, 693, 698, 703, 705, 707, 712, 720, 721, 722, 723, 724, 725, 732, 733, 734, 735, 749, 750, 751, 752, 753, 754, 755, 759, 760, 761, 762, 763, 764, 770, 771, 772, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 717, 718, 719, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 756, 757, 758, 765, 766, 767, 768, 769, 773, 774, 775, 776, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.7 Giac

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 107, 108, 109, 110, 119, 120, 122, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 455, 470, 471, 472, 480, 481, 482, 488, 489, 491, 493, 494, 495, 497, 499, 500, 501, 502, 504, 505, 506, 507, 508, 512, 514, 516, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 737, 738, 739, 740, 741, 763, 773, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 4, 5, 6, 14, 15, 93, 94, 95, 96, 97, 98, 103, 104, 106, 111, 112, 113, 114, 116, 117, 118, 121, 124, 125, 132, 133, 140, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 490, 492, 496, 498, 503, 509, 510, 511, 513, 515, 517, 518, 519, 520 }

C grade: { 141 }

F grade: { 105, 115, 123, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 732, 733, 734, 735, 736, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 764, 765, 766, 767, 768, 769, 770, 771, 772, 774, 775, 776, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.8 Mupad

A grade: { 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 116, 221, 222, 223, 224, 229, 230, 231, 232, 238, 239, 240, 241, 242, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816 }

C grade: { }

F grade: { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 226, 227, 228, 233, 234, 235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	95	99	0	110	130
normalized size	1	1.00	0.89	1.08	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.059	0.182	0.698	0.776	0.794	0.000	0.463	3.941

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	70	88	0	96	102
normalized size	1	1.00	0.95	1.14	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.046	0.155	0.702	0.488	0.717	0.000	0.499	2.511

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	58	74	0	80	75
normalized size	1	1.00	1.00	1.09	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.042	0.022	0.720	0.319	0.612	0.000	0.434	1.057

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	29	60	37	63	47
normalized size	1	1.00	1.00	1.33	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.023	0.012	0.520	0.632	0.667	4.739	0.370	0.685

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	20
normalized size	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	1.25
time (sec)	N/A	0.007	0.002	0.028	0.654	1.644	2.024	0.338	0.610

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	20	17	15	39	15
normalized size	1	1.00	1.73	1.40	1.33	1.13	1.00	2.60	1.00
time (sec)	N/A	0.019	0.009	0.478	0.359	0.686	2.089	0.391	0.600

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	38	34	29	0	56	50
normalized size	1	1.00	0.84	1.00	0.89	0.76	0.00	1.47	1.32
time (sec)	N/A	0.035	0.056	0.579	0.970	1.663	0.000	0.363	1.063

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	0	72	55
normalized size	1	1.00	1.06	0.91	0.85	0.78	0.00	1.33	1.02
time (sec)	N/A	0.040	0.070	0.820	0.324	0.782	0.000	0.390	0.654

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	0	86	79
normalized size	1	1.00	0.96	0.79	0.75	0.70	0.00	1.13	1.04
time (sec)	N/A	0.055	0.125	0.957	0.751	0.653	0.000	0.381	4.170

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	487	124	133	124	0	138	170
normalized size	1	1.00	3.99	1.02	1.09	1.02	0.00	1.13	1.39
time (sec)	N/A	0.095	1.724	0.907	0.340	0.598	0.000	0.474	5.700

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	877	102	145	111	0	122	141
normalized size	1	1.00	9.14	1.06	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.084	6.463	0.870	0.772	0.796	0.000	0.479	3.960

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	318	78	85	96	0	106	112
normalized size	1	1.00	4.30	1.05	1.15	1.30	0.00	1.43	1.51
time (sec)	N/A	0.081	0.650	0.852	0.659	0.653	0.000	0.549	2.466

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	219	58	81	83	0	90	83
normalized size	1	1.00	4.06	1.07	1.50	1.54	0.00	1.67	1.54
time (sec)	N/A	0.047	0.610	0.711	0.645	0.694	0.000	0.478	1.175

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	171	50	41	76	0	79	56
normalized size	1	1.00	5.03	1.47	1.21	2.24	0.00	2.32	1.65
time (sec)	N/A	0.024	0.468	0.523	0.477	0.886	0.000	0.376	0.710

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	47	51	52	53	0	79	33
normalized size	1	1.00	1.38	1.50	1.53	1.56	0.00	2.32	0.97
time (sec)	N/A	0.051	0.012	0.536	0.835	0.639	0.000	0.410	0.700

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	48	36	0	64	57
normalized size	1	1.00	0.76	1.16	1.07	0.80	0.00	1.42	1.27
time (sec)	N/A	0.060	0.038	0.518	0.788	0.728	0.000	0.473	1.074

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	64	61	49	0	80	61
normalized size	1	1.00	0.72	1.12	1.07	0.86	0.00	1.40	1.07
time (sec)	N/A	0.082	0.085	0.902	0.726	0.554	0.000	1.800	0.656

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	90	83	63	0	96	89
normalized size	1	1.00	0.61	1.03	0.95	0.72	0.00	1.10	1.02
time (sec)	N/A	0.079	0.140	0.982	0.903	0.721	0.000	3.918	4.193

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	96	95	76	0	112	105
normalized size	1	1.00	0.59	0.93	0.92	0.74	0.00	1.09	1.02
time (sec)	N/A	0.110	0.136	1.254	0.657	0.544	0.000	1.221	4.419

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	487	124	179	124	0	138	170
normalized size	1	1.00	4.27	1.09	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.125	1.507	1.056	0.821	0.642	0.000	1.660	5.482

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	877	101	156	111	0	122	141
normalized size	1	1.00	9.43	1.09	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.114	6.422	1.016	0.621	0.816	0.000	1.874	4.053

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	154	80	104	98	0	106	112
normalized size	1	1.00	2.14	1.11	1.44	1.36	0.00	1.47	1.56
time (sec)	N/A	0.075	5.669	0.854	0.319	0.707	0.000	3.908	2.574

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	235	71	91	98	0	100	88
normalized size	1	1.00	3.56	1.08	1.38	1.48	0.00	1.52	1.33
time (sec)	N/A	0.047	0.926	0.692	0.406	0.830	0.000	1.971	0.737

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	211	65	64	91	0	80	57
normalized size	1	1.00	4.40	1.35	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.058	0.865	0.649	0.740	0.774	0.000	2.012	0.700

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	72	74	65	0	100	88
normalized size	1	1.00	1.37	1.22	1.25	1.10	0.00	1.69	1.49
time (sec)	N/A	0.067	0.071	0.486	0.520	0.604	0.000	0.486	0.725

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	71	50	0	80	63
normalized size	1	1.00	0.70	1.17	1.13	0.79	0.00	1.27	1.00
time (sec)	N/A	0.075	0.061	0.730	0.583	0.431	0.000	0.480	0.670

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	100	94	63	0	96	89
normalized size	1	1.00	0.60	1.18	1.11	0.74	0.00	1.13	1.05
time (sec)	N/A	0.097	0.125	0.791	0.508	0.579	0.000	0.691	4.173

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	121	117	76	0	112	105
normalized size	1	1.00	0.60	1.15	1.11	0.72	0.00	1.07	1.00
time (sec)	N/A	0.112	0.149	1.272	0.445	0.660	0.000	0.567	4.377

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	73	143	143	89	0	128	121
normalized size	1	1.00	0.57	1.11	1.11	0.69	0.00	0.99	0.94
time (sec)	N/A	0.134	0.209	1.432	0.530	0.637	0.000	2.027	3.492

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	211	146	270	137	0	154	199
normalized size	1	1.00	1.55	1.07	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.172	0.819	1.186	0.712	1.317	0.000	3.640	4.649

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	498	123	190	124	0	138	170
normalized size	1	1.00	4.49	1.11	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.135	1.523	1.392	0.687	0.707	0.000	0.974	5.470

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	877	102	175	111	0	122	141
normalized size	1	1.00	9.14	1.06	1.82	1.16	0.00	1.27	1.47
time (sec)	N/A	0.109	6.422	1.162	0.539	0.533	0.000	0.779	3.989

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	773	93	116	110	0	116	117
normalized size	1	1.00	8.49	1.02	1.27	1.21	0.00	1.27	1.29
time (sec)	N/A	0.090	6.272	0.849	0.438	7.104	0.000	0.410	0.878

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	272	86	110	111	0	129	115
normalized size	1	1.00	3.73	1.18	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.076	1.383	0.827	0.634	0.814	0.000	0.594	0.905

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	241	86	85	105	0	129	117
normalized size	1	1.00	3.30	1.18	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.079	1.804	0.663	0.352	0.895	0.000	0.818	0.891

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	91	94	97	80	0	116	93
normalized size	1	1.00	1.25	1.29	1.33	1.10	0.00	1.59	1.27
time (sec)	N/A	0.082	0.106	0.734	0.932	0.811	0.000	0.457	0.687

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	111	104	63	0	96	89
normalized size	1	1.00	0.64	1.28	1.20	0.72	0.00	1.10	1.02
time (sec)	N/A	0.098	0.103	0.896	0.749	0.899	0.000	1.229	4.125

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	133	128	76	0	112	105
normalized size	1	1.00	0.62	1.30	1.25	0.75	0.00	1.10	1.03
time (sec)	N/A	0.114	0.153	1.217	0.449	1.516	0.000	0.621	4.428

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	169	165	89	0	128	121
normalized size	1	1.00	0.57	1.33	1.30	0.70	0.00	1.01	0.95
time (sec)	N/A	0.145	0.220	1.357	0.717	0.782	0.000	1.242	3.418

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	83	185	187	102	0	144	137
normalized size	1	1.00	0.56	1.26	1.27	0.69	0.00	0.98	0.93
time (sec)	N/A	0.155	0.267	1.583	0.715	0.733	0.000	0.645	3.636

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	229	168	314	150	0	170	228
normalized size	1	1.00	1.47	1.08	2.01	0.96	0.00	1.09	1.46
time (sec)	N/A	0.198	1.394	1.338	0.735	0.787	0.000	2.237	4.843

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	374	183	205	124	0	114	96
normalized size	1	1.00	3.63	1.78	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.098	3.485	0.358	0.652	0.628	0.000	0.931	0.933

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	250	143	162	112	0	101	95
normalized size	1	1.00	2.94	1.68	1.91	1.32	0.00	1.19	1.12
time (sec)	N/A	0.093	1.457	0.412	0.589	0.752	0.000	0.853	0.727

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	194	99	119	97	0	84	67
normalized size	1	1.00	3.80	1.94	2.33	1.90	0.00	1.65	1.31
time (sec)	N/A	0.106	0.798	0.354	0.411	0.669	0.000	3.706	0.679

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	109	58	75	65	0	54	31
normalized size	1	1.00	2.87	1.53	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.068	0.180	0.332	0.666	1.713	0.000	1.994	0.647

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	0	16	16
normalized size	1	1.00	0.77	0.77	1.05	1.00	0.00	0.73	0.73
time (sec)	N/A	0.024	0.025	0.362	0.392	0.689	0.000	3.890	0.589

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	37	49	37	0	28	23
normalized size	1	1.00	2.00	1.28	1.69	1.28	0.00	0.97	0.79
time (sec)	N/A	0.014	0.128	0.405	1.154	0.650	0.000	2.130	0.638

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	89	68	92	46	0	58	66
normalized size	1	1.00	2.02	1.55	2.09	1.05	0.00	1.32	1.50
time (sec)	N/A	0.057	0.233	0.548	0.610	0.510	0.000	0.382	0.662

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	117	103	133	57	0	73	89
normalized size	1	1.00	1.58	1.39	1.80	0.77	0.00	0.99	1.20
time (sec)	N/A	0.082	0.246	0.511	0.686	0.816	0.000	0.328	0.725

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	143	136	176	70	0	88	70
normalized size	1	1.00	1.52	1.45	1.87	0.74	0.00	0.94	0.74
time (sec)	N/A	0.090	0.323	0.585	1.070	0.482	0.000	1.928	0.907

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	173	171	217	79	0	101	98
normalized size	1	1.00	1.47	1.45	1.84	0.67	0.00	0.86	0.83
time (sec)	N/A	0.101	0.316	0.556	1.174	0.693	0.000	3.916	2.454

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	300	162	190	162	0	122	122
normalized size	1	1.00	2.44	1.32	1.54	1.32	0.00	0.99	0.99
time (sec)	N/A	0.179	1.905	0.359	1.172	0.707	0.000	1.949	0.735

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	247	120	145	146	0	106	92
normalized size	1	1.00	2.78	1.35	1.63	1.64	0.00	1.19	1.03
time (sec)	N/A	0.155	1.262	0.396	0.488	0.753	0.000	0.428	0.688

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	160	77	98	114	0	77	43
normalized size	1	1.00	2.42	1.17	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.118	0.368	0.350	0.435	1.372	0.000	0.826	0.643

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	32	46	49	0	31	30
normalized size	1	1.00	0.82	0.58	0.84	0.89	0.00	0.56	0.55
time (sec)	N/A	0.066	0.069	0.409	0.497	0.593	0.000	0.695	0.596

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	32	47	51	0	31	30
normalized size	1	1.00	1.09	0.58	0.85	0.93	0.00	0.56	0.55
time (sec)	N/A	0.049	0.143	0.334	0.878	0.790	0.000	0.495	0.597

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	112	56	72	80	0	50	35
normalized size	1	1.00	1.96	0.98	1.26	1.40	0.00	0.88	0.61
time (sec)	N/A	0.069	0.283	0.458	1.415	1.370	0.000	0.370	0.630

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	151	88	118	90	0	79	91
normalized size	1	1.00	2.10	1.22	1.64	1.25	0.00	1.10	1.26
time (sec)	N/A	0.129	0.525	0.502	0.916	0.821	0.000	0.343	0.698

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	177	122	164	99	0	95	113
normalized size	1	1.00	1.61	1.11	1.49	0.90	0.00	0.86	1.03
time (sec)	N/A	0.175	0.390	0.598	1.133	0.948	0.000	0.385	0.735

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	199	156	207	108	0	108	135
normalized size	1	1.00	1.60	1.26	1.67	0.87	0.00	0.87	1.09
time (sec)	N/A	0.190	0.451	0.599	1.119	1.402	0.000	0.912	0.782

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	351	181	211	206	0	139	141
normalized size	1	1.00	2.17	1.12	1.30	1.27	0.00	0.86	0.87
time (sec)	N/A	0.292	1.051	0.424	0.619	0.809	0.000	3.531	0.679

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	294	139	165	190	0	122	111
normalized size	1	1.00	2.30	1.09	1.29	1.48	0.00	0.95	0.87
time (sec)	N/A	0.265	1.338	0.357	0.394	0.873	0.000	1.985	0.682

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	209	96	119	158	0	94	58
normalized size	1	1.00	1.99	0.91	1.13	1.50	0.00	0.90	0.55
time (sec)	N/A	0.222	0.512	0.410	0.747	0.732	0.000	1.934	0.688

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	45	67	75	0	46	45
normalized size	1	1.00	0.69	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.123	0.110	0.409	0.892	0.584	0.000	1.572	0.616

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	32	47	73	0	31	30
normalized size	1	1.00	0.86	0.39	0.57	0.88	0.00	0.37	0.36
time (sec)	N/A	0.097	0.164	0.357	0.725	0.565	0.000	1.255	0.597

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	45	67	75	0	46	45
normalized size	1	1.00	1.04	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.081	0.224	0.386	0.763	0.580	0.000	1.000	0.619

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	162	75	92	116	0	68	81
normalized size	1	1.00	1.84	0.85	1.05	1.32	0.00	0.77	0.92
time (sec)	N/A	0.112	0.278	0.420	0.963	0.560	0.000	0.510	0.692

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	169	107	137	126	0	96	113
normalized size	1	1.00	1.64	1.04	1.33	1.22	0.00	0.93	1.10
time (sec)	N/A	0.221	0.581	0.614	1.212	1.298	0.000	4.363	0.728

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	181	141	184	135	0	113	137
normalized size	1	1.00	1.23	0.96	1.25	0.92	0.00	0.77	0.93
time (sec)	N/A	0.290	0.577	0.557	1.023	0.680	0.000	1.814	0.758

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	403	200	231	250	0	155	160
normalized size	1	1.00	2.09	1.04	1.20	1.30	0.00	0.80	0.83
time (sec)	N/A	0.393	1.558	0.441	1.076	0.527	0.000	0.500	0.754

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	349	158	186	234	0	139	130
normalized size	1	1.00	2.19	0.99	1.17	1.47	0.00	0.87	0.82
time (sec)	N/A	0.369	1.287	0.507	1.009	1.054	0.000	0.496	0.702

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	115	139	202	0	110	83
normalized size	1	1.00	1.42	0.85	1.02	1.49	0.00	0.81	0.61
time (sec)	N/A	0.323	0.936	0.402	0.785	0.592	0.000	2.868	0.668

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	69	56	87	99	0	59	58
normalized size	1	1.00	0.58	0.47	0.72	0.82	0.00	0.49	0.48
time (sec)	N/A	0.169	0.200	0.414	1.022	1.428	0.000	2.872	0.673

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	58	87	99	0	59	58
normalized size	1	1.00	0.78	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.154	0.243	0.362	1.032	0.899	0.000	0.879	0.662

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	58	87	99	0	59	58
normalized size	1	1.00	0.88	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.125	0.268	0.411	0.792	0.806	0.000	0.586	0.675

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	58	87	99	0	59	58
normalized size	1	1.00	1.00	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.111	0.257	0.341	0.636	0.648	0.000	0.504	0.669

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	224	94	112	152	0	83	102
normalized size	1	1.00	2.02	0.85	1.01	1.37	0.00	0.75	0.92
time (sec)	N/A	0.160	0.395	0.472	1.193	1.039	0.000	0.413	0.725

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	263	126	158	162	0	112	137
normalized size	1	1.00	2.09	1.00	1.25	1.29	0.00	0.89	1.09
time (sec)	N/A	0.304	0.464	0.605	0.802	0.893	0.000	0.505	0.755

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	289	160	204	171	0	128	159
normalized size	1	1.00	1.64	0.91	1.16	0.97	0.00	0.73	0.90
time (sec)	N/A	0.393	0.585	0.644	1.094	0.658	0.000	0.454	0.811

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	401	177	206	278	0	155	149
normalized size	1	1.00	2.00	0.88	1.03	1.39	0.00	0.78	0.74
time (sec)	N/A	0.480	1.971	0.363	0.762	1.305	0.000	0.977	0.715

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	219	134	159	246	0	126	99
normalized size	1	1.00	1.24	0.76	0.90	1.39	0.00	0.71	0.56
time (sec)	N/A	0.427	1.960	0.416	0.690	0.660	0.000	0.876	0.691

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	71	107	123	0	72	127
normalized size	1	1.00	0.61	0.45	0.67	0.77	0.00	0.45	0.80
time (sec)	N/A	0.216	0.187	0.409	0.706	1.696	0.000	1.462	0.764

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	58	87	123	0	59	58
normalized size	1	1.00	0.61	0.36	0.55	0.77	0.00	0.37	0.36
time (sec)	N/A	0.215	0.207	0.407	0.599	0.669	0.000	1.427	0.720

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	110	45	67	123	0	46	45
normalized size	1	1.00	0.79	0.32	0.48	0.88	0.00	0.33	0.32
time (sec)	N/A	0.182	0.246	0.403	0.597	0.575	0.000	0.529	0.656

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	125	58	87	123	0	59	58
normalized size	1	1.00	0.87	0.41	0.61	0.86	0.00	0.41	0.41
time (sec)	N/A	0.157	0.250	0.357	0.502	0.840	0.000	3.301	0.689

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	138	71	107	123	0	72	127
normalized size	1	1.00	0.97	0.50	0.75	0.86	0.00	0.50	0.89
time (sec)	N/A	0.142	0.289	0.389	0.455	0.660	0.000	0.822	0.763

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	280	113	132	188	0	100	125
normalized size	1	1.00	1.94	0.78	0.92	1.31	0.00	0.69	0.87
time (sec)	N/A	0.207	0.558	0.474	0.677	0.599	0.000	1.014	0.798

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	319	145	178	198	0	129	159
normalized size	1	1.00	2.01	0.91	1.12	1.25	0.00	0.81	1.00
time (sec)	N/A	0.397	0.695	0.646	1.056	0.579	0.000	2.123	0.832

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	345	179	224	207	0	145	181
normalized size	1	1.00	1.60	0.83	1.04	0.96	0.00	0.67	0.84
time (sec)	N/A	0.509	0.778	0.665	1.130	3.027	0.000	0.864	0.950

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	82	0	82	0	120	331
normalized size	1	1.00	0.48	0.67	0.00	0.67	0.00	0.98	2.71
time (sec)	N/A	0.207	0.138	1.104	0.000	1.255	0.000	20.570	5.925

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	72	0	72	0	101	115
normalized size	1	1.00	0.56	0.84	0.00	0.84	0.00	1.17	1.34
time (sec)	N/A	0.152	0.112	1.011	0.000	0.695	0.000	3.672	4.458

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	36	62	0	60	0	82	108
normalized size	1	1.00	0.64	1.11	0.00	1.07	0.00	1.46	1.93
time (sec)	N/A	0.083	0.100	0.927	0.000	0.653	0.000	3.110	1.389

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	42	0	41	0	62	41
normalized size	1	1.00	1.12	1.62	0.00	1.58	0.00	2.38	1.58
time (sec)	N/A	0.029	0.070	0.902	0.000	0.752	0.000	7.084	0.191

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	89	146	133	0	130	-1
normalized size	1	1.00	1.62	2.41	3.95	3.59	0.00	3.51	-0.03
time (sec)	N/A	0.023	0.096	1.014	1.329	0.876	0.000	9.016	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	123	791	242	0	282	-1
normalized size	1	1.00	1.00	1.98	12.76	3.90	0.00	4.55	-0.02
time (sec)	N/A	0.063	0.216	1.096	1.575	0.759	0.000	3.323	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	47	221	1059	270	0	378	-1
normalized size	1	1.00	0.46	2.17	10.38	2.65	0.00	3.71	-0.01
time (sec)	N/A	0.118	0.106	1.132	1.179	0.623	0.000	13.770	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	47	310	1921	290	0	475	-1
normalized size	1	1.00	0.34	2.25	13.92	2.10	0.00	3.44	-0.01
time (sec)	N/A	0.178	0.099	1.228	2.075	1.594	0.000	4.279	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	47	399	6638	310	0	571	-1
normalized size	1	1.00	0.27	2.29	38.15	1.78	0.00	3.28	-0.01
time (sec)	N/A	0.235	0.099	1.260	2.827	0.737	0.000	4.459	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	70	93	0	98	0	180	429
normalized size	1	1.00	0.43	0.57	0.00	0.60	0.00	1.11	2.65
time (sec)	N/A	0.275	0.554	0.986	0.000	1.649	0.000	8.343	6.701

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	83	0	87	0	151	346
normalized size	1	1.00	0.52	0.72	0.00	0.75	0.00	1.30	2.98
time (sec)	N/A	0.194	0.180	0.844	0.000	0.676	0.000	9.719	5.084

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	73	0	74	0	121	116
normalized size	1	1.00	0.56	0.85	0.00	0.86	0.00	1.41	1.35
time (sec)	N/A	0.120	0.134	0.841	0.000	0.524	0.000	7.766	4.364

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	63	0	61	0	93	111
normalized size	1	1.00	0.64	1.07	0.00	1.03	0.00	1.58	1.88
time (sec)	N/A	0.061	0.091	0.862	0.000	0.577	0.000	9.427	1.237

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	180	997	235	0	195	-1
normalized size	1	1.00	1.14	2.73	15.11	3.56	0.00	2.95	-0.02
time (sec)	N/A	0.037	0.213	0.925	1.002	0.879	0.000	16.034	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	89	125	803	248	0	278	-1
normalized size	1	1.00	1.37	1.92	12.35	3.82	0.00	4.28	-0.02
time (sec)	N/A	0.118	0.206	0.991	1.194	0.611	0.000	7.162	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	222	0	278	0	0	-1
normalized size	1	1.00	1.02	2.09	0.00	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.377	1.037	0.000	0.637	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	311	0	300	0	535	-1
normalized size	1	1.00	0.83	2.16	0.00	2.08	0.00	3.72	-0.01
time (sec)	N/A	0.187	0.550	1.084	0.000	0.839	0.000	6.920	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	80	105	0	121	0	209	542
normalized size	1	1.00	0.39	0.52	0.00	0.60	0.00	1.03	2.67
time (sec)	N/A	0.374	0.214	0.964	0.000	0.598	0.000	6.219	9.325

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	70	95	0	108	0	180	456
normalized size	1	1.00	0.48	0.65	0.00	0.74	0.00	1.23	3.12
time (sec)	N/A	0.230	0.534	0.872	0.000	0.694	0.000	5.908	8.176

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	85	0	95	0	151	349
normalized size	1	1.00	0.52	0.73	0.00	0.82	0.00	1.30	3.01
time (sec)	N/A	0.155	0.182	0.819	0.000	0.668	0.000	5.110	4.580

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	50	75	0	82	0	122	146
normalized size	1	1.00	0.56	0.84	0.00	0.92	0.00	1.37	1.64
time (sec)	N/A	0.095	0.099	0.951	0.000	0.830	0.000	6.116	4.506

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	360	214	1395	310	0	225	-1
normalized size	1	1.00	3.67	2.18	14.23	3.16	0.00	2.30	-0.01
time (sec)	N/A	0.102	6.356	1.135	0.918	2.808	0.000	8.773	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	189	128	1383	276	0	368	-1
normalized size	1	1.00	2.01	1.36	14.71	2.94	0.00	3.91	-0.01
time (sec)	N/A	0.157	2.513	1.015	1.012	0.749	0.000	7.651	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	150	224	0	294	0	364	-1
normalized size	1	1.00	1.42	2.11	0.00	2.77	0.00	3.43	-0.01
time (sec)	N/A	0.164	0.562	1.031	0.000	0.560	0.000	8.775	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	313	0	320	0	539	-1
normalized size	1	1.00	1.05	2.17	0.00	2.22	0.00	3.74	-0.01
time (sec)	N/A	0.235	0.817	1.071	0.000	0.750	0.000	7.323	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	161	402	0	346	0	0	-1
normalized size	1	1.00	0.88	2.21	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.294	0.819	1.142	0.000	0.525	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	42	0	44	0	57	36
normalized size	1	1.00	1.11	1.56	0.00	1.63	0.00	2.11	1.33
time (sec)	N/A	0.030	0.117	1.120	0.000	0.957	0.000	1.444	0.787

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	188	91	146	182	0	65	-1
normalized size	1	1.00	4.95	2.39	3.84	4.79	0.00	1.71	-0.03
time (sec)	N/A	0.022	0.621	1.051	1.681	0.881	0.000	1.472	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	260	103	791	294	0	134	-1
normalized size	1	1.00	4.00	1.58	12.17	4.52	0.00	2.06	-0.02
time (sec)	N/A	0.064	0.936	1.277	1.279	0.899	0.000	0.786	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	314	0	347	0	205	-1
normalized size	1	1.00	0.76	2.24	0.00	2.48	0.00	1.46	-0.01
time (sec)	N/A	0.276	0.224	1.145	0.000	0.824	0.000	9.896	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	221	0	316	0	136	-1
normalized size	1	1.00	0.83	2.12	0.00	3.04	0.00	1.31	-0.01
time (sec)	N/A	0.158	0.158	1.108	0.000	0.804	0.000	5.134	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	83	121	0	262	0	132	-1
normalized size	1	1.00	1.14	1.66	0.00	3.59	0.00	1.81	-0.01
time (sec)	N/A	0.088	0.083	0.867	0.000	0.775	0.000	6.770	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	95	0	158	0	65	-1
normalized size	1	1.00	1.39	2.07	0.00	3.43	0.00	1.41	-0.02
time (sec)	N/A	0.036	0.050	0.813	0.000	0.830	0.000	6.411	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	5402	141	0	294	0	0	-1
normalized size	1	1.00	63.55	1.66	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.066	24.071	0.905	0.000	0.696	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	201	0	417	0	365	-1
normalized size	1	1.00	0.97	1.86	0.00	3.86	0.00	3.38	-0.01
time (sec)	N/A	0.179	0.164	1.099	0.000	1.089	0.000	6.022	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	118	380	0	446	0	462	-1
normalized size	1	1.00	0.80	2.59	0.00	3.03	0.00	3.14	-0.01
time (sec)	N/A	0.248	0.319	1.156	0.000	0.924	0.000	4.708	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	124	417	0	414	0	245	-1
normalized size	1	1.00	0.68	2.28	0.00	2.26	0.00	1.34	-0.01
time (sec)	N/A	0.425	0.576	1.031	0.000	0.902	0.000	8.535	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	114	322	0	387	0	207	-1
normalized size	1	1.00	0.79	2.22	0.00	2.67	0.00	1.43	-0.01
time (sec)	N/A	0.288	0.362	1.029	0.000	3.566	0.000	5.767	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	104	225	0	336	0	159	-1
normalized size	1	1.00	0.99	2.14	0.00	3.20	0.00	1.51	-0.01
time (sec)	N/A	0.168	0.394	0.993	0.000	2.265	0.000	20.502	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	94	222	0	329	0	122	-1
normalized size	1	1.00	1.22	2.88	0.00	4.27	0.00	1.58	-0.01
time (sec)	N/A	0.097	0.262	0.822	0.000	0.708	0.000	8.561	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	93	222	0	327	0	122	-1
normalized size	1	1.00	1.21	2.88	0.00	4.25	0.00	1.58	-0.01
time (sec)	N/A	0.072	0.144	0.789	0.000	0.628	0.000	8.758	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	5524	370	0	491	0	54	-1
normalized size	1	1.00	48.46	3.25	0.00	4.31	0.00	0.47	-0.01
time (sec)	N/A	0.116	24.844	0.900	0.000	1.153	0.000	2.998	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	384	0	518	0	415	-1
normalized size	1	1.00	0.90	2.67	0.00	3.60	0.00	2.88	-0.01
time (sec)	N/A	0.258	0.981	1.067	0.000	1.080	0.000	4.291	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	197	560	0	536	0	501	-1
normalized size	1	1.00	1.06	3.03	0.00	2.90	0.00	2.71	-0.01
time (sec)	N/A	0.391	3.373	1.134	0.000	0.869	0.000	6.462	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	135	417	0	455	0	254	-1
normalized size	1	1.00	0.74	2.28	0.00	2.49	0.00	1.39	-0.01
time (sec)	N/A	0.424	1.546	1.031	0.000	0.546	0.000	5.424	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	125	316	0	404	0	216	-1
normalized size	1	1.00	0.86	2.18	0.00	2.79	0.00	1.49	-0.01
time (sec)	N/A	0.309	0.848	0.980	0.000	0.759	0.000	10.270	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	116	323	0	403	0	160	-1
normalized size	1	1.00	1.08	3.02	0.00	3.77	0.00	1.50	-0.01
time (sec)	N/A	0.176	0.777	1.020	0.000	1.196	0.000	7.536	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	315	0	399	0	160	-1
normalized size	1	1.00	1.07	2.94	0.00	3.73	0.00	1.50	-0.01
time (sec)	N/A	0.137	0.742	0.852	0.000	0.741	0.000	10.299	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	52	315	0	403	0	160	-1
normalized size	1	1.00	0.49	2.94	0.00	3.77	0.00	1.50	-0.01
time (sec)	N/A	0.113	0.065	0.777	0.000	0.690	0.000	22.799	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	5564	550	0	585	0	92	-1
normalized size	1	1.00	38.64	3.82	0.00	4.06	0.00	0.64	-0.01
time (sec)	N/A	0.176	25.202	1.019	0.000	1.007	0.000	2.693	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	552	0	606	0	459	-1
normalized size	1	1.00	0.97	3.17	0.00	3.48	0.00	2.64	-0.01
time (sec)	N/A	0.372	2.231	1.095	0.000	0.976	0.000	22.079	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	94	83	0	161	0	81	-1
normalized size	1	1.00	1.96	1.73	0.00	3.35	0.00	1.69	-0.02
time (sec)	N/A	0.038	0.461	1.017	0.000	0.835	0.000	2.196	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	127	119	0	301	0	0	-1
normalized size	1	1.00	1.46	1.37	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.540	1.069	0.000	1.249	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	105	0	0	0	0	0	-1
normalized size	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	0.340	0.720	0.000	1.589	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	85	0	0	0	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.150	0.639	0.000	0.780	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	66	0	0	0	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.061	0.619	0.000	0.717	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	694	0	0	0	0	0	-1
normalized size	1	1.00	9.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	8.976	0.794	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	2700	0	0	0	0	0	-1
normalized size	1	1.00	35.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	16.363	0.827	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	96	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	0.335	0.717	0.000	1.217	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	106	0	0	0	0	0	-1
normalized size	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.451	0.727	0.000	0.990	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	66	0	0	0	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.085	0.697	0.000	0.886	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	2694	0	0	0	0	0	-1
normalized size	1	1.00	31.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	16.089	0.707	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	2700	0	0	0	0	0	-1
normalized size	1	1.00	31.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	16.334	0.882	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	155	0	0	0	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	0.407	0.783	0.000	1.069	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	95	0	0	0	0	0	-1
normalized size	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	0.183	0.817	0.000	1.075	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	85	0	0	0	0	0	-1
normalized size	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	0.126	0.694	0.000	0.967	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	65	0	0	0	0	0	-1
normalized size	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.083	0.710	0.000	1.220	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	718	0	0	0	0	0	-1
normalized size	1	1.00	9.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	5.331	0.776	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	240	0	0	0	0	0	-1
normalized size	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	2.041	0.853	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	766	766	111	0	0	0	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.059	0.561	0.789	0.000	1.223	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	98	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.771	0.282	0.767	0.000	0.829	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	90	0	0	0	0	0	-1
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	0.358	0.708	0.000	1.143	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	744	744	68	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	0.076	0.663	0.000	0.849	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	3007	0	0	0	0	0	-1
normalized size	1	1.00	33.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	16.719	0.699	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	3011	0	0	0	0	0	-1
normalized size	1	1.00	33.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	16.584	0.778	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	115	384	0	0	0	0	-1
normalized size	1	1.00	0.76	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.246	5.935	0.000	1.334	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	369	0	0	0	0	-1
normalized size	1	1.00	0.67	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.221	5.777	0.000	1.062	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	146	0	0	0	0	-1
normalized size	1	1.00	0.70	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.167	3.893	0.000	0.679	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	150	0	0	0	0	-1
normalized size	1	1.00	0.65	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.098	2.823	0.000	1.915	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	225	0	0	0	0	-1
normalized size	1	1.00	0.72	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.153	3.334	0.000	0.634	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	93	219	0	0	0	0	-1
normalized size	1	1.00	0.73	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.222	3.078	0.000	2.024	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	103	270	0	0	0	0	-1
normalized size	1	1.00	0.68	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.294	3.514	0.000	1.009	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	269	439	0	0	0	0	-1
normalized size	1	1.00	1.44	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	2.646	6.524	0.000	1.776	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	269	386	0	0	0	0	-1
normalized size	1	1.00	1.67	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.580	5.977	0.000	0.847	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	264	371	0	0	0	0	-1
normalized size	1	1.00	2.02	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	1.254	5.522	0.000	0.855	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	104	0	0	0	0	-1
normalized size	1	1.00	0.75	1.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.158	4.125	0.000	0.884	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	156	228	0	0	0	0	-1
normalized size	1	1.00	1.46	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.854	3.395	0.000	0.708	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	0	0	0	-1
normalized size	1	1.00	1.01	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.142	3.474	0.000	0.693	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	0	0	0	-1
normalized size	1	1.00	0.93	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.376	3.596	0.000	0.668	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	0	0	0	-1
normalized size	1	1.00	1.53	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	2.261	6.367	0.000	0.650	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	267	386	0	0	0	0	-1
normalized size	1	1.00	1.70	2.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	1.910	6.512	0.000	0.766	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	187	371	0	0	0	0	-1
normalized size	1	1.00	1.43	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	1.413	5.599	0.000	0.997	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	169	172	0	0	0	0	-1
normalized size	1	1.00	1.29	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	1.093	3.683	0.000	0.529	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	171	250	0	0	0	0	-1
normalized size	1	1.00	1.31	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	1.068	3.755	0.000	1.019	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0	-1
normalized size	1	1.00	0.91	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	1.519	3.565	0.000	0.607	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0	-1
normalized size	1	1.00	0.83	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	1.984	4.049	0.000	0.642	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	271	492	0	0	0	0	-1
normalized size	1	1.00	1.27	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	5.089	7.016	0.000	0.670	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	0	0	0	-1
normalized size	1	1.00	1.49	2.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	2.725	6.307	0.000	1.021	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	286	386	0	0	0	0	-1
normalized size	1	1.00	1.78	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	3.268	6.321	0.000	1.706	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	0	0	0	-1
normalized size	1	1.00	0.59	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.322	4.750	0.000	0.731	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	184	194	0	0	0	0	-1
normalized size	1	1.00	1.16	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	1.243	3.592	0.000	0.740	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	180	272	0	0	0	0	-1
normalized size	1	1.00	1.12	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.323	3.292	0.000	0.666	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0	-1
normalized size	1	1.00	0.83	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	1.854	3.214	0.000	0.764	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	296	273	0	0	0	0	-1
normalized size	1	1.00	1.39	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	3.375	3.745	0.000	0.673	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	291	413	0	0	0	0	-1
normalized size	1	1.00	1.77	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	3.209	6.279	0.000	0.617	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	262	253	0	0	0	0	-1
normalized size	1	1.00	1.93	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.780	4.247	0.000	0.937	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	201	200	0	0	0	0	-1
normalized size	1	1.00	1.83	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.592	3.465	0.000	0.730	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	202	198	0	0	0	0	-1
normalized size	1	1.00	1.84	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.562	3.295	0.000	0.501	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	317	199	0	0	0	0	-1
normalized size	1	1.00	2.83	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	1.570	3.720	0.000	0.567	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	318	215	0	0	0	0	-1
normalized size	1	1.00	2.27	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	4.436	3.614	0.000	0.585	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	347	229	0	0	0	0	-1
normalized size	1	1.00	2.07	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	2.583	3.776	0.000	0.617	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	0	0	0	-1
normalized size	1	1.00	1.42	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	3.841	6.520	0.000	0.736	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	0	0	0	-1
normalized size	1	1.00	1.43	2.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	1.408	4.005	0.000	0.792	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	0	0	0	-1
normalized size	1	1.00	1.62	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.267	3.553	0.000	1.441	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	0	0	0	-1
normalized size	1	1.00	1.27	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.426	3.397	0.000	0.558	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	0	0	0	-1
normalized size	1	1.00	1.60	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	1.499	3.976	0.000	0.831	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	260	257	0	0	0	0	-1
normalized size	1	1.00	1.71	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.952	4.019	0.000	0.757	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	0	0	0	-1
normalized size	1	1.00	1.44	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	1.992	3.993	0.000	0.669	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	0	0	0	-1
normalized size	1	1.00	1.36	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	1.975	4.286	0.000	1.475	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	378	453	0	0	0	0	-1
normalized size	1	1.00	1.53	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	5.337	6.158	0.000	0.704	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	371	555	0	0	0	0	-1
normalized size	1	1.00	1.68	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	2.120	4.245	0.000	0.571	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	0	0	0	-1
normalized size	1	1.00	1.41	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.325	4.840	3.893	0.000	0.639	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	0	0	0	-1
normalized size	1	1.00	1.90	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	1.815	4.120	0.000	0.762	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	0	0	0	-1
normalized size	1	1.00	1.90	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.319	1.966	4.043	0.000	0.728	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	0	0	0	-1
normalized size	1	1.00	1.39	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	5.623	4.668	0.000	1.398	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	386	270	0	0	0	0	-1
normalized size	1	1.00	1.98	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	2.067	4.275	0.000	0.576	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	0	0	0	-1
normalized size	1	1.00	1.29	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	2.466	4.532	0.000	0.760	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	297	296	0	0	0	0	-1
normalized size	1	1.00	1.20	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	2.859	4.236	0.000	0.796	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	219	1264	356	0	0	-1
normalized size	1	1.00	0.86	1.89	10.90	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.557	1.766	1.062	0.818	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	188	662	302	0	0	-1
normalized size	1	1.00	1.04	2.61	9.19	4.19	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.250	1.627	0.705	0.640	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	150	241	189	0	0	-1
normalized size	1	1.00	1.46	4.05	6.51	5.11	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.112	1.422	0.651	0.644	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	52	20	49	0	0	53
normalized size	1	1.00	1.08	1.44	0.56	1.36	0.00	0.00	1.47
time (sec)	N/A	0.054	0.094	1.474	0.622	0.540	0.000	0.000	0.537

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	68	113	66	0	0	69
normalized size	1	1.00	0.64	0.88	1.47	0.86	0.00	0.00	0.90
time (sec)	N/A	0.109	0.164	1.601	0.583	0.554	0.000	0.000	1.297

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	80	203	78	0	0	82
normalized size	1	1.00	0.53	0.70	1.77	0.68	0.00	0.00	0.71
time (sec)	N/A	0.164	0.194	1.544	0.670	0.589	0.000	0.000	1.668

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	90	293	88	0	0	93
normalized size	1	1.00	0.46	0.59	1.92	0.58	0.00	0.00	0.61
time (sec)	N/A	0.216	0.271	1.548	0.755	0.803	0.000	0.000	2.202

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	112	244	2361	400	0	0	-1
normalized size	1	1.00	0.70	1.52	14.76	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.591	1.518	0.907	1.670	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	99	214	2244	370	0	0	-1
normalized size	1	1.00	0.82	1.78	18.70	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.453	1.533	0.880	0.660	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	182	1143	310	0	0	-1
normalized size	1	1.00	1.00	2.43	15.24	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.294	1.492	0.681	0.648	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	174	274	307	0	0	-1
normalized size	1	1.00	1.13	2.29	3.61	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.381	1.511	0.873	0.623	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	71	38	69	0	0	70
normalized size	1	1.00	0.63	0.90	0.48	0.87	0.00	0.00	0.89
time (sec)	N/A	0.109	0.219	1.559	0.566	0.626	0.000	0.000	1.406

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	81	210	80	0	0	81
normalized size	1	1.00	0.52	0.70	1.81	0.69	0.00	0.00	0.70
time (sec)	N/A	0.172	0.290	1.671	0.870	0.564	0.000	0.000	1.773

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	93	303	92	0	0	94
normalized size	1	1.00	0.45	0.58	1.88	0.57	0.00	0.00	0.58
time (sec)	N/A	0.232	0.371	1.704	0.614	1.478	0.000	0.000	2.325

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	80	103	396	103	0	0	105
normalized size	1	1.00	0.40	0.51	1.97	0.51	0.00	0.00	0.52
time (sec)	N/A	0.292	0.565	1.692	1.105	0.465	0.000	0.000	3.022

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	582	286	3860	446	0	0	-1
normalized size	1	1.00	2.91	1.43	19.30	2.23	0.00	0.00	-0.00
time (sec)	N/A	0.337	8.502	1.755	1.578	0.651	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	458	254	3469	420	0	0	-1
normalized size	1	1.00	2.86	1.59	21.68	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.275	8.058	1.651	0.990	1.602	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	224	2826	386	0	0	-1
normalized size	1	1.00	0.88	1.87	23.55	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.509	1.443	5.135	0.691	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	199	0	346	0	0	-1
normalized size	1	1.00	0.81	1.78	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.749	1.589	0.000	1.394	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	195	593	364	0	0	-1
normalized size	1	1.00	0.87	1.65	5.03	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.455	1.655	0.656	1.255	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	64	85	60	87	0	0	85
normalized size	1	1.00	0.54	0.71	0.50	0.73	0.00	0.00	0.71
time (sec)	N/A	0.169	0.330	1.681	0.588	0.835	0.000	0.000	1.835

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	74	95	323	100	0	0	96
normalized size	1	1.00	0.47	0.61	2.07	0.64	0.00	0.00	0.62
time (sec)	N/A	0.236	0.371	1.735	0.751	0.686	0.000	0.000	2.316

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	80	105	422	113	0	0	107
normalized size	1	1.00	0.40	0.52	2.10	0.56	0.00	0.00	0.53
time (sec)	N/A	0.333	0.603	1.749	0.720	0.546	0.000	0.000	2.958

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	90	115	521	126	0	0	356
normalized size	1	1.00	0.37	0.48	2.16	0.52	0.00	0.00	1.48
time (sec)	N/A	0.403	0.402	1.742	0.761	1.514	0.000	0.000	6.925

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	45	0	121	50	0	0	55
normalized size	1	1.00	1.18	0.00	3.18	1.32	0.00	0.00	1.45
time (sec)	N/A	0.056	0.093	1.336	0.594	0.695	0.000	0.000	0.746

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	150	241	189	0	0	-1
normalized size	1	1.00	1.46	4.05	6.51	5.11	0.00	0.00	-0.03
time (sec)	N/A	0.058	0.150	1.607	0.620	0.768	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	299	126	353	215	0	0	-1
normalized size	1	1.00	7.87	3.32	9.29	5.66	0.00	0.00	-0.03
time (sec)	N/A	0.063	2.061	1.704	0.988	0.698	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	125	224	876	481	0	0	-1
normalized size	1	1.00	0.98	1.75	6.84	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.261	1.577	0.690	1.157	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	184	476	351	0	0	-1
normalized size	1	1.00	0.94	1.94	5.01	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.104	1.546	0.820	0.643	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	99	90	160	0	0	-1
normalized size	1	1.00	1.34	1.77	1.61	2.86	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.074	1.381	0.672	0.679	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	102	100	104	281	0	0	-1
normalized size	1	1.00	1.10	1.08	1.12	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.235	1.520	0.594	1.473	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	120	120	282	318	0	0	-1
normalized size	1	1.00	0.92	0.92	2.15	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.265	1.627	0.664	0.511	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	117	130	357	342	0	0	-1
normalized size	1	1.00	0.69	0.77	2.11	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.345	1.209	1.753	0.840	0.630	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	252	282	4934	579	0	0	-1
normalized size	1	1.00	1.45	1.62	28.36	3.33	0.00	0.00	-0.01
time (sec)	N/A	0.419	0.676	1.530	1.026	0.606	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	220	240	2122	559	0	0	-1
normalized size	1	1.00	1.64	1.79	15.84	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.619	1.486	1.486	0.811	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	220	146	0	338	0	0	-1
normalized size	1	1.00	2.27	1.51	0.00	3.48	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.644	1.466	0.000	0.553	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	120	146	1031	340	0	0	-1
normalized size	1	1.00	1.24	1.51	10.63	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.189	1.383	1.505	0.718	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	145	175	7176	378	0	0	-1
normalized size	1	1.00	1.06	1.28	52.38	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.555	1.659	1.005	0.546	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	193	0	398	0	0	-1
normalized size	1	1.00	0.85	1.09	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.367	0.984	1.756	0.000	0.732	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	203	0	418	0	0	-1
normalized size	1	1.00	0.75	0.94	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.516	1.383	1.817	0.000	0.682	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	340	454	9048	667	0	0	-1
normalized size	1	1.00	1.59	2.12	42.28	3.12	0.00	0.00	-0.00
time (sec)	N/A	0.562	1.345	1.617	6.100	1.418	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	308	406	4988	665	0	0	-1
normalized size	1	1.00	1.77	2.33	28.67	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.430	0.734	1.583	2.560	0.990	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	308	210	0	426	0	0	-1
normalized size	1	1.00	2.25	1.53	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.804	1.507	0.000	1.149	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	266	210	2875	422	0	0	-1
normalized size	1	1.00	1.94	1.53	20.99	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.188	1.980	1.383	1.469	0.767	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	208	3049	426	0	0	-1
normalized size	1	1.00	1.07	1.52	22.26	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.242	1.002	1.343	1.059	1.055	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	186	236	0	446	0	0	-1
normalized size	1	1.00	1.05	1.33	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.380	1.392	1.657	0.000	1.536	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	165	254	0	466	0	0	-1
normalized size	1	1.00	0.76	1.17	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.507	2.492	1.704	0.000	0.970	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	140	253	1643	337	0	0	-1
normalized size	1	1.00	1.11	2.01	13.04	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.458	1.457	1.083	0.871	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	111	220	873	297	0	0	-1
normalized size	1	1.00	1.31	2.59	10.27	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.309	1.424	1.246	0.971	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	76	180	473	223	0	0	-1
normalized size	1	1.00	1.41	3.33	8.76	4.13	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.088	1.544	0.930	0.922	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	95	87	88	0	0	-1
normalized size	1	1.00	1.48	3.52	3.22	3.26	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.031	1.362	1.290	0.625	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	90	98	101	144	0	0	-1
normalized size	1	1.00	1.45	1.58	1.63	2.32	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.231	1.497	1.091	0.697	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	116	279	163	0	0	-1
normalized size	1	1.00	1.20	1.18	2.85	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.256	1.637	0.915	1.216	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	126	354	174	0	0	-1
normalized size	1	1.00	0.91	0.94	2.64	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.306	1.748	0.759	0.473	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	71	0	0	0	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.278	1.240	0.000	0.486	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	71	0	0	0	0	0	-1
normalized size	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.163	1.603	0.000	0.501	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	71	0	0	0	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.171	1.331	0.000	0.523	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	71	0	0	0	0	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	0.261	1.255	0.000	0.628	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	71	0	0	0	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	0.254	1.256	0.000	0.592	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	71	0	0	0	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.159	1.248	0.000	0.544	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	71	0	0	0	0	0	-1
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	0.164	1.306	0.000	0.925	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	71	0	0	0	0	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	0.203	1.213	0.000	0.585	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	750	0	0	0	0	0	-1
normalized size	1	1.00	9.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	9.246	1.157	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	749	0	0	0	0	0	-1
normalized size	1	1.00	9.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	8.124	1.122	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	3346	0	0	0	0	0	-1
normalized size	1	1.00	44.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	20.588	1.184	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	585	0	0	0	0	0	-1
normalized size	1	1.00	7.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	7.472	1.188	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1982	0	0	0	0	0	-1
normalized size	1	1.00	25.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	15.006	1.411	0.000	0.975	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	2618	0	0	0	0	0	-1
normalized size	1	1.00	33.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	20.391	1.281	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	79	274	0	0	0	0	0	-1
normalized size	1	0.24	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	8.331	1.298	0.000	0.490	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2325	0	0	0	0	0	-1
normalized size	1	1.00	29.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	15.178	1.215	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	0.660	2.920	0.000	0.662	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	286	0	0	0	0	0	-1
normalized size	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	1.543	7.981	0.000	0.954	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	222	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	1.094	5.173	0.000	0.979	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.163	2.498	0.000	1.193	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	1.050	2.699	0.000	1.383	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	10.144	0.938	0.000	0.903	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	398	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	58.402	1.135	0.000	0.549	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.456	0.992	0.000	0.672	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.046	1.085	0.000	0.587	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	2938	0	0	0	0	0	-1
normalized size	1	1.00	49.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	16.087	0.957	0.000	0.529	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	2990	0	0	0	0	0	-1
normalized size	1	1.00	48.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	17.208	0.962	0.000	0.606	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.437	1.083	0.000	0.606	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.058	0.871	0.000	0.571	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	-1
normalized size	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	6.240	0.933	0.000	0.589	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	-1
normalized size	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	6.282	0.976	0.000	0.548	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.407	1.068	0.000	0.597	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.045	0.886	0.000	0.558	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	-1
normalized size	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	6.221	0.843	0.000	0.577	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	-1
normalized size	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	6.236	0.762	0.000	0.543	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	400	0	0	0	0	0	-1
normalized size	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.295	8.122	1.192	0.000	0.474	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.426	1.089	0.000	0.528	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.101	1.231	0.000	0.520	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	2964	0	0	0	0	0	-1
normalized size	1	1.00	48.59	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.134	6.238	1.141	0.000	0.581	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	2992	0	0	0	0	0	-1
normalized size	1	1.00	44.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	6.257	1.070	0.000	0.570	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.378	0.948	0.000	0.513	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.140	1.033	0.000	0.512	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	-1
normalized size	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	6.234	0.915	0.000	0.508	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	-1
normalized size	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	6.223	0.843	0.000	0.570	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.344	0.960	0.000	0.474	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.129	1.071	0.000	0.463	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	-1
normalized size	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	6.210	0.917	0.000	0.899	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	-1
normalized size	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	6.225	0.870	0.000	0.641	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	429	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	24.645	1.277	0.000	0.500	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	346	0	0	0	0	0	-1
normalized size	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	13.716	1.101	0.000	0.547	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	213	0	0	0	0	0	-1
normalized size	1	1.00	4.53	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	72.779	1.128	0.000	0.518	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.154	1.328	1.023	0.000	0.466	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.167	75.592	0.945	0.000	0.546	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	332	0	0	0	0	0	-1
normalized size	1	1.00	2.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	2.101	1.286	0.000	0.467	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	185	0	0	0	0	0	-1
normalized size	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.404	1.258	0.000	0.531	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	346	0	0	0	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.162	1.184	0.000	0.515	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	213	0	0	0	0	0	-1
normalized size	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.581	1.174	0.000	0.456	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	2246	0	0	0	0	0	-1
normalized size	1	1.00	31.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	14.367	2.438	0.000	0.525	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	255	0	0	0	0	0	-1
normalized size	1	1.00	2.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.345	2.656	0.000	0.508	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	2248	0	0	0	0	0	-1
normalized size	1	1.00	25.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	6.237	2.674	0.000	0.535	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.105	2.659	0.000	0.487	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2248	0	0	0	0	0	-1
normalized size	1	1.00	26.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	6.229	2.358	0.000	0.552	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	257	0	0	0	0	0	-1
normalized size	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.306	2.491	0.000	0.542	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	2250	0	0	0	0	0	-1
normalized size	1	1.00	25.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	6.223	2.885	0.000	0.543	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.260	2.886	0.000	0.566	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	2248	0	0	0	0	0	-1
normalized size	1	1.00	28.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	6.212	2.414	0.000	0.564	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	257	0	0	0	0	0	-1
normalized size	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.278	2.384	0.000	0.575	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	2250	0	0	0	0	0	-1
normalized size	1	1.00	23.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	6.224	2.428	0.000	0.491	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.241	2.500	0.000	0.568	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	154	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	1.359	1.161	0.000	0.548	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.631	1.064	0.000	0.508	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.218	0.917	0.000	0.540	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.106	0.989	0.000	0.579	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	711	0	0	0	0	0	-1
normalized size	1	1.00	8.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	6.795	0.737	0.000	0.695	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	3781	0	0	0	0	0	-1
normalized size	1	1.00	45.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	16.663	1.410	0.000	0.632	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	2529	0	0	0	0	0	-1
normalized size	1	1.00	25.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	15.013	1.100	0.000	0.522	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	2225	0	0	0	0	0	-1
normalized size	1	1.00	23.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	14.644	1.177	0.000	0.523	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	2424	0	0	0	0	0	-1
normalized size	1	1.00	25.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	14.948	1.155	0.000	0.532	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	3349	0	0	0	0	0	-1
normalized size	1	1.00	34.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	19.417	1.107	0.000	0.538	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	490	270	0	0	0	0	87
normalized size	1	1.00	4.41	2.43	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.090	6.102	3.646	0.000	0.450	0.000	0.000	1.178

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	0	0	0	80
normalized size	1	1.00	2.67	2.52	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.080	5.674	3.599	0.000	0.445	0.000	0.000	0.783

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	0	0	0	53
normalized size	1	1.00	3.64	3.69	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.069	5.214	3.407	0.000	0.473	0.000	0.000	0.763

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	0	0	0	27
normalized size	1	1.00	4.43	4.29	0.00	0.00	0.00	0.00	0.77
time (sec)	N/A	0.059	1.855	3.023	0.000	0.505	0.000	0.000	0.196

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	209	146	0	0	0	0	60
normalized size	1	1.00	3.67	2.56	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.067	4.971	3.630	0.000	0.526	0.000	0.000	1.032

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	444	369	0	0	0	0	87
normalized size	1	1.00	5.35	4.45	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.079	6.134	5.690	0.000	0.509	0.000	0.000	1.171

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	0	0	0	87
normalized size	1	1.00	4.30	3.46	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.091	6.162	5.626	0.000	0.500	0.000	0.000	1.282

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	294	437	0	0	0	0	87
normalized size	1	1.00	2.18	3.24	0.00	0.00	0.00	0.00	0.64
time (sec)	N/A	0.102	4.607	6.009	0.000	0.488	0.000	0.000	1.377

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	0	0	0	136
normalized size	1	1.00	3.73	1.77	0.00	0.00	0.00	0.00	0.93
time (sec)	N/A	0.174	6.123	3.616	0.000	0.475	0.000	0.000	1.184

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	0	0	0	129
normalized size	1	1.00	4.26	2.25	0.00	0.00	0.00	0.00	1.07
time (sec)	N/A	0.158	6.117	3.440	0.000	0.501	0.000	0.000	1.039

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	0	0	0	104
normalized size	1	1.00	2.47	2.63	0.00	0.00	0.00	0.00	1.09
time (sec)	N/A	0.143	5.932	3.751	0.000	0.529	0.000	0.000	0.919

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	0	0	0	59
normalized size	1	1.00	3.34	3.40	0.00	0.00	0.00	0.00	0.88
time (sec)	N/A	0.128	5.558	3.685	0.000	0.629	0.000	0.000	0.885

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	104	0	0	0	0	82
normalized size	1	1.00	0.89	2.36	0.00	0.00	0.00	0.00	1.86
time (sec)	N/A	0.109	0.161	3.410	0.000	0.549	0.000	0.000	1.214

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	470	371	0	0	0	0	109
normalized size	1	1.00	5.16	4.08	0.00	0.00	0.00	0.00	1.20
time (sec)	N/A	0.139	6.155	5.791	0.000	0.607	0.000	0.000	1.272

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	503	386	0	0	0	0	114
normalized size	1	1.00	4.16	3.19	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.154	6.201	6.000	0.000	0.462	0.000	0.000	1.378

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	0	0	0	114
normalized size	1	1.00	3.61	2.99	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.173	6.243	6.890	0.000	0.545	0.000	0.000	1.456

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	0	0	0	206
normalized size	1	1.00	3.73	1.77	0.00	0.00	0.00	0.00	1.40
time (sec)	N/A	0.252	6.130	3.886	0.000	0.631	0.000	0.000	1.150

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	0	0	0	143
normalized size	1	1.00	4.26	2.25	0.00	0.00	0.00	0.00	1.18
time (sec)	N/A	0.220	6.128	3.998	0.000	0.459	0.000	0.000	1.026

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	0	0	0	104
normalized size	1	1.00	2.56	2.75	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.186	6.113	3.300	0.000	0.527	0.000	0.000	0.975

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	240	172	0	0	0	0	104
normalized size	1	1.00	2.64	1.89	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.197	4.914	4.089	0.000	0.654	0.000	0.000	1.016

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	479	371	0	0	0	0	126
normalized size	1	1.00	5.26	4.08	0.00	0.00	0.00	0.00	1.38
time (sec)	N/A	0.194	6.186	5.915	0.000	0.518	0.000	0.000	1.640

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	501	386	0	0	0	0	154
normalized size	1	1.00	4.28	3.30	0.00	0.00	0.00	0.00	1.32
time (sec)	N/A	0.223	6.211	5.700	0.000	0.494	0.000	0.000	1.585

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	0	0	0	145
normalized size	1	1.00	3.61	2.99	0.00	0.00	0.00	0.00	0.99
time (sec)	N/A	0.239	6.243	6.614	0.000	0.524	0.000	0.000	1.642

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	314	229	0	0	0	0	-1
normalized size	1	1.00	2.45	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	1.766	3.639	0.000	0.545	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	292	215	0	0	0	0	-1
normalized size	1	1.00	2.92	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	2.155	3.931	0.000	0.698	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	270	199	0	0	0	0	-1
normalized size	1	1.00	3.75	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	1.745	3.316	0.000	0.642	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	262	198	0	0	0	0	-1
normalized size	1	1.00	3.74	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.089	3.347	0.000	0.549	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	263	200	0	0	0	0	-1
normalized size	1	1.00	3.76	2.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.106	3.387	0.000	0.691	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	303	253	0	0	0	0	-1
normalized size	1	1.00	3.16	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	1.753	4.319	0.000	0.887	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	338	413	0	0	0	0	-1
normalized size	1	1.00	2.73	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	3.748	6.559	0.000	0.972	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	366	283	0	0	0	0	-1
normalized size	1	1.00	2.29	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	2.588	3.986	0.000	1.141	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	341	270	0	0	0	0	-1
normalized size	1	1.00	2.47	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	1.799	3.864	0.000	0.911	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	374	257	0	0	0	0	-1
normalized size	1	1.00	3.34	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	6.191	3.765	0.000	0.853	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	656	257	0	0	0	0	-1
normalized size	1	1.00	6.02	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	6.184	3.967	0.000	0.705	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	63	188	0	0	0	0	-1
normalized size	1	1.00	1.11	3.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.237	3.721	0.000	0.770	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	312	257	0	0	0	0	-1
normalized size	1	1.00	2.86	2.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	5.125	3.363	0.000	0.731	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	393	405	0	0	0	0	-1
normalized size	1	1.00	2.89	2.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.263	6.258	4.147	0.000	0.587	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	372	413	0	0	0	0	-1
normalized size	1	1.00	2.30	2.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	2.136	6.661	0.000	0.698	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	391	296	0	0	0	0	-1
normalized size	1	1.00	1.89	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	2.571	3.724	0.000	0.537	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	375	283	0	0	0	0	-1
normalized size	1	1.00	2.07	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.396	2.054	3.641	0.000	0.907	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	357	270	0	0	0	0	-1
normalized size	1	1.00	2.30	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.370	1.727	3.760	0.000	0.995	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	721	270	0	0	0	0	-1
normalized size	1	1.00	4.65	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	6.261	3.608	0.000	0.661	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	0	0	0	-1
normalized size	1	1.00	2.21	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.378	1.567	4.184	0.000	0.609	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	0	0	0	-1
normalized size	1	1.00	2.21	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	1.603	3.556	0.000	0.750	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	721	268	0	0	0	0	-1
normalized size	1	1.00	4.65	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.390	6.253	3.793	0.000	0.954	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	372	555	0	0	0	0	-1
normalized size	1	1.00	2.06	3.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	1.912	4.089	0.000	0.786	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	402	453	0	0	0	0	-1
normalized size	1	1.00	1.94	2.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	2.777	6.666	0.000	0.567	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	80	80	293	79	0	0	-1
normalized size	1	1.00	0.52	0.52	1.92	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.213	1.150	0.866	0.662	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	70	203	69	0	0	-1
normalized size	1	1.00	0.53	0.61	1.77	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.178	1.243	0.966	0.845	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	49	58	113	57	0	0	-1
normalized size	1	1.00	0.64	0.75	1.47	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.115	1.230	1.234	1.153	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	20	49	0	0	-1
normalized size	1	1.00	1.08	1.39	0.56	1.36	0.00	0.00	-0.03
time (sec)	N/A	0.110	0.101	0.965	0.589	1.266	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	142	241	180	0	0	-1
normalized size	1	1.00	1.30	2.49	4.23	3.16	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.157	1.029	1.549	0.732	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	178	662	325	0	0	-1
normalized size	1	1.00	0.98	1.93	7.20	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.243	1.001	1.504	0.543	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	100	213	1264	355	0	0	-1
normalized size	1	1.00	0.74	1.57	9.29	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.486	1.012	0.721	0.549	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	72	83	303	84	0	0	-1
normalized size	1	1.00	0.45	0.52	1.88	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.311	0.276	1.170	0.564	0.568	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	60	71	210	72	0	0	-1
normalized size	1	1.00	0.52	0.61	1.81	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.215	1.168	1.564	0.584	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	61	38	61	0	0	-1
normalized size	1	1.00	0.63	0.77	0.48	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.165	1.024	1.578	1.011	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	172	274	298	0	0	-1
normalized size	1	1.00	0.84	1.79	2.85	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.141	1.138	1.294	0.466	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	182	1143	337	0	0	-1
normalized size	1	1.00	0.97	1.92	12.03	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.311	1.082	1.792	0.747	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	99	212	2244	369	0	0	-1
normalized size	1	1.00	0.71	1.51	16.03	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.430	0.982	1.435	0.555	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	112	242	2361	391	0	0	-1
normalized size	1	1.00	0.62	1.34	13.12	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.523	1.014	1.402	0.573	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	90	95	422	105	0	0	-1
normalized size	1	1.00	0.45	0.47	2.10	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.243	1.109	1.300	0.547	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	74	85	323	92	0	0	-1
normalized size	1	1.00	0.47	0.54	2.07	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.256	1.176	0.918	0.511	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	64	75	60	79	0	0	-1
normalized size	1	1.00	0.54	0.63	0.50	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.239	1.817	1.263	1.086	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	93	185	593	339	0	0	-1
normalized size	1	1.00	0.67	1.34	4.30	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.282	0.244	1.060	1.452	0.700	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	90	197	0	373	0	0	-1
normalized size	1	1.00	0.68	1.49	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.281	0.290	1.069	0.000	0.590	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	214	2826	385	0	0	-1
normalized size	1	1.00	0.68	1.53	20.19	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.649	1.043	10.408	0.675	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	180	244	3469	411	0	0	-1
normalized size	1	1.00	1.00	1.36	19.27	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.343	5.473	1.031	0.952	0.705	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	190	276	3860	437	0	0	-1
normalized size	1	1.00	0.86	1.25	17.55	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.403	5.567	1.074	1.661	0.577	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	136	120	357	324	0	0	-1
normalized size	1	1.00	0.72	0.63	1.89	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.414	0.308	1.135	1.536	0.953	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	116	110	282	300	0	0	-1
normalized size	1	1.00	0.77	0.73	1.87	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.215	1.086	1.478	0.860	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	98	104	281	0	0	-1
normalized size	1	1.00	0.88	0.87	0.92	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.100	0.967	1.266	0.717	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	76	95	91	90	160	0	0	-1
normalized size	1	1.36	1.70	1.62	1.61	2.86	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.072	1.069	1.282	0.622	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	109	174	476	342	0	0	-1
normalized size	1	1.00	0.81	1.29	3.53	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.092	0.982	0.748	0.468	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	145	214	876	520	0	0	-1
normalized size	1	1.00	0.86	1.27	5.21	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.340	0.236	1.064	2.800	0.577	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	178	247	1646	550	0	0	-1
normalized size	1	1.00	0.84	1.17	7.80	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.392	1.050	1.904	0.876	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	152	193	0	400	0	0	-1
normalized size	1	1.00	0.64	0.81	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.925	1.145	0.000	0.489	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	133	183	0	380	0	0	-1
normalized size	1	1.00	0.68	0.93	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.446	0.648	1.111	0.000	0.663	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	173	7176	360	0	0	-1
normalized size	1	1.00	0.88	1.10	45.71	2.29	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.928	1.059	0.725	0.715	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	138	1031	340	0	0	-1
normalized size	1	1.00	1.12	1.18	8.81	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.443	1.125	0.600	0.500	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	248	136	0	338	0	0	-1
normalized size	1	1.00	2.12	1.16	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.867	0.993	0.000	0.471	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	248	230	2122	550	0	0	-1
normalized size	1	1.00	1.43	1.32	12.20	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.356	0.838	1.012	0.693	0.507	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	242	270	4934	614	0	0	-1
normalized size	1	1.00	1.13	1.26	23.06	2.87	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.972	1.011	1.688	0.499	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	144	244	0	448	0	0	-1
normalized size	1	1.00	0.61	1.03	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.596	1.169	1.176	0.000	0.483	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	141	234	0	428	0	0	-1
normalized size	1	1.00	0.72	1.19	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.458	0.955	1.161	0.000	0.463	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	168	200	3049	408	0	0	-1
normalized size	1	1.00	1.07	1.27	19.42	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.315	1.318	1.184	1.827	0.528	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	224	198	2875	408	0	0	-1
normalized size	1	1.00	1.43	1.26	18.31	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.260	3.718	1.123	2.501	0.500	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	341	200	0	408	0	0	-1
normalized size	1	1.00	2.17	1.27	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.259	1.353	1.120	0.000	0.485	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	328	396	4988	638	0	0	-1
normalized size	1	1.00	1.53	1.85	23.31	2.98	0.00	0.00	-0.00
time (sec)	N/A	0.503	1.209	1.105	1.003	0.529	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	348	444	9048	702	0	0	-1
normalized size	1	1.00	1.37	1.75	35.62	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.646	1.540	1.144	3.570	0.532	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	308	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	1.590	8.529	0.000	0.465	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	266	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	1.287	5.463	0.000	0.488	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	105	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.142	2.722	0.000	0.461	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.976	2.769	0.000	0.465	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	9.050	1.189	0.000	0.483	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	95	99	0	164	152
normalized size	1	1.00	0.89	1.08	1.12	1.16	0.00	1.93	1.79
time (sec)	N/A	0.067	0.248	0.743	0.326	0.469	0.000	0.220	3.574

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	70	88	0	122	109
normalized size	1	1.00	0.95	1.14	1.11	1.40	0.00	1.94	1.73
time (sec)	N/A	0.053	0.161	0.740	0.411	0.502	0.000	0.230	2.809

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	58	74	0	107	85
normalized size	1	1.00	1.00	1.09	1.23	1.57	0.00	2.28	1.81
time (sec)	N/A	0.049	0.019	0.735	0.332	0.514	0.000	0.213	1.476

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	29	60	37	63	47
normalized size	1	1.00	1.00	1.33	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.026	0.012	0.557	0.545	0.460	4.274	0.199	0.793

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	57
normalized size	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	3.56
time (sec)	N/A	0.008	0.002	0.035	0.516	0.473	1.803	0.142	0.803

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	20	17	15	39	17
normalized size	1	1.00	1.73	1.40	1.33	1.13	1.00	2.60	1.13
time (sec)	N/A	0.023	0.008	0.501	0.379	0.538	1.861	0.159	0.744

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	38	34	29	0	82	31
normalized size	1	1.00	0.92	1.00	0.89	0.76	0.00	2.16	0.82
time (sec)	N/A	0.038	0.064	0.619	0.321	0.448	0.000	0.164	0.813

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	46	42	0	98	55
normalized size	1	1.00	1.06	0.91	0.85	0.78	0.00	1.81	1.02
time (sec)	N/A	0.046	0.068	1.000	0.323	0.482	0.000	0.148	0.831

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	57	53	0	140	75
normalized size	1	1.00	0.96	0.79	0.75	0.70	0.00	1.84	0.99
time (sec)	N/A	0.056	0.128	1.021	0.332	0.467	0.000	0.170	0.832

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	70	69	64	0	154	113
normalized size	1	1.00	0.97	0.76	0.75	0.70	0.00	1.67	1.23
time (sec)	N/A	0.058	0.117	1.032	0.336	0.536	0.000	0.180	4.748

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	118	157	132	136	0	272	221
normalized size	1	1.00	0.87	1.16	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.106	0.583	0.911	0.339	0.512	0.000	0.232	3.823

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	142	144	133	0	258	184
normalized size	1	1.00	0.75	1.29	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.094	0.284	0.924	0.373	0.502	0.000	0.270	3.664

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	89	84	100	0	178	141
normalized size	1	1.00	0.89	1.11	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.090	0.224	0.918	0.340	0.483	0.000	0.236	3.066

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	78	80	93	0	129	99
normalized size	1	1.00	0.76	1.32	1.36	1.58	0.00	2.19	1.68
time (sec)	N/A	0.054	0.108	0.736	0.348	0.485	0.000	0.246	1.529

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	49	40	74	0	77	181
normalized size	1	1.00	0.97	1.48	1.21	2.24	0.00	2.33	5.48
time (sec)	N/A	0.026	0.076	0.559	0.334	0.468	0.000	0.190	0.866

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	49	51	52	0	78	73
normalized size	1	1.00	1.39	1.48	1.55	1.58	0.00	2.36	2.21
time (sec)	N/A	0.055	0.017	0.568	0.394	0.454	0.000	0.217	0.846

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	47	40	0	96	42
normalized size	1	1.00	0.92	1.02	0.94	0.80	0.00	1.92	0.84
time (sec)	N/A	0.066	0.076	0.556	0.469	0.446	0.000	0.199	0.854

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	63	60	52	0	153	72
normalized size	1	1.00	1.02	1.09	1.03	0.90	0.00	2.64	1.24
time (sec)	N/A	0.089	0.154	0.908	0.344	0.454	0.000	0.212	0.825

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	89	82	77	0	224	93
normalized size	1	1.00	0.85	0.88	0.81	0.76	0.00	2.22	0.92
time (sec)	N/A	0.088	0.173	1.053	0.335	0.460	0.000	0.190	0.866

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	95	94	86	0	247	117
normalized size	1	1.00	0.77	0.86	0.85	0.77	0.00	2.23	1.05
time (sec)	N/A	0.122	0.162	1.319	0.339	0.454	0.000	0.216	0.894

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	120	206	181	170	0	367	258
normalized size	1	1.00	0.63	1.09	0.96	0.90	0.00	1.94	1.37
time (sec)	N/A	0.312	0.899	1.082	0.466	0.462	0.000	0.295	4.766

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	90	160	158	140	0	330	226
normalized size	1	1.00	0.69	1.23	1.22	1.08	0.00	2.54	1.74
time (sec)	N/A	0.198	0.450	1.098	0.353	0.481	0.000	0.280	4.798

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	118	106	126	0	205	157
normalized size	1	1.00	0.71	1.19	1.07	1.27	0.00	2.07	1.59
time (sec)	N/A	0.131	0.251	0.908	0.344	0.451	0.000	0.275	3.151

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	95	93	112	0	145	136
normalized size	1	1.00	0.75	1.30	1.27	1.53	0.00	1.99	1.86
time (sec)	N/A	0.049	0.163	0.731	0.356	0.489	0.000	0.199	0.955

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	68	66	94	0	131	97
normalized size	1	1.00	1.31	1.01	0.99	1.40	0.00	1.96	1.45
time (sec)	N/A	0.112	0.342	0.641	0.373	0.486	0.000	0.231	0.923

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	105	90	76	72	0	137	123
normalized size	1	1.00	1.33	1.14	0.96	0.91	0.00	1.73	1.56
time (sec)	N/A	0.119	0.143	0.499	0.381	0.482	0.000	0.248	1.025

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	76	73	66	0	170	77
normalized size	1	1.00	0.80	0.76	0.73	0.66	0.00	1.70	0.77
time (sec)	N/A	0.150	0.121	0.763	0.341	0.461	0.000	0.230	0.862

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	100	102	95	84	0	297	250
normalized size	1	1.00	0.81	0.83	0.77	0.68	0.00	2.41	2.03
time (sec)	N/A	0.183	0.282	0.997	0.371	0.439	0.000	0.245	3.834

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	123	119	110	0	332	287
normalized size	1	1.00	0.81	0.77	0.74	0.69	0.00	2.08	1.79
time (sec)	N/A	0.192	0.300	1.275	0.455	0.488	0.000	0.228	3.926

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	159	145	145	132	0	431	350
normalized size	1	1.00	0.86	0.78	0.78	0.71	0.00	2.33	1.89
time (sec)	N/A	0.232	0.340	1.489	0.384	0.494	0.000	0.268	3.298

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	154	302	275	217	0	592	370
normalized size	1	1.00	0.63	1.24	1.13	0.89	0.00	2.43	1.52
time (sec)	N/A	0.450	0.963	1.239	0.385	0.505	0.000	0.310	4.878

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	125	225	195	182	0	461	304
normalized size	1	1.00	0.70	1.26	1.09	1.02	0.00	2.58	1.70
time (sec)	N/A	0.302	0.789	1.336	0.390	0.489	0.000	0.292	4.970

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	101	188	180	163	0	360	245
normalized size	1	1.00	0.69	1.29	1.23	1.12	0.00	2.47	1.68
time (sec)	N/A	0.243	0.543	1.125	0.355	0.511	0.000	0.292	4.922

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	77	135	121	138	0	221	185
normalized size	1	1.00	0.72	1.26	1.13	1.29	0.00	2.07	1.73
time (sec)	N/A	0.116	0.303	0.982	0.390	0.535	0.000	0.198	1.029

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	280	114	115	130	0	179	152
normalized size	1	1.00	2.69	1.10	1.11	1.25	0.00	1.72	1.46
time (sec)	N/A	0.212	0.541	0.954	0.356	0.495	0.000	0.260	1.040

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	119	109	90	116	0	170	150
normalized size	1	1.00	1.10	1.01	0.83	1.07	0.00	1.57	1.39
time (sec)	N/A	0.217	0.684	0.764	0.337	0.485	0.000	0.269	1.034

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	131	102	98	0	212	158
normalized size	1	1.00	1.11	1.14	0.89	0.85	0.00	1.84	1.37
time (sec)	N/A	0.240	0.165	0.839	0.337	0.483	0.000	0.238	1.115

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	104	116	109	96	0	318	123
normalized size	1	1.00	0.72	0.80	0.75	0.66	0.00	2.19	0.85
time (sec)	N/A	0.313	0.236	0.948	0.457	0.471	0.000	0.242	0.884

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	133	138	133	121	0	425	330
normalized size	1	1.00	0.77	0.80	0.77	0.70	0.00	2.46	1.91
time (sec)	N/A	0.352	0.518	1.507	0.425	0.477	0.000	0.238	3.821

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	156	174	170	150	0	550	214
normalized size	1	1.00	0.73	0.82	0.80	0.70	0.00	2.58	1.00
time (sec)	N/A	0.380	0.458	1.596	0.368	0.537	0.000	0.252	1.094

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	114	205	198	183	0	380	274
normalized size	1	1.00	0.72	1.30	1.25	1.16	0.00	2.41	1.73
time (sec)	N/A	0.236	0.621	1.103	0.360	0.487	0.000	0.222	1.364

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	258	400	0	557	0	286	1021
normalized size	1	1.00	1.64	2.55	0.00	3.55	0.00	1.82	6.50
time (sec)	N/A	0.485	2.701	0.345	0.000	0.664	0.000	0.275	2.453

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	238	262	0	485	0	211	1002
normalized size	1	1.00	2.00	2.20	0.00	4.08	0.00	1.77	8.42
time (sec)	N/A	0.275	1.140	0.423	0.000	0.670	0.000	0.259	1.646

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	115	134	0	392	0	152	119
normalized size	1	1.00	1.35	1.58	0.00	4.61	0.00	1.79	1.40
time (sec)	N/A	0.163	0.394	0.354	0.000	0.549	0.000	0.295	1.213

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	102	88	0	290	0	120	186
normalized size	1	1.00	1.50	1.29	0.00	4.26	0.00	1.76	2.74
time (sec)	N/A	0.109	0.085	0.353	0.000	0.573	0.000	0.236	1.063

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	185	0	77	40
normalized size	1	1.00	0.98	0.90	0.00	3.78	0.00	1.57	0.82
time (sec)	N/A	0.059	0.039	0.385	0.000	0.463	0.000	0.207	0.894

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	67	0	230	0	218	186
normalized size	1	1.00	1.02	1.14	0.00	3.90	0.00	3.69	3.15
time (sec)	N/A	0.051	0.088	0.433	0.000	0.526	0.000	0.194	1.089

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	102	0	277	0	126	395
normalized size	1	1.00	0.95	1.34	0.00	3.64	0.00	1.66	5.20
time (sec)	N/A	0.102	0.154	0.638	0.000	0.491	0.000	0.217	1.310

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	222	0	334	0	178	592
normalized size	1	1.00	0.88	2.02	0.00	3.04	0.00	1.62	5.38
time (sec)	N/A	0.278	0.242	0.639	0.000	0.503	0.000	0.204	1.811

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	122	367	0	401	0	249	654
normalized size	1	1.00	0.82	2.48	0.00	2.71	0.00	1.68	4.42
time (sec)	N/A	0.459	0.333	0.718	0.000	0.569	0.000	0.221	2.064

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	153	672	0	482	0	393	2678
normalized size	1	1.00	0.79	3.48	0.00	2.50	0.00	2.04	13.88
time (sec)	N/A	0.687	0.592	0.657	0.000	0.511	0.000	0.238	3.425

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	357	405	0	909	0	299	3685
normalized size	1	1.00	1.61	1.82	0.00	4.09	0.00	1.35	16.60
time (sec)	N/A	0.612	6.132	0.390	0.000	1.247	0.000	0.306	8.010

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	275	0	760	0	331	3159
normalized size	1	1.00	0.99	1.68	0.00	4.63	0.00	2.02	19.26
time (sec)	N/A	0.346	1.554	0.451	0.000	0.844	0.000	0.296	6.811

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	146	225	0	596	0	203	2848
normalized size	1	1.00	1.25	1.92	0.00	5.09	0.00	1.74	24.34
time (sec)	N/A	0.219	0.387	0.378	0.000	0.812	0.000	0.276	6.727

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	118	0	329	0	150	92
normalized size	1	1.00	0.98	1.39	0.00	3.87	0.00	1.76	1.08
time (sec)	N/A	0.127	0.200	0.397	0.000	0.490	0.000	0.233	1.118

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	118	0	332	0	150	92
normalized size	1	1.00	0.97	1.37	0.00	3.86	0.00	1.74	1.07
time (sec)	N/A	0.103	0.229	0.339	0.000	0.515	0.000	0.239	1.043

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	138	204	0	484	0	179	2886
normalized size	1	1.00	1.27	1.87	0.00	4.44	0.00	1.64	26.48
time (sec)	N/A	0.169	0.476	0.486	0.000	0.539	0.000	0.199	6.672

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	172	242	0	565	0	837	3169
normalized size	1	1.00	1.18	1.66	0.00	3.87	0.00	5.73	21.71
time (sec)	N/A	0.328	0.756	0.558	0.000	0.571	0.000	0.387	7.011

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	144	362	0	660	0	264	3738
normalized size	1	1.00	0.69	1.74	0.00	3.17	0.00	1.27	17.97
time (sec)	N/A	0.585	0.769	0.734	0.000	0.561	0.000	0.214	8.323

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	176	508	0	757	0	335	3839
normalized size	1	1.00	0.67	1.95	0.00	2.90	0.00	1.28	14.71
time (sec)	N/A	0.835	1.070	0.676	0.000	0.565	0.000	0.243	9.085

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	205	735	0	1354	0	383	5332
normalized size	1	1.00	0.89	3.20	0.00	5.89	0.00	1.67	23.18
time (sec)	N/A	0.707	4.851	0.394	0.000	1.447	0.000	0.404	9.290

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	194	685	0	1153	0	347	5078
normalized size	1	1.00	1.03	3.64	0.00	6.13	0.00	1.85	27.01
time (sec)	N/A	0.410	1.362	0.427	0.000	1.449	0.000	0.367	9.497

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	113	184	0	594	0	253	204
normalized size	1	1.00	0.76	1.23	0.00	3.99	0.00	1.70	1.37
time (sec)	N/A	0.232	0.436	0.401	0.000	0.520	0.000	0.348	3.235

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	195	0	565	0	277	210
normalized size	1	1.00	0.86	1.46	0.00	4.22	0.00	2.07	1.57
time (sec)	N/A	0.191	0.397	0.353	0.000	0.531	0.000	0.319	3.374

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	115	186	0	595	0	254	204
normalized size	1	1.00	0.86	1.40	0.00	4.47	0.00	1.91	1.53
time (sec)	N/A	0.175	0.470	0.377	0.000	0.509	0.000	0.330	3.183

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	205	664	0	919	0	322	5090
normalized size	1	1.00	1.18	3.84	0.00	5.31	0.00	1.86	29.42
time (sec)	N/A	0.310	0.765	0.422	0.000	0.564	0.000	0.213	9.222

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	229	702	0	1037	0	357	5338
normalized size	1	1.00	1.03	3.15	0.00	4.65	0.00	1.60	23.94
time (sec)	N/A	0.623	0.880	0.683	0.000	0.602	0.000	0.313	8.999

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	199	827	0	1158	0	1723	5950
normalized size	1	1.00	0.67	2.79	0.00	3.91	0.00	5.82	20.10
time (sec)	N/A	0.998	2.042	0.608	0.000	0.609	0.000	0.830	9.285

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	416	1481	0	2058	0	592	7476
normalized size	1	1.00	1.32	4.69	0.00	6.51	0.00	1.87	23.66
time (sec)	N/A	1.106	6.235	0.432	0.000	3.028	0.000	0.389	10.358

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	250	1429	0	1822	0	559	7222
normalized size	1	1.00	0.97	5.52	0.00	7.03	0.00	2.16	27.88
time (sec)	N/A	0.752	4.144	0.362	0.000	3.203	0.000	0.468	12.448

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	158	285	0	903	0	403	378
normalized size	1	1.00	0.71	1.28	0.00	4.07	0.00	1.82	1.70
time (sec)	N/A	0.421	1.027	0.406	0.000	0.609	0.000	0.318	4.413

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	165	294	0	902	0	431	380
normalized size	1	1.00	0.80	1.43	0.00	4.38	0.00	2.09	1.84
time (sec)	N/A	0.353	1.098	0.347	0.000	0.559	0.000	0.377	4.379

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	164	297	0	901	0	431	382
normalized size	1	1.00	0.85	1.55	0.00	4.69	0.00	2.24	1.99
time (sec)	N/A	0.307	1.283	0.428	0.000	0.578	0.000	0.353	4.343

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	163	284	0	905	0	403	378
normalized size	1	1.00	0.89	1.54	0.00	4.92	0.00	2.19	2.05
time (sec)	N/A	0.307	1.166	0.346	0.000	0.591	0.000	0.356	4.320

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	268	1408	0	1456	0	532	7234
normalized size	1	1.00	1.11	5.82	0.00	6.02	0.00	2.20	29.89
time (sec)	N/A	0.535	1.495	0.527	0.000	0.654	0.000	0.266	12.790

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	293	1448	0	1603	0	564	7534
normalized size	1	1.00	0.98	4.84	0.00	5.36	0.00	1.89	25.20
time (sec)	N/A	1.037	1.700	0.684	0.000	0.682	0.000	0.323	10.384

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	326	1576	0	1767	0	615	8133
normalized size	1	1.00	0.84	4.07	0.00	4.57	0.00	1.59	21.02
time (sec)	N/A	1.456	6.384	0.797	0.000	0.748	0.000	0.348	10.803

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	34	47	33	0	30	21
normalized size	1	1.00	0.97	1.10	1.52	1.06	0.00	0.97	0.68
time (sec)	N/A	0.031	0.053	0.399	0.782	0.469	0.000	0.163	0.824

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	73	63	88	73	0	59	52
normalized size	1	1.00	1.30	1.12	1.57	1.30	0.00	1.05	0.93
time (sec)	N/A	0.080	0.172	0.500	0.450	0.466	0.000	0.170	0.859

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	108	94	131	116	0	75	79
normalized size	1	1.00	1.33	1.16	1.62	1.43	0.00	0.93	0.98
time (sec)	N/A	0.116	0.346	0.432	0.471	0.544	0.000	0.176	0.907

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	141	125	171	159	0	88	105
normalized size	1	1.00	1.33	1.18	1.61	1.50	0.00	0.83	0.99
time (sec)	N/A	0.158	0.534	0.551	1.127	0.470	0.000	0.186	1.094

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	51	70	52	0	43	21
normalized size	1	1.00	0.99	0.73	1.00	0.74	0.00	0.61	0.30
time (sec)	N/A	0.035	0.068	0.423	0.483	0.507	0.000	0.214	0.875

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	162	87	111	102	0	69	52
normalized size	1	1.00	1.71	0.92	1.17	1.07	0.00	0.73	0.55
time (sec)	N/A	0.093	0.162	0.481	0.470	0.459	0.000	0.186	0.881

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	241	123	155	155	0	85	78
normalized size	1	1.00	2.01	1.02	1.29	1.29	0.00	0.71	0.65
time (sec)	N/A	0.133	0.315	0.434	0.488	0.489	0.000	0.175	0.953

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	344	159	194	208	0	98	105
normalized size	1	1.00	2.37	1.10	1.34	1.43	0.00	0.68	0.72
time (sec)	N/A	0.180	0.516	0.552	0.520	0.525	0.000	0.186	1.099

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	401	1584	0	0	0	0	-1
normalized size	1	1.00	1.37	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	13.501	1.793	0.000	0.469	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	293	913	0	0	0	0	-1
normalized size	1	1.00	1.22	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	10.793	1.446	0.000	0.445	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	232	814	0	0	0	0	-1
normalized size	1	1.00	1.11	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	10.344	1.360	0.000	0.504	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
normalized size	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	1.612	1.421	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	2713	826	0	0	0	0	-1
normalized size	1	1.00	8.22	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	18.251	1.657	0.000	2.036	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	1173	1254	0	0	0	0	-1
normalized size	1	1.00	2.96	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	18.772	1.433	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	550	2522	0	0	0	0	-1
normalized size	1	1.00	1.36	6.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	17.827	2.125	0.000	1.261	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	471	1852	0	0	0	0	-1
normalized size	1	1.00	1.38	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	14.047	1.675	0.000	0.656	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	408	1566	0	0	0	0	-1
normalized size	1	1.00	1.45	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	13.260	1.453	0.000	0.778	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	304	1106	0	0	0	0	-1
normalized size	1	1.00	1.22	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	10.510	1.269	0.000	0.899	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0	-1
normalized size	1	1.00	2.85	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	18.195	1.351	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	439	1026	0	0	0	0	-1
normalized size	1	1.00	1.31	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	11.998	1.263	0.000	2.259	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	1159	1440	0	0	0	0	-1
normalized size	1	1.00	2.97	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	18.587	1.233	0.000	1.840	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	615	2807	0	0	0	0	-1
normalized size	1	1.00	1.33	6.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	16.962	2.301	0.000	0.608	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	552	2523	0	0	0	0	-1
normalized size	1	1.00	1.38	6.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	16.568	1.875	0.000	2.676	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	474	1852	0	0	0	0	-1
normalized size	1	1.00	1.42	5.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	13.958	1.528	0.000	0.707	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	440	1775	0	0	0	0	-1
normalized size	1	1.00	1.49	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	16.252	1.353	0.000	0.766	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	713	1514	0	0	0	0	-1
normalized size	1	1.00	2.03	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	17.917	1.281	0.000	0.852	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	780	1640	0	0	0	0	-1
normalized size	1	1.00	2.21	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	17.088	1.363	0.000	25.398	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	4588	1646	0	0	0	0	-1
normalized size	1	1.00	11.50	4.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.632	23.170	1.233	0.000	1.239	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	1018	1881	0	0	0	0	-1
normalized size	1	1.00	2.21	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.933	17.491	1.278	0.000	28.415	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	1688	2330	0	0	0	0	-1
normalized size	1	1.00	3.18	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.300	16.863	1.467	0.000	34.337	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	1150	2185	0	0	0	0	-1
normalized size	1	1.00	2.85	5.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	15.698	1.501	0.000	1.026	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	463	1852	0	0	0	0	-1
normalized size	1	1.00	1.29	5.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.673	14.668	1.761	0.000	0.529	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	365	1584	0	0	0	0	-1
normalized size	1	1.00	1.21	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	14.586	1.665	0.000	0.496	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	341	919	0	0	0	0	-1
normalized size	1	1.00	1.40	3.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	13.646	1.607	0.000	0.723	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	2189	639	0	0	0	0	-1
normalized size	1	1.00	10.73	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	19.304	1.395	0.000	0.453	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	143	0	0	0	0	-1
normalized size	1	1.00	0.94	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	1.663	1.118	0.000	0.907	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	178	0	0	0	0	-1
normalized size	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	1.570	1.290	0.000	25.969	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	5060	649	0	0	0	0	-1
normalized size	1	1.00	14.97	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	24.422	1.419	0.000	0.811	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	1195	1259	0	0	0	0	-1
normalized size	1	1.00	2.98	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	19.379	1.392	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	498	2480	0	0	0	0	-1
normalized size	1	1.00	1.25	6.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	17.078	2.041	0.000	0.673	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	470	1792	0	0	0	0	-1
normalized size	1	1.00	1.45	5.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	15.896	1.655	0.000	0.941	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	440	1451	0	0	0	0	-1
normalized size	1	1.00	1.71	5.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	14.326	1.510	0.000	0.809	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	249	837	0	0	0	0	-1
normalized size	1	1.00	1.05	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	9.426	1.213	0.000	0.518	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	244	817	0	0	0	0	-1
normalized size	1	1.00	1.03	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	10.312	1.258	0.000	0.565	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
normalized size	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	18.731	1.331	0.000	25.837	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	1069	1662	0	0	0	0	-1
normalized size	1	1.00	2.70	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.498	15.620	1.391	0.000	24.226	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	1745	2298	0	0	0	0	-1
normalized size	1	1.00	3.71	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	14.659	1.437	0.000	31.414	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	578	4176	0	0	0	0	-1
normalized size	1	1.00	1.35	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.933	19.088	2.040	0.000	0.562	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	3345	3674	0	0	0	0	-1
normalized size	1	1.00	9.24	10.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	23.527	1.778	0.000	0.536	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	503	2733	0	0	0	0	-1
normalized size	1	1.00	1.49	8.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	16.086	1.326	0.000	0.542	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	486	2411	0	0	0	0	-1
normalized size	1	1.00	1.53	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	13.942	1.263	0.000	0.605	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	360	1781	0	0	0	0	-1
normalized size	1	1.00	1.18	5.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	8.835	1.224	0.000	0.674	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	1798	3888	0	0	0	0	-1
normalized size	1	1.00	4.01	8.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	15.593	1.239	0.000	26.379	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	1481	4580	0	0	0	0	-1
normalized size	1	1.00	2.90	8.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	17.666	1.849	0.000	32.015	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	2285	5638	0	0	0	0	-1
normalized size	1	1.00	4.07	10.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.167	16.379	1.838	0.000	1.287	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	2346	7838	0	0	0	0	-1
normalized size	1	1.00	4.39	14.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	16.531	1.566	0.000	1.133	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	0	0	0	-1
normalized size	1	1.00	0.64	3.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.365	9.102	0.000	0.455	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	0	0	0	-1
normalized size	1	1.00	0.69	3.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.269	8.109	0.000	0.518	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	148	0	0	0	0	-1
normalized size	1	1.00	0.73	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.128	3.888	0.000	0.640	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	0	0	0	-1
normalized size	1	1.00	0.69	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.086	3.225	0.000	0.493	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	228	0	0	0	0	-1
normalized size	1	1.00	0.75	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.158	3.146	0.000	0.566	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	0	0	0	-1
normalized size	1	1.00	0.69	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.399	3.792	0.000	0.512	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	0	0	0	-1
normalized size	1	1.00	0.66	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.621	3.623	0.000	1.042	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	0	0	0	-1
normalized size	1	1.00	0.70	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.987	10.770	0.000	1.209	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	126	660	0	0	0	0	-1
normalized size	1	1.00	0.72	3.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.400	10.226	0.000	0.924	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	93	514	0	0	0	0	-1
normalized size	1	1.00	0.69	3.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.362	8.635	0.000	0.733	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	82	202	0	0	0	0	-1
normalized size	1	1.00	0.76	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.204	4.451	0.000	0.991	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	0	0	0	-1
normalized size	1	1.00	0.78	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.206	3.734	0.000	0.838	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	321	0	0	0	0	-1
normalized size	1	1.00	0.71	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.482	4.089	0.000	0.655	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	0	0	0	-1
normalized size	1	1.00	0.69	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.815	4.144	0.000	1.779	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	177	847	0	0	0	0	-1
normalized size	1	1.00	0.76	3.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	3.741	12.798	0.000	1.033	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	738	0	0	0	0	-1
normalized size	1	1.00	0.71	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	1.654	11.117	0.000	0.471	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	106	631	0	0	0	0	-1
normalized size	1	1.00	0.67	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.540	8.744	0.000	1.186	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	0	0	0	-1
normalized size	1	1.00	0.65	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.626	3.982	0.000	0.793	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	106	376	0	0	0	0	-1
normalized size	1	1.00	0.68	2.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.499	3.860	0.000	0.449	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	0	0	0	-1
normalized size	1	1.00	0.66	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	1.074	3.756	0.000	0.484	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	0	0	0	-1
normalized size	1	1.00	0.68	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	1.442	3.532	0.000	0.627	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	256	1174	0	0	0	0	-1
normalized size	1	1.00	0.89	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	2.430	15.926	0.000	1.064	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	168	925	0	0	0	0	-1
normalized size	1	1.00	0.68	3.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.715	12.663	0.000	0.519	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	146	907	0	0	0	0	-1
normalized size	1	1.00	0.70	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	2.384	10.298	0.000	0.729	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	130	777	0	0	0	0	-1
normalized size	1	1.00	0.62	3.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	1.236	5.015	0.000	0.551	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	138	619	0	0	0	0	-1
normalized size	1	1.00	0.67	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.666	4.591	0.000	0.515	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	142	476	0	0	0	0	-1
normalized size	1	1.00	0.67	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.935	3.728	0.000	0.515	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	168	529	0	0	0	0	-1
normalized size	1	1.00	0.69	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	1.694	4.096	0.000	0.854	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	199	586	0	0	0	0	-1
normalized size	1	1.00	0.69	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	2.115	4.105	0.000	0.653	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	165	450	0	0	0	0	-1
normalized size	1	1.00	0.88	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.511	3.250	9.504	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	353	0	0	0	0	-1
normalized size	1	1.00	0.71	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	5.227	4.223	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	-1
normalized size	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.373	3.234	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	47	187	0	0	0	0	-1
normalized size	1	1.00	0.51	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.278	3.863	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	226	0	0	0	0	-1
normalized size	1	1.00	1.30	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	7.198	3.829	0.000	67.687	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	194	516	0	0	0	0	-1
normalized size	1	1.00	1.13	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.366	7.133	4.481	0.000	69.793	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	653	1002	0	0	0	0	-1
normalized size	1	1.00	1.91	2.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	6.867	14.653	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	351	868	0	0	0	0	-1
normalized size	1	1.00	1.26	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.643	6.361	11.059	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	582	608	0	0	0	0	-1
normalized size	1	1.00	2.72	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	6.723	7.158	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	628	707	0	0	0	0	-1
normalized size	1	1.00	3.02	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.358	6.725	8.325	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	251	788	0	0	0	0	-1
normalized size	1	1.00	1.11	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	5.137	8.391	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	603	809	0	0	0	0	-1
normalized size	1	1.00	2.47	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	6.769	10.325	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	634	1064	0	0	0	0	-1
normalized size	1	1.00	2.09	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	6.851	11.489	0.000	116.578	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	721	2014	0	0	0	0	-1
normalized size	1	1.00	1.86	5.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.961	6.885	16.754	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	692	1203	0	0	0	0	-1
normalized size	1	1.00	2.20	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	6.819	7.985	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	728	1760	0	0	0	0	-1
normalized size	1	1.00	2.33	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	6.809	13.495	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	719	1858	0	0	0	0	-1
normalized size	1	1.00	2.35	6.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.646	6.773	13.654	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	744	1936	0	0	0	0	-1
normalized size	1	1.00	2.30	5.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.667	6.855	13.740	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	707	1957	0	0	0	0	-1
normalized size	1	1.00	2.07	5.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.754	6.915	15.081	0.000	135.357	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	731	2216	0	0	0	0	-1
normalized size	1	1.00	1.80	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.021	6.970	16.279	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	321	789	0	0	0	0	-1
normalized size	1	1.00	1.35	3.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.653	5.642	1.818	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	96	283	0	0	0	0	-1
normalized size	1	1.00	0.70	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.354	2.309	1.490	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	925	0	0	0	0	-1
normalized size	1	1.00	1.00	13.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.118	1.750	0.000	0.896	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	156	1021	0	0	0	0	-1
normalized size	1	1.00	0.81	5.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.377	0.630	1.838	0.000	1.263	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	203	1736	0	0	0	0	-1
normalized size	1	1.00	0.83	7.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	0.947	1.817	0.000	1.034	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	237	2050	0	0	0	0	-1
normalized size	1	1.00	0.78	6.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.847	1.341	1.902	0.000	2.509	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	549	1744	0	0	0	0	-1
normalized size	1	1.00	1.84	5.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.994	6.602	1.737	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	394	1207	0	0	0	0	-1
normalized size	1	1.00	1.58	4.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	8.384	1.599	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	129	1367	0	0	0	0	-1
normalized size	1	1.00	0.62	6.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	2.554	1.838	0.000	5.240	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	1219	0	0	0	0	-1
normalized size	1	1.00	0.83	6.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	0.749	1.894	0.000	2.181	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	197	1707	0	0	0	0	-1
normalized size	1	1.00	0.82	7.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.615	1.233	1.800	0.000	0.947	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	0	0	0	-1
normalized size	1	1.00	0.78	6.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.871	1.971	1.967	0.000	0.926	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	602	2295	0	0	0	0	-1
normalized size	1	1.00	1.63	6.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.348	6.686	1.811	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	560	1982	0	0	0	0	-1
normalized size	1	1.00	1.78	6.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.071	6.597	1.536	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	538	1949	0	0	0	0	-1
normalized size	1	1.00	2.05	7.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	6.623	1.789	0.000	4.103	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	409	1661	0	0	0	0	-1
normalized size	1	1.00	1.56	6.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	6.507	1.789	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	200	1931	0	0	0	0	-1
normalized size	1	1.00	0.84	8.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.690	1.798	1.916	0.000	0.724	0.000	0.000	0.000

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	0	0	0	-1
normalized size	1	1.00	0.78	6.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.936	2.480	1.968	0.000	0.831	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	286	2788	0	0	0	0	-1
normalized size	1	1.00	0.79	7.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.254	2.890	2.093	0.000	0.750	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	397	1755	0	0	0	0	-1
normalized size	1	1.00	1.27	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	6.441	1.712	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	329	996	0	0	0	0	-1
normalized size	1	1.00	1.34	4.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	9.425	1.749	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	216	0	0	0	0	-1
normalized size	1	1.00	1.00	3.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.130	1.692	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	171	0	0	0	0	-1
normalized size	1	1.00	1.00	2.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.074	1.596	0.000	0.991	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	96	736	0	0	0	0	-1
normalized size	1	1.00	0.68	5.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	2.957	1.827	0.000	1.031	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	147	1024	0	0	0	0	-1
normalized size	1	1.00	0.75	5.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	0.703	1.983	0.000	1.393	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	193	1736	0	0	0	0	-1
normalized size	1	1.00	0.78	6.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	0.845	2.114	0.000	0.520	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	478	1501	0	0	0	0	-1
normalized size	1	1.00	1.39	4.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.997	4.851	1.857	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	557	1144	0	0	0	0	-1
normalized size	1	1.00	2.70	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	6.524	1.574	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	103	501	0	0	0	0	-1
normalized size	1	1.00	0.82	3.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.322	1.922	0.000	0.772	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	156	510	0	0	0	0	-1
normalized size	1	1.00	0.78	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.624	1.728	0.000	0.876	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	165	999	0	0	0	0	-1
normalized size	1	1.00	0.77	4.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.716	1.794	0.000	0.959	0.000	0.000	0.000

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	203	1315	0	0	0	0	-1
normalized size	1	1.00	0.70	4.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	0.958	2.137	0.000	0.530	0.000	0.000	0.000

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	250	1861	0	0	0	0	-1
normalized size	1	1.00	0.69	5.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	1.464	1.891	0.000	1.278	0.000	0.000	0.000

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	677	4591	0	0	0	0	-1
normalized size	1	1.00	1.48	10.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.413	6.817	1.773	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	487	3854	0	0	0	0	-1
normalized size	1	1.00	1.32	10.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.103	5.931	1.563	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	169	1343	0	0	0	0	-1
normalized size	1	1.00	0.61	4.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	1.154	1.701	0.000	1.190	0.000	0.000	0.000

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	178	1822	0	0	0	0	-1
normalized size	1	1.00	0.63	6.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	1.106	1.561	0.000	1.331	0.000	0.000	0.000

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	196	2070	0	0	0	0	-1
normalized size	1	1.00	0.65	6.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	1.307	1.519	0.000	1.861	0.000	0.000	0.000

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	208	3103	0	0	0	0	-1
normalized size	1	1.00	0.66	9.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	1.382	1.873	0.000	1.202	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	257	3615	0	0	0	0	-1
normalized size	1	1.00	0.66	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.022	1.794	2.042	0.000	0.745	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	292	4586	0	0	0	0	-1
normalized size	1	1.00	0.62	9.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.353	2.205	2.073	0.000	0.622	0.000	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	78	405	0	0	0	0	-1
normalized size	1	1.00	0.64	3.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.153	1.770	0.000	0.470	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	374	0	0	0	0	-1
normalized size	1	1.00	0.62	3.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.127	1.883	0.000	0.461	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	68	405	0	0	0	0	-1
normalized size	1	1.00	0.63	3.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.085	1.849	0.000	0.454	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	78	390	0	0	0	0	-1
normalized size	1	1.00	0.63	3.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.102	1.863	0.000	0.445	0.000	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	81	409	0	0	0	0	-1
normalized size	1	1.00	0.64	3.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.160	1.803	0.000	0.440	0.000	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	72	381	0	0	0	0	-1
normalized size	1	1.00	0.64	3.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.129	1.909	0.000	0.501	0.000	0.000	0.000

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	72	370	0	0	0	0	-1
normalized size	1	1.00	0.56	2.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.086	1.785	0.000	0.504	0.000	0.000	0.000

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	394	0	0	0	0	-1
normalized size	1	1.00	0.70	3.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.118	1.877	0.000	0.508	0.000	0.000	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	142	0	0	0	0	-1
normalized size	1	1.00	1.00	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.060	1.494	0.000	0.481	0.000	0.000	0.000

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	137	0	0	0	0	-1
normalized size	1	1.00	1.00	2.54	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.068	1.583	0.000	0.492	0.000	0.000	0.000

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	144	0	0	0	0	-1
normalized size	1	1.00	1.00	2.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.040	1.533	0.000	0.542	0.000	0.000	0.000

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	139	0	0	0	0	-1
normalized size	1	1.00	1.00	2.28	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.046	1.651	0.000	0.455	0.000	0.000	0.000

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	145	0	0	0	0	-1
normalized size	1	1.00	1.00	2.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.062	1.464	0.000	0.472	0.000	0.000	0.000

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	138	0	0	0	0	-1
normalized size	1	1.00	1.00	2.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.061	1.564	0.000	0.520	0.000	0.000	0.000

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	136	0	0	0	0	-1
normalized size	1	1.00	0.87	2.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.046	1.608	0.000	0.519	0.000	0.000	0.000

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	142	0	0	0	0	-1
normalized size	1	1.00	1.11	2.58	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.047	1.678	0.000	0.565	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7160	0	0	0	0	0	-1
normalized size	1	1.00	68.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	26.858	0.718	0.000	0.506	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	2.089	0.959	0.000	0.000	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	21877	0	0	0	0	0	-1
normalized size	1	1.00	60.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.668	26.952	0.892	0.000	0.702	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	18991	0	0	0	0	0	-1
normalized size	1	1.00	62.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	26.775	0.800	0.000	0.656	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	2505	0	0	0	0	0	-1
normalized size	1	1.00	9.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	18.897	0.737	0.000	0.751	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	7142	0	0	0	0	0	-1
normalized size	1	1.00	68.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	26.400	0.708	0.000	0.482	0.000	0.000	0.000

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.012	2.058	0.825	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	8660	0	0	0	0	0	-1
normalized size	1	1.00	80.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	27.297	0.728	0.000	1.014	0.000	0.000	0.000

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	20.446	0.737	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	28057	0	0	0	0	0	-1
normalized size	1	1.00	68.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	27.410	0.810	0.000	1.165	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	21890	0	0	0	0	0	-1
normalized size	1	1.00	61.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.653	27.340	0.891	0.000	1.272	0.000	0.000	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	19016	0	0	0	0	0	-1
normalized size	1	1.00	63.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	27.276	0.749	0.000	0.856	0.000	0.000	0.000

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	8668	0	0	0	0	0	-1
normalized size	1	1.00	80.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	27.774	0.684	0.000	1.114	0.000	0.000	0.000

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	26.489	0.732	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	19015	0	0	0	0	0	-1
normalized size	1	1.00	60.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	26.602	0.817	0.000	1.106	0.000	0.000	0.000

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	7195	0	0	0	0	0	-1
normalized size	1	1.00	27.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	26.783	0.820	0.000	0.512	0.000	0.000	0.000

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	2759	0	0	0	0	0	-1
normalized size	1	1.00	12.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	19.189	0.714	0.000	1.142	0.000	0.000	0.000

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	-1
normalized size	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	2.067	0.818	0.000	0.580	0.000	0.000	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	1.025	0.793	0.000	0.000	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	-1
normalized size	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.969	0.802	0.000	0.510	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	1.064	0.732	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	10343	0	0	0	0	0	-1
normalized size	1	1.00	94.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	27.293	0.740	0.000	1.016	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	27.358	0.751	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	21987	0	0	0	0	0	-1
normalized size	1	1.00	58.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	26.831	0.895	0.000	0.618	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	19126	0	0	0	0	0	-1
normalized size	1	1.00	62.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	26.697	0.891	0.000	0.933	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	7325	0	0	0	0	0	-1
normalized size	1	1.00	25.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	27.004	0.769	0.000	0.580	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	10363	0	0	0	0	0	-1
normalized size	1	1.00	94.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	27.276	0.746	0.000	0.524	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	23.062	0.758	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	4543	0	0	0	0	0	-1
normalized size	1	1.00	26.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	21.739	0.949	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	4544	0	0	0	0	0	-1
normalized size	1	1.00	26.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	21.544	0.916	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	7542	0	0	0	0	0	-1
normalized size	1	1.00	43.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	29.326	0.914	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	7588	0	0	0	0	0	-1
normalized size	1	1.00	43.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	29.256	0.951	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	37.726	1.660	0.000	0.915	0.000	0.000	0.000

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	49.590	1.703	0.000	2.474	0.000	0.000	0.000

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	32.117	1.648	0.000	0.959	0.000	0.000	0.000

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	39.849	1.477	0.000	3.627	0.000	0.000	0.000

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	2.675	1.994	0.000	1.446	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	9.387	1.503	0.000	4.670	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	18.335	1.772	0.000	1.513	0.000	0.000	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	24.433	1.774	0.000	2.103	0.000	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	35.510	1.486	0.000	1.613	0.000	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	41.611	1.509	0.000	1.957	0.000	0.000	0.000

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	39.329	1.444	0.000	0.771	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	41.269	1.556	0.000	0.777	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	36.060	1.532	0.000	0.727	0.000	0.000	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	42.131	1.598	0.000	2.147	0.000	0.000	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	31.839	1.495	0.000	1.606	0.000	0.000	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	42.649	1.421	0.000	2.676	0.000	0.000	0.000

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	23.903	1.505	0.000	2.456	0.000	0.000	0.000

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	28.979	1.355	0.000	3.211	0.000	0.000	0.000

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	31.465	1.303	0.000	1.379	0.000	0.000	0.000

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	39.219	1.331	0.000	2.262	0.000	0.000	0.000

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	43.769	1.572	0.000	0.848	0.000	0.000	0.000

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	48.442	1.492	0.000	0.808	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	42.233	1.370	0.000	0.776	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	49.100	1.288	0.000	2.322	0.000	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	36.396	1.315	0.000	1.701	0.000	0.000	0.000

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	50.144	1.282	0.000	2.040	0.000	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	37.707	1.418	0.000	1.758	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	43.839	1.432	0.000	2.176	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	36.205	1.470	0.000	1.283	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	42.938	1.389	0.000	2.347	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	30.284	1.450	0.000	0.625	0.000	0.000	0.000

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	36.104	1.348	0.000	0.692	0.000	0.000	0.000

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	1.948	1.471	0.000	0.555	0.000	0.000	0.000

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	2.233	1.531	0.000	2.636	0.000	0.000	0.000

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	1.471	1.598	0.000	1.057	0.000	0.000	0.000

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	3.201	1.589	0.000	3.761	0.000	0.000	0.000

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	26.025	1.754	0.000	2.596	0.000	0.000	0.000

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	31.242	1.660	0.000	2.338	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	34.480	1.611	0.000	1.439	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	41.795	1.560	0.000	2.077	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	34.966	1.484	0.000	0.726	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	42.036	1.384	0.000	1.218	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	36.217	1.325	0.000	0.717	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	45.110	1.335	0.000	2.290	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	37.499	1.250	0.000	1.986	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	44.293	1.401	0.000	2.538	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	42.311	1.445	0.000	1.561	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	49.101	1.374	0.000	1.931	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	18.833	1.716	0.000	1.824	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	51.493	1.600	0.000	2.162	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	41.305	1.432	0.000	1.121	0.000	0.000	0.000

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	49.258	1.441	0.000	0.984	0.000	0.000	0.000

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	40.695	1.503	0.000	0.901	0.000	0.000	0.000

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	46.502	1.318	0.000	4.000	0.000	0.000	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	44.455	1.356	0.000	3.186	0.000	0.000	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	51.297	1.560	0.000	2.633	0.000	0.000	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	49.970	1.577	0.000	2.888	0.000	0.000	0.000

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	58.913	1.575	0.000	2.492	0.000	0.000	0.000

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	23.276	1.618	0.000	4.987	0.000	0.000	0.000

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	65.961	1.647	0.000	2.256	0.000	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	231	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.967	11.293	0.000	0.662	0.000	0.000	0.000

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	171	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.361	8.194	0.000	0.498	0.000	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.173	2.598	0.000	0.641	0.000	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	5280	0	0	0	0	0	-1
normalized size	1	1.00	27.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.302	25.845	2.829	0.000	0.714	0.000	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	13940	0	0	0	0	0	-1
normalized size	1	1.00	46.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	46.576	0.865	0.000	0.522	0.000	0.000	0.000

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	16.820	1.241	0.000	0.568	0.000	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	0.536	1.252	0.000	0.589	0.000	0.000	0.000

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.070	3.008	1.071	0.000	0.820	0.000	0.000	0.000

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	2.741	1.117	0.000	0.700	0.000	0.000	0.000

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	2.696	2.995	0.000	0.595	0.000	0.000	0.000

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.643	2.530	0.000	0.517	0.000	0.000	0.000

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	8899	0	0	0	0	0	-1
normalized size	1	1.00	32.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	26.775	1.143	0.000	0.667	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	5564	0	0	0	0	0	-1
normalized size	1	1.00	25.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	23.164	0.808	0.000	0.687	0.000	0.000	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	2828	0	0	0	0	0	-1
normalized size	1	1.00	27.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	14.897	0.921	0.000	0.577	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	2.084	0.783	0.000	0.471	0.000	0.000	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.033	6.724	1.278	0.000	0.569	0.000	0.000	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	5.895	1.941	0.000	0.544	0.000	0.000	0.000

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	90	318	0	0	0	0	87
normalized size	1	1.00	0.67	2.36	0.00	0.00	0.00	0.00	0.64
time (sec)	N/A	0.104	0.391	4.016	0.000	0.505	0.000	0.000	1.310

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0	87
normalized size	1	1.00	0.69	2.61	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.089	0.564	3.384	0.000	0.562	0.000	0.000	1.142

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0	80
normalized size	1	1.00	0.76	3.01	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.076	0.274	3.640	0.000	0.531	0.000	0.000	1.039

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	228	0	0	0	0	53
normalized size	1	1.00	0.87	3.74	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.067	0.122	3.513	0.000	0.570	0.000	0.000	0.173

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	152	0	0	0	0	33
normalized size	1	1.00	0.91	4.34	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.057	0.087	2.958	0.000	0.852	0.000	0.000	0.229

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0	60
normalized size	1	1.00	0.89	2.60	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.067	0.167	3.959	0.000	0.447	0.000	0.000	1.249

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0	87
normalized size	1	1.00	0.78	4.78	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.075	0.447	7.964	0.000	0.478	0.000	0.000	1.544

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0	87
normalized size	1	1.00	0.86	4.52	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.087	0.349	8.999	0.000	0.566	0.000	0.000	1.696

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	0	0	0	135
normalized size	1	1.00	0.71	2.49	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.187	0.870	3.952	0.000	0.522	0.000	0.000	1.313

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	0	0	0	128
normalized size	1	1.00	0.73	2.68	0.00	0.00	0.00	0.00	0.95
time (sec)	N/A	0.172	0.653	3.899	0.000	0.500	0.000	0.000	1.191

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	79	321	0	0	0	0	102
normalized size	1	1.00	0.78	3.18	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.153	0.325	3.680	0.000	0.499	0.000	0.000	1.110

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	0	0	0	76
normalized size	1	1.00	0.89	3.93	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.139	0.173	4.243	0.000	0.570	0.000	0.000	1.046

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	0	0	0	81
normalized size	1	1.00	0.91	2.97	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.136	0.355	4.142	0.000	0.691	0.000	0.000	1.398

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	514	0	0	0	0	108
normalized size	1	1.00	0.77	5.41	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.145	0.673	8.343	0.000	0.554	0.000	0.000	1.520

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	660	0	0	0	0	113
normalized size	1	1.00	0.92	4.89	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.164	0.444	10.707	0.000	1.009	0.000	0.000	1.666

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	142	689	0	0	0	0	113
normalized size	1	1.00	0.89	4.31	0.00	0.00	0.00	0.00	0.71
time (sec)	N/A	0.183	0.621	10.986	0.000	0.599	0.000	0.000	1.789

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	0	0	0	178
normalized size	1	1.00	0.71	2.42	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.286	1.089	3.875	0.000	0.968	0.000	0.000	1.344

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	0	0	0	146
normalized size	1	1.00	0.69	2.65	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.260	0.851	4.124	0.000	0.910	0.000	0.000	1.219

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	84	376	0	0	0	0	125
normalized size	1	1.00	0.72	3.24	0.00	0.00	0.00	0.00	1.08
time (sec)	N/A	0.228	0.423	3.699	0.000	0.484	0.000	0.000	1.180

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	303	0	0	0	0	124
normalized size	1	1.00	0.69	2.40	0.00	0.00	0.00	0.00	0.98
time (sec)	N/A	0.231	0.599	4.556	0.000	0.627	0.000	0.000	1.241

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	84	630	0	0	0	0	128
normalized size	1	1.00	0.71	5.34	0.00	0.00	0.00	0.00	1.08
time (sec)	N/A	0.230	1.286	8.455	0.000	0.712	0.000	0.000	2.097

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	738	0	0	0	0	156
normalized size	1	1.00	0.84	4.95	0.00	0.00	0.00	0.00	1.05
time (sec)	N/A	0.246	1.093	10.750	0.000	0.986	0.000	0.000	2.167

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	177	847	0	0	0	0	147
normalized size	1	1.00	0.91	4.37	0.00	0.00	0.00	0.00	0.76
time (sec)	N/A	0.285	0.937	12.604	0.000	0.461	0.000	0.000	2.240

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	226	668	0	0	0	0	-1
normalized size	1	1.00	1.49	4.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.602	1.796	4.520	0.000	43.627	0.000	0.000	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	158	516	0	0	0	0	-1
normalized size	1	1.00	1.41	4.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.390	1.929	4.552	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	226	0	0	0	0	-1
normalized size	1	1.00	1.08	3.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.337	3.685	0.000	83.107	0.000	0.000	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	187	0	0	0	0	-1
normalized size	1	1.00	0.91	3.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.189	0.080	3.864	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	-1
normalized size	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	0.090	3.071	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	195	353	0	0	0	0	-1
normalized size	1	1.00	2.53	4.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	3.341	4.956	0.000	0.000	0.000	0.000	0.000

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	210	450	0	0	0	0	-1
normalized size	1	1.00	1.64	3.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.540	4.516	10.055	0.000	0.000	0.000	0.000	0.000

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	266	1063	0	0	0	0	-1
normalized size	1	1.00	1.09	4.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	2.128	11.366	0.000	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	252	809	0	0	0	0	-1
normalized size	1	1.00	1.37	4.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.479	2.085	10.569	0.000	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	194	788	0	0	0	0	-1
normalized size	1	1.00	1.16	4.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.419	4.214	8.365	0.000	0.000	0.000	0.000	0.000

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	229	707	0	0	0	0	-1
normalized size	1	1.00	1.55	4.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.394	3.267	8.278	0.000	0.000	0.000	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	239	608	0	0	0	0	-1
normalized size	1	1.00	1.55	3.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.445	3.835	6.375	0.000	0.000	0.000	0.000	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	278	868	0	0	0	0	-1
normalized size	1	1.00	1.27	3.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.684	3.569	10.828	0.000	0.000	0.000	0.000	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	353	2216	0	0	0	0	-1
normalized size	1	1.00	1.02	6.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.057	4.347	17.272	0.000	0.000	0.000	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	311	1957	0	0	0	0	-1
normalized size	1	1.00	1.10	6.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.807	3.266	15.624	0.000	136.713	0.000	0.000	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	286	1936	0	0	0	0	-1
normalized size	1	1.00	1.09	7.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	3.148	13.445	0.000	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	272	1858	0	0	0	0	-1
normalized size	1	1.00	1.11	7.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	2.233	13.896	0.000	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	289	1760	0	0	0	0	-1
normalized size	1	1.00	1.14	6.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.721	3.187	13.650	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	297	1203	0	0	0	0	-1
normalized size	1	1.00	1.16	4.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.746	3.162	8.638	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	334	2014	0	0	0	0	-1
normalized size	1	1.00	1.02	6.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.000	3.743	17.374	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	340	1724	0	0	0	0	-1
normalized size	1	1.00	1.39	7.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.659	9.580	1.450	0.000	1.454	0.000	0.000	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	273	1011	0	0	0	0	-1
normalized size	1	1.00	1.42	5.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.432	8.897	1.487	0.000	1.051	0.000	0.000	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	198	923	0	0	0	0	-1
normalized size	1	1.00	2.96	13.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	3.876	1.286	0.000	0.778	0.000	0.000	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	14885	275	0	0	0	0	-1
normalized size	1	1.00	107.86	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	28.841	1.219	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	33277	780	0	0	0	0	-1
normalized size	1	1.00	140.41	3.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.690	31.537	1.267	0.000	0.000	0.000	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	383	2040	0	0	0	0	-1
normalized size	1	1.00	1.26	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.940	11.587	1.416	0.000	1.894	0.000	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	344	1695	0	0	0	0	-1
normalized size	1	1.00	1.43	7.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.681	9.261	1.490	0.000	2.346	0.000	0.000	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	284	1209	0	0	0	0	-1
normalized size	1	1.00	1.52	6.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.469	7.441	1.474	0.000	0.606	0.000	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	25369	1365	0	0	0	0	-1
normalized size	1	1.00	121.38	6.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	30.214	1.317	0.000	2.608	0.000	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	34674	1205	0	0	0	0	-1
normalized size	1	1.00	139.25	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	31.925	1.286	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	51315	1742	0	0	0	0	-1
normalized size	1	1.00	171.62	5.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	32.868	1.180	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	477	2778	0	0	0	0	-1
normalized size	1	1.00	1.31	7.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.328	14.580	1.525	0.000	1.235	0.000	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	419	2040	0	0	0	0	-1
normalized size	1	1.00	1.38	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.006	12.137	1.276	0.000	1.561	0.000	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	391	1921	0	0	0	0	-1
normalized size	1	1.00	1.64	8.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.756	12.492	1.337	0.000	0.953	0.000	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	36372	1651	0	0	0	0	-1
normalized size	1	1.00	138.82	6.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.853	32.885	1.262	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	44191	1947	0	0	0	0	-1
normalized size	1	1.00	168.03	7.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.849	32.170	1.273	0.000	2.867	0.000	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	52888	1972	0	0	0	0	-1
normalized size	1	1.00	168.43	6.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.152	32.331	1.305	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	61979	2285	0	0	0	0	-1
normalized size	1	1.00	167.96	6.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.421	32.641	1.169	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	340	1726	0	0	0	0	-1
normalized size	1	1.00	1.37	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.651	9.677	1.292	0.000	0.926	0.000	0.000	0.000

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	265	1014	0	0	0	0	-1
normalized size	1	1.00	1.36	5.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.439	7.643	1.469	0.000	0.875	0.000	0.000	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	216	732	0	0	0	0	-1
normalized size	1	1.00	1.52	5.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.307	4.026	1.415	0.000	1.234	0.000	0.000	0.000

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	102	163	0	0	0	0	-1
normalized size	1	1.00	1.52	2.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.893	1.344	0.000	0.700	0.000	0.000	0.000

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	14986	206	0	0	0	0	-1
normalized size	1	1.00	220.38	3.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	29.052	1.298	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	21698	986	0	0	0	0	-1
normalized size	1	1.00	88.20	4.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	30.992	1.457	0.000	0.000	0.000	0.000	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	51323	1745	0	0	0	0	-1
normalized size	1	1.00	164.50	5.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.952	32.307	1.339	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	419	1851	0	0	0	0	-1
normalized size	1	1.00	1.16	5.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.050	12.751	1.431	0.000	1.384	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	382	1307	0	0	0	0	-1
normalized size	1	1.00	1.32	4.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.752	9.316	1.543	0.000	1.074	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	330	997	0	0	0	0	-1
normalized size	1	1.00	1.54	4.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	9.659	1.486	0.000	1.015	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	245	502	0	0	0	0	-1
normalized size	1	1.00	1.22	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	7.801	1.409	0.000	0.928	0.000	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	260	491	0	0	0	0	-1
normalized size	1	1.00	2.06	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	7.845	1.255	0.000	0.984	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	47811	1134	0	0	0	0	-1
normalized size	1	1.00	232.09	5.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	32.271	1.350	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	51610	1492	0	0	0	0	-1
normalized size	1	1.00	149.59	4.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.089	32.565	1.423	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	527	3604	0	0	0	0	-1
normalized size	1	1.00	1.35	9.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.095	14.711	1.523	0.000	1.221	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	507	3101	0	0	0	0	-1
normalized size	1	1.00	1.60	9.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.822	14.591	1.505	0.000	1.161	0.000	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	398	2062	0	0	0	0	-1
normalized size	1	1.00	1.32	6.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.740	9.243	1.528	0.000	1.346	0.000	0.000	0.000

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	447	1804	0	0	0	0	-1
normalized size	1	1.00	1.59	6.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	11.704	1.312	0.000	1.221	0.000	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	311	1333	0	0	0	0	-1
normalized size	1	1.00	1.12	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	9.518	1.512	0.000	1.390	0.000	0.000	0.000

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	92128	3844	0	0	0	0	-1
normalized size	1	1.00	248.99	10.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.205	33.468	1.385	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	222	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.745	11.273	0.000	0.449	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	161	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.430	8.810	0.000	2.008	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.129	3.038	0.000	1.113	0.000	0.000	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5216	0	0	0	0	0	-1
normalized size	1	1.00	26.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	25.728	2.541	0.000	0.437	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	14144	0	0	0	0	0	-1
normalized size	1	1.00	45.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	45.576	1.104	0.000	0.681	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [661] had the largest ratio of [.5600]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	19	0.210
2	A	5	4	1.00	19	0.210
3	A	5	5	1.00	19	0.263
4	A	4	4	1.00	17	0.235
5	A	2	1	1.00	10	0.100
6	A	3	3	1.00	17	0.176
7	A	4	4	1.00	19	0.210
8	A	5	4	1.00	19	0.210
9	A	6	4	1.00	19	0.210
10	A	7	5	1.00	21	0.238
11	A	6	5	1.00	21	0.238
12	A	6	6	1.00	21	0.286
13	A	5	5	1.00	19	0.263
14	A	4	4	1.00	12	0.333
15	A	4	4	1.00	19	0.210
16	A	4	4	1.00	21	0.190
17	A	6	5	1.00	21	0.238
18	A	6	5	1.00	21	0.238
19	A	8	6	1.00	21	0.286
20	A	11	4	1.00	21	0.190
21	A	11	5	1.00	21	0.238
22	A	9	5	1.00	19	0.263
23	A	5	5	1.00	12	0.417
24	A	6	5	1.00	19	0.263
25	A	6	5	1.00	21	0.238
26	A	7	5	1.00	21	0.238
27	A	10	5	1.00	21	0.238
28	A	11	4	1.00	21	0.190
29	A	13	4	1.00	21	0.190
30	A	15	4	1.00	21	0.190
31	A	13	5	1.00	21	0.238
32	A	12	5	1.00	19	0.263
33	A	6	6	1.00	12	0.500
34	A	8	6	1.00	19	0.316
35	A	8	6	1.00	21	0.286

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	8	6	1.00	21	0.286
37	A	10	5	1.00	21	0.238
38	A	12	5	1.00	21	0.238
39	A	15	4	1.00	21	0.190
40	A	15	4	1.00	21	0.190
41	A	17	4	1.00	21	0.190
42	A	6	5	1.00	21	0.238
43	A	6	6	1.00	21	0.286
44	A	4	4	1.00	21	0.190
45	A	3	3	1.00	21	0.143
46	A	1	1	1.00	19	0.053
47	A	2	2	1.00	12	0.167
48	A	4	4	1.00	19	0.210
49	A	5	5	1.00	21	0.238
50	A	6	5	1.00	21	0.238
51	A	7	5	1.00	21	0.238
52	A	7	7	1.00	21	0.333
53	A	6	6	1.00	21	0.286
54	A	4	4	1.00	21	0.190
55	A	2	2	1.00	21	0.095
56	A	2	2	1.00	19	0.105
57	A	3	3	1.00	12	0.250
58	A	5	5	1.00	19	0.263
59	A	6	6	1.00	21	0.286
60	A	7	6	1.00	21	0.286
61	A	8	7	1.00	21	0.333
62	A	7	7	1.00	21	0.333
63	A	5	5	1.00	21	0.238
64	A	3	3	1.00	21	0.143
65	A	3	3	1.00	21	0.143
66	A	3	2	1.00	19	0.105
67	A	4	4	1.00	12	0.333
68	A	6	5	1.00	19	0.263
69	A	7	6	1.00	21	0.286
70	A	9	7	1.00	21	0.333
71	A	8	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	6	6	1.00	21	0.286
73	A	4	4	1.00	21	0.190
74	A	4	4	1.00	21	0.190
75	A	4	3	1.00	21	0.143
76	A	4	2	1.00	19	0.105
77	A	5	4	1.00	12	0.333
78	A	7	5	1.00	19	0.263
79	A	8	6	1.00	21	0.286
80	A	9	7	1.00	21	0.333
81	A	7	6	1.00	21	0.286
82	A	5	4	1.00	21	0.190
83	A	5	5	1.00	21	0.238
84	A	5	4	1.00	21	0.190
85	A	5	3	1.00	21	0.143
86	A	5	2	1.00	19	0.105
87	A	6	4	1.00	12	0.333
88	A	8	5	1.00	19	0.263
89	A	9	6	1.00	21	0.286
90	A	4	4	1.00	23	0.174
91	A	3	3	1.00	23	0.130
92	A	2	2	1.00	23	0.087
93	A	1	1	1.00	21	0.048
94	A	2	2	1.00	14	0.143
95	A	3	3	1.00	21	0.143
96	A	4	3	1.00	23	0.130
97	A	5	3	1.00	23	0.130
98	A	6	3	1.00	23	0.130
99	A	6	6	1.00	23	0.261
100	A	4	4	1.00	23	0.174
101	A	3	3	1.00	23	0.130
102	A	2	2	1.00	21	0.095
103	A	4	4	1.00	14	0.286
104	A	5	5	1.00	21	0.238
105	A	5	5	1.00	23	0.217
106	A	6	5	1.00	23	0.217
107	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
108	A	5	4	1.00	23	0.174
109	A	4	3	1.00	23	0.130
110	A	3	2	1.00	21	0.095
111	A	5	5	1.00	14	0.357
112	A	4	4	1.00	21	0.190
113	A	4	4	1.00	23	0.174
114	A	5	5	1.00	23	0.217
115	A	6	5	1.00	23	0.217
116	A	1	1	1.00	22	0.045
117	A	2	2	1.00	15	0.133
118	A	3	3	1.00	22	0.136
119	A	5	5	1.00	23	0.217
120	A	4	4	1.00	23	0.174
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	21	0.095
123	A	5	4	1.00	14	0.286
124	A	6	5	1.00	21	0.238
125	A	7	6	1.00	23	0.261
126	A	6	6	1.00	23	0.261
127	A	5	5	1.00	23	0.217
128	A	4	4	1.00	23	0.174
129	A	3	3	1.00	23	0.130
130	A	3	3	1.00	21	0.143
131	A	6	5	1.00	14	0.357
132	A	7	6	1.00	21	0.286
133	A	8	6	1.00	23	0.261
134	A	6	6	1.00	23	0.261
135	A	5	5	1.00	23	0.217
136	A	4	4	1.00	23	0.174
137	A	4	4	1.00	23	0.174
138	A	4	3	1.00	21	0.143
139	A	7	6	1.00	14	0.429
140	A	8	7	1.00	21	0.333
141	A	2	2	1.00	22	0.091
142	A	5	4	1.00	15	0.267
143	A	7	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	6	6	1.00	23	0.261
145	A	5	5	1.00	21	0.238
146	A	3	3	1.00	14	0.214
147	A	3	3	1.00	21	0.143
148	A	8	7	1.00	23	0.304
149	A	7	6	1.00	23	0.261
150	A	6	5	1.00	21	0.238
151	A	3	3	1.00	14	0.214
152	A	3	3	1.00	21	0.143
153	A	7	7	1.00	23	0.304
154	A	6	6	1.00	23	0.261
155	A	5	5	1.00	23	0.217
156	A	4	4	1.00	21	0.190
157	A	3	3	1.00	14	0.214
158	A	3	3	1.00	21	0.143
159	A	9	9	1.00	23	0.391
160	A	8	8	1.00	23	0.348
161	A	8	8	1.00	23	0.348
162	A	8	7	1.00	21	0.333
163	A	3	3	1.00	14	0.214
164	A	3	3	1.00	21	0.143
165	A	8	5	1.00	21	0.238
166	A	7	5	1.00	21	0.238
167	A	6	5	1.00	21	0.238
168	A	5	4	1.00	21	0.190
169	A	6	5	1.00	21	0.238
170	A	7	5	1.00	21	0.238
171	A	8	5	1.00	21	0.238
172	A	9	6	1.00	23	0.261
173	A	8	6	1.00	23	0.261
174	A	7	6	1.00	23	0.261
175	A	4	4	1.00	23	0.174
176	A	6	5	1.00	23	0.217
177	A	7	6	1.00	23	0.261
178	A	8	6	1.00	23	0.261
179	A	16	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	14	5	1.00	23	0.217
181	A	12	5	1.00	23	0.217
182	A	12	6	1.00	23	0.261
183	A	12	5	1.00	23	0.217
184	A	14	5	1.00	23	0.217
185	A	16	5	1.00	23	0.217
186	A	21	5	1.00	23	0.217
187	A	18	5	1.00	23	0.217
188	A	16	5	1.00	23	0.217
189	A	15	6	1.00	23	0.261
190	A	15	6	1.00	23	0.261
191	A	16	5	1.00	23	0.217
192	A	18	5	1.00	23	0.217
193	A	21	5	1.00	23	0.217
194	A	8	6	1.00	23	0.261
195	A	7	6	1.00	23	0.261
196	A	6	5	1.00	23	0.217
197	A	6	5	1.00	23	0.217
198	A	6	5	1.00	23	0.217
199	A	7	6	1.00	23	0.261
200	A	8	6	1.00	23	0.261
201	A	9	7	1.00	23	0.304
202	A	8	7	1.00	23	0.304
203	A	7	6	1.00	23	0.261
204	A	4	4	1.00	23	0.174
205	A	7	6	1.00	23	0.261
206	A	7	6	1.00	23	0.261
207	A	8	7	1.00	23	0.304
208	A	9	7	1.00	23	0.304
209	A	10	7	1.00	23	0.304
210	A	9	7	1.00	23	0.304
211	A	8	6	1.00	23	0.261
212	A	8	7	1.00	23	0.304
213	A	8	7	1.00	23	0.304
214	A	8	7	1.00	23	0.304
215	A	8	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	9	7	1.00	23	0.304
217	A	10	7	1.00	23	0.304
218	A	4	3	1.00	25	0.120
219	A	3	3	1.00	25	0.120
220	A	2	2	1.00	25	0.080
221	A	1	1	1.00	25	0.040
222	A	2	2	1.00	25	0.080
223	A	3	2	1.00	25	0.080
224	A	4	2	1.00	25	0.080
225	A	6	5	1.00	25	0.200
226	A	5	5	1.00	25	0.200
227	A	4	4	1.00	25	0.160
228	A	4	4	1.00	25	0.160
229	A	2	2	1.00	25	0.080
230	A	3	3	1.00	25	0.120
231	A	5	4	1.00	25	0.160
232	A	6	4	1.00	25	0.160
233	A	6	5	1.00	25	0.200
234	A	5	5	1.00	25	0.200
235	A	4	4	1.00	25	0.160
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	25	0.160
238	A	3	2	1.00	25	0.080
239	A	4	3	1.00	25	0.120
240	A	5	4	1.00	25	0.160
241	A	6	4	1.00	25	0.160
242	A	2	2	1.00	25	0.080
243	A	2	2	1.00	25	0.080
244	A	2	2	1.00	28	0.071
245	A	6	6	1.00	25	0.240
246	A	5	5	1.00	25	0.200
247	A	2	2	1.00	25	0.080
248	A	3	3	1.00	25	0.120
249	A	4	4	1.00	25	0.160
250	A	5	5	1.00	25	0.200
251	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
252	A	6	6	1.00	25	0.240
253	A	3	3	1.00	25	0.120
254	A	3	3	1.00	25	0.120
255	A	4	4	1.00	25	0.160
256	A	5	5	1.00	25	0.200
257	A	6	5	1.00	25	0.200
258	A	8	8	1.00	25	0.320
259	A	7	7	1.00	25	0.280
260	A	4	3	1.00	25	0.120
261	A	4	4	1.00	25	0.160
262	A	4	4	1.00	25	0.160
263	A	5	5	1.00	25	0.200
264	A	6	6	1.00	25	0.240
265	A	7	6	1.00	23	0.261
266	A	6	5	1.00	23	0.217
267	A	5	4	1.00	23	0.174
268	A	2	2	1.00	23	0.087
269	A	3	3	1.00	23	0.130
270	A	4	4	1.00	23	0.174
271	A	5	5	1.00	23	0.217
272	A	4	4	1.00	27	0.148
273	A	3	3	1.00	27	0.111
274	A	4	4	1.00	27	0.148
275	A	7	6	1.00	27	0.222
276	A	6	6	1.00	27	0.222
277	A	5	5	1.00	27	0.185
278	A	6	6	1.00	27	0.222
279	A	7	6	1.00	27	0.222
280	A	4	4	1.00	27	0.148
281	A	4	4	1.00	27	0.148
282	A	4	4	1.00	27	0.148
283	A	4	4	1.00	27	0.148
284	A	3	3	1.00	25	0.120
285	A	3	3	1.00	25	0.120
286	C	3	3	0.24	25	0.120
287	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	8	6	1.00	21	0.286
289	A	7	5	1.00	21	0.238
290	A	6	4	1.00	21	0.190
291	A	5	3	1.00	19	0.158
292	A	6	4	1.00	21	0.190
293	A	7	5	1.00	21	0.238
294	A	4	4	1.00	21	0.190
295	A	4	4	1.00	21	0.190
296	A	2	2	1.00	21	0.095
297	A	3	3	1.00	21	0.143
298	A	3	3	1.00	21	0.143
299	A	4	4	1.00	23	0.174
300	A	2	2	1.00	23	0.087
301	A	2	2	1.00	23	0.087
302	A	2	2	1.00	23	0.087
303	A	4	4	1.00	23	0.174
304	A	2	2	1.00	23	0.087
305	A	2	2	1.00	23	0.087
306	A	2	2	1.00	23	0.087
307	A	4	4	1.00	23	0.174
308	A	4	4	1.00	23	0.174
309	A	2	2	1.00	23	0.087
310	A	4	4	1.00	23	0.174
311	A	4	4	1.00	23	0.174
312	A	5	5	1.00	25	0.200
313	A	3	3	1.00	25	0.120
314	A	3	3	1.00	25	0.120
315	A	3	3	1.00	25	0.120
316	A	5	5	1.00	25	0.200
317	A	3	3	1.00	25	0.120
318	A	3	3	1.00	25	0.120
319	A	3	3	1.00	25	0.120
320	A	4	4	1.00	26	0.154
321	A	4	4	1.00	26	0.154
322	A	2	2	1.00	26	0.077
323	A	4	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
324	A	4	4	1.00	26	0.154
325	A	5	5	1.00	24	0.208
326	A	3	3	1.00	24	0.125
327	A	5	5	1.00	26	0.192
328	A	3	3	1.00	26	0.115
329	A	2	2	1.00	19	0.105
330	A	2	2	1.00	21	0.095
331	A	3	3	1.00	21	0.143
332	A	3	3	1.00	22	0.136
333	A	2	2	1.00	21	0.095
334	A	2	2	1.00	23	0.087
335	A	3	3	1.00	23	0.130
336	A	3	3	1.00	24	0.125
337	A	2	2	1.00	21	0.095
338	A	2	2	1.00	23	0.087
339	A	3	3	1.00	23	0.130
340	A	3	3	1.00	24	0.125
341	A	6	6	1.00	21	0.286
342	A	5	5	1.00	21	0.238
343	A	4	4	1.00	21	0.190
344	A	3	3	1.00	19	0.158
345	A	3	3	1.00	12	0.250
346	A	3	3	1.00	19	0.158
347	A	3	3	1.00	25	0.120
348	A	3	3	1.00	25	0.120
349	A	3	3	1.00	25	0.120
350	A	3	3	1.00	25	0.120
351	A	7	5	1.00	21	0.238
352	A	6	5	1.00	21	0.238
353	A	5	5	1.00	21	0.238
354	A	4	4	1.00	21	0.190
355	A	5	5	1.00	21	0.238
356	A	6	5	1.00	21	0.238
357	A	7	5	1.00	21	0.238
358	A	8	5	1.00	21	0.238
359	A	10	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	9	7	1.00	23	0.304
361	A	8	7	1.00	23	0.304
362	A	7	6	1.00	23	0.261
363	A	5	5	1.00	23	0.217
364	A	8	7	1.00	23	0.304
365	A	9	7	1.00	23	0.304
366	A	10	7	1.00	23	0.304
367	A	17	6	1.00	23	0.261
368	A	15	6	1.00	23	0.261
369	A	13	6	1.00	23	0.261
370	A	13	7	1.00	23	0.304
371	A	13	6	1.00	23	0.261
372	A	15	6	1.00	23	0.261
373	A	17	6	1.00	23	0.261
374	A	9	7	1.00	23	0.304
375	A	8	7	1.00	23	0.304
376	A	7	6	1.00	23	0.261
377	A	7	6	1.00	23	0.261
378	A	7	6	1.00	23	0.261
379	A	8	7	1.00	23	0.304
380	A	9	7	1.00	23	0.304
381	A	10	8	1.00	23	0.348
382	A	9	8	1.00	23	0.348
383	A	8	7	1.00	23	0.304
384	A	8	7	1.00	23	0.304
385	A	5	5	1.00	23	0.217
386	A	8	7	1.00	23	0.304
387	A	9	8	1.00	23	0.348
388	A	10	8	1.00	23	0.348
389	A	11	8	1.00	23	0.348
390	A	10	8	1.00	23	0.348
391	A	9	7	1.00	23	0.304
392	A	9	8	1.00	23	0.348
393	A	9	8	1.00	23	0.348
394	A	9	8	1.00	23	0.348
395	A	9	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	10	8	1.00	23	0.348
397	A	11	8	1.00	23	0.348
398	A	5	3	1.00	25	0.120
399	A	4	3	1.00	25	0.120
400	A	3	3	1.00	25	0.120
401	A	2	2	1.00	25	0.080
402	A	3	3	1.00	25	0.120
403	A	4	4	1.00	25	0.160
404	A	5	4	1.00	25	0.160
405	A	6	5	1.00	25	0.200
406	A	4	4	1.00	25	0.160
407	A	3	3	1.00	25	0.120
408	A	5	5	1.00	25	0.200
409	A	5	5	1.00	25	0.200
410	A	6	6	1.00	25	0.240
411	A	7	6	1.00	25	0.240
412	A	6	5	1.00	25	0.200
413	A	5	4	1.00	25	0.160
414	A	4	3	1.00	25	0.120
415	A	5	5	1.00	25	0.200
416	A	5	5	1.00	25	0.200
417	A	5	5	1.00	25	0.200
418	A	6	6	1.00	25	0.240
419	A	7	6	1.00	25	0.240
420	A	6	6	1.00	25	0.240
421	A	5	5	1.00	25	0.200
422	A	4	4	1.00	25	0.160
423	A	3	3	1.36	25	0.120
424	A	6	6	1.00	25	0.240
425	A	7	7	1.00	25	0.280
426	A	8	8	1.00	25	0.320
427	A	7	6	1.00	25	0.240
428	A	6	6	1.00	25	0.240
429	A	5	5	1.00	25	0.200
430	A	4	4	1.00	25	0.160
431	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	7	7	1.00	25	0.280
433	A	8	8	1.00	25	0.320
434	A	7	7	1.00	25	0.280
435	A	6	6	1.00	25	0.240
436	A	5	5	1.00	25	0.200
437	A	5	5	1.00	25	0.200
438	A	5	4	1.00	25	0.160
439	A	8	8	1.00	25	0.320
440	A	9	9	1.00	25	0.360
441	A	8	6	1.00	23	0.261
442	A	7	5	1.00	23	0.217
443	A	5	4	1.00	21	0.190
444	A	7	5	1.00	23	0.217
445	A	8	6	1.00	23	0.261
446	A	6	4	1.00	19	0.210
447	A	5	4	1.00	19	0.210
448	A	5	5	1.00	19	0.263
449	A	4	4	1.00	17	0.235
450	A	2	1	1.00	10	0.100
451	A	3	3	1.00	17	0.176
452	A	4	4	1.00	19	0.210
453	A	5	4	1.00	19	0.210
454	A	6	4	1.00	19	0.210
455	A	6	4	1.00	19	0.210
456	A	7	5	1.00	21	0.238
457	A	6	5	1.00	21	0.238
458	A	6	6	1.00	21	0.286
459	A	5	5	1.00	19	0.263
460	A	4	4	1.00	12	0.333
461	A	4	4	1.00	19	0.210
462	A	4	4	1.00	21	0.190
463	A	6	5	1.00	21	0.238
464	A	6	5	1.00	21	0.238
465	A	8	6	1.00	21	0.286
466	A	8	7	1.00	21	0.333
467	A	7	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
468	A	6	6	1.00	19	0.316
469	A	5	4	1.00	12	0.333
470	A	5	5	1.00	19	0.263
471	A	5	5	1.00	21	0.238
472	A	5	5	1.00	21	0.238
473	A	7	6	1.00	21	0.286
474	A	7	6	1.00	21	0.286
475	A	9	7	1.00	21	0.333
476	A	9	7	1.00	21	0.333
477	A	8	7	1.00	21	0.333
478	A	7	7	1.00	19	0.368
479	A	6	5	1.00	12	0.417
480	A	6	6	1.00	19	0.316
481	A	6	6	1.00	21	0.286
482	A	6	6	1.00	21	0.286
483	A	6	6	1.00	21	0.286
484	A	8	7	1.00	21	0.333
485	A	8	7	1.00	21	0.333
486	A	7	6	1.00	12	0.500
487	A	8	8	1.00	21	0.381
488	A	7	7	1.00	21	0.333
489	A	6	6	1.00	21	0.286
490	A	5	5	1.00	21	0.238
491	A	3	3	1.00	19	0.158
492	A	3	3	1.00	12	0.250
493	A	5	5	1.00	19	0.263
494	A	6	6	1.00	21	0.286
495	A	7	6	1.00	21	0.286
496	A	8	6	1.00	21	0.286
497	A	8	8	1.00	21	0.381
498	A	7	7	1.00	21	0.333
499	A	6	6	1.00	21	0.286
500	A	5	5	1.00	21	0.238
501	A	5	5	1.00	19	0.263
502	A	5	5	1.00	12	0.417
503	A	6	6	1.00	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	7	6	1.00	21	0.286
505	A	8	6	1.00	21	0.286
506	A	8	8	1.00	21	0.381
507	A	7	7	1.00	21	0.333
508	A	6	6	1.00	21	0.286
509	A	6	6	1.00	21	0.286
510	A	6	6	1.00	19	0.316
511	A	6	6	1.00	12	0.500
512	A	7	7	1.00	19	0.368
513	A	8	7	1.00	21	0.333
514	A	9	9	1.00	21	0.429
515	A	8	8	1.00	21	0.381
516	A	7	7	1.00	21	0.333
517	A	7	6	1.00	21	0.286
518	A	7	6	1.00	21	0.286
519	A	7	6	1.00	19	0.316
520	A	7	6	1.00	12	0.500
521	A	8	7	1.00	19	0.368
522	A	9	7	1.00	21	0.333
523	A	2	2	1.00	12	0.167
524	A	4	4	1.00	12	0.333
525	A	5	5	1.00	12	0.417
526	A	6	5	1.00	12	0.417
527	A	3	3	1.00	12	0.250
528	A	5	5	1.00	12	0.417
529	A	6	6	1.00	12	0.500
530	A	7	6	1.00	12	0.500
531	A	5	5	1.00	23	0.217
532	A	4	4	1.00	23	0.174
533	A	3	3	1.00	21	0.143
534	A	1	1	1.00	14	0.071
535	A	6	6	1.00	21	0.286
536	A	7	7	1.00	23	0.304
537	A	7	6	1.00	23	0.261
538	A	6	5	1.00	23	0.217
539	A	5	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
540	A	4	4	1.00	21	0.190
541	A	5	5	1.00	14	0.357
542	A	6	6	1.00	21	0.286
543	A	7	7	1.00	23	0.304
544	A	8	6	1.00	23	0.261
545	A	7	5	1.00	23	0.217
546	A	6	5	1.00	23	0.217
547	A	5	5	1.00	21	0.238
548	A	6	6	1.00	14	0.429
549	A	6	6	1.00	21	0.286
550	A	7	7	1.00	23	0.304
551	A	8	7	1.00	23	0.304
552	A	9	7	1.00	23	0.304
553	A	7	7	1.00	14	0.500
554	A	6	6	1.00	23	0.261
555	A	5	5	1.00	23	0.217
556	A	4	4	1.00	23	0.174
557	A	3	3	1.00	23	0.130
558	A	1	1	1.00	21	0.048
559	A	1	1	1.00	14	0.071
560	A	6	6	1.00	21	0.286
561	A	7	7	1.00	23	0.304
562	A	6	6	1.00	23	0.261
563	A	5	5	1.00	23	0.217
564	A	4	4	1.00	23	0.174
565	A	4	4	1.00	23	0.174
566	A	5	5	1.00	21	0.238
567	A	6	6	1.00	14	0.429
568	A	7	7	1.00	21	0.333
569	A	8	8	1.00	23	0.348
570	A	6	6	1.00	23	0.261
571	A	5	5	1.00	23	0.217
572	A	5	5	1.00	23	0.217
573	A	5	5	1.00	23	0.217
574	A	5	5	1.00	21	0.238
575	A	7	7	1.00	14	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
576	A	8	8	1.00	21	0.381
577	A	9	8	1.00	23	0.348
578	A	8	7	1.00	14	0.500
579	A	8	5	1.00	21	0.238
580	A	7	5	1.00	21	0.238
581	A	6	5	1.00	21	0.238
582	A	5	4	1.00	21	0.190
583	A	6	5	1.00	21	0.238
584	A	7	5	1.00	21	0.238
585	A	8	5	1.00	21	0.238
586	A	9	6	1.00	23	0.261
587	A	8	6	1.00	23	0.261
588	A	7	6	1.00	23	0.261
589	A	6	5	1.00	23	0.217
590	A	6	5	1.00	23	0.217
591	A	7	6	1.00	23	0.261
592	A	8	6	1.00	23	0.261
593	A	9	7	1.00	23	0.304
594	A	8	7	1.00	23	0.304
595	A	7	6	1.00	23	0.261
596	A	7	6	1.00	23	0.261
597	A	7	6	1.00	23	0.261
598	A	8	7	1.00	23	0.304
599	A	9	7	1.00	23	0.304
600	A	10	8	1.00	23	0.348
601	A	9	8	1.00	23	0.348
602	A	8	7	1.00	23	0.304
603	A	8	7	1.00	23	0.304
604	A	8	7	1.00	23	0.304
605	A	8	7	1.00	23	0.304
606	A	9	8	1.00	23	0.348
607	A	10	8	1.00	23	0.348
608	A	10	9	1.00	23	0.391
609	A	6	6	1.00	23	0.261
610	A	2	2	1.00	23	0.087
611	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
612	A	8	7	1.00	23	0.304
613	A	9	8	1.00	23	0.348
614	A	11	9	1.00	23	0.391
615	A	10	9	1.00	23	0.391
616	A	9	8	1.00	23	0.348
617	A	9	8	1.00	23	0.348
618	A	9	8	1.00	23	0.348
619	A	9	8	1.00	23	0.348
620	A	10	9	1.00	23	0.391
621	A	11	10	1.00	23	0.435
622	A	10	9	1.00	23	0.391
623	A	10	9	1.00	23	0.391
624	A	10	9	1.00	23	0.391
625	A	10	9	1.00	23	0.391
626	A	10	9	1.00	23	0.391
627	A	11	10	1.00	23	0.435
628	A	12	12	1.00	25	0.480
629	A	7	7	1.00	25	0.280
630	A	3	3	1.00	25	0.120
631	A	8	8	1.00	25	0.320
632	A	9	9	1.00	25	0.360
633	A	10	9	1.00	25	0.360
634	A	13	13	1.00	25	0.520
635	A	12	12	1.00	25	0.480
636	A	11	11	1.00	25	0.440
637	A	8	8	1.00	25	0.320
638	A	9	9	1.00	25	0.360
639	A	10	9	1.00	25	0.360
640	A	14	13	1.00	25	0.520
641	A	13	13	1.00	25	0.520
642	A	12	12	1.00	25	0.480
643	A	12	12	1.00	25	0.480
644	A	9	9	1.00	25	0.360
645	A	10	9	1.00	25	0.360
646	A	11	9	1.00	25	0.360
647	A	13	13	1.00	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
648	A	12	12	1.00	25	0.480
649	A	3	3	1.00	25	0.120
650	A	3	3	1.00	25	0.120
651	A	7	7	1.00	25	0.280
652	A	8	8	1.00	25	0.320
653	A	9	9	1.00	25	0.360
654	A	13	13	1.00	25	0.520
655	A	9	9	1.00	25	0.360
656	A	5	5	1.00	25	0.200
657	A	8	8	1.00	25	0.320
658	A	8	8	1.00	25	0.320
659	A	9	9	1.00	25	0.360
660	A	10	9	1.00	25	0.360
661	A	14	14	1.00	25	0.560
662	A	13	13	1.00	25	0.520
663	A	9	9	1.00	25	0.360
664	A	9	9	1.00	25	0.360
665	A	9	9	1.00	25	0.360
666	A	9	9	1.00	25	0.360
667	A	10	10	1.00	25	0.400
668	A	11	10	1.00	25	0.400
669	A	5	5	1.00	25	0.200
670	A	5	5	1.00	25	0.200
671	A	7	7	1.00	25	0.280
672	A	7	7	1.00	25	0.280
673	A	5	5	1.00	25	0.200
674	A	5	5	1.00	25	0.200
675	A	5	5	1.00	25	0.200
676	A	5	5	1.00	25	0.200
677	A	2	2	1.00	25	0.080
678	A	2	2	1.00	25	0.080
679	A	3	3	1.00	25	0.120
680	A	3	3	1.00	25	0.120
681	A	2	2	1.00	25	0.080
682	A	2	2	1.00	25	0.080
683	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
684	A	2	2	1.00	25	0.080
685	A	3	3	1.00	21	0.143
686	A	0	0	0.00	0	0.000
687	A	10	7	1.00	23	0.304
688	A	9	6	1.00	23	0.261
689	A	8	5	1.00	23	0.217
690	A	3	3	1.00	21	0.143
691	A	0	0	0.00	0	0.000
692	A	3	3	1.00	21	0.143
693	A	0	0	0.00	0	0.000
694	A	11	7	1.00	23	0.304
695	A	10	6	1.00	23	0.261
696	A	9	6	1.00	23	0.261
697	A	3	3	1.00	21	0.143
698	A	0	0	0.00	0	0.000
699	A	9	6	1.00	23	0.261
700	A	8	5	1.00	23	0.217
701	A	7	4	1.00	23	0.174
702	A	3	3	1.00	21	0.143
703	A	0	0	0.00	0	0.000
704	A	3	3	1.00	21	0.143
705	A	0	0	0.00	0	0.000
706	A	3	3	1.00	21	0.143
707	A	0	0	0.00	0	0.000
708	A	9	6	1.00	23	0.261
709	A	8	5	1.00	23	0.217
710	A	8	5	1.00	23	0.217
711	A	3	3	1.00	21	0.143
712	A	0	0	0.00	0	0.000
713	A	6	4	1.00	23	0.174
714	A	6	4	1.00	23	0.174
715	A	6	4	1.00	23	0.174
716	A	6	4	1.00	23	0.174
717	A	0	0	0.00	0	0.000
718	A	0	0	0.00	0	0.000
719	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
720	A	0	0	0.00	0	0.000
721	A	0	0	0.00	0	0.000
722	A	0	0	0.00	0	0.000
723	A	0	0	0.00	0	0.000
724	A	0	0	0.00	0	0.000
725	A	0	0	0.00	0	0.000
726	A	0	0	0.00	0	0.000
727	A	0	0	0.00	0	0.000
728	A	0	0	0.00	0	0.000
729	A	0	0	0.00	0	0.000
730	A	0	0	0.00	0	0.000
731	A	0	0	0.00	0	0.000
732	A	0	0	0.00	0	0.000
733	A	0	0	0.00	0	0.000
734	A	0	0	0.00	0	0.000
735	A	0	0	0.00	0	0.000
736	A	0	0	0.00	0	0.000
737	A	0	0	0.00	0	0.000
738	A	0	0	0.00	0	0.000
739	A	0	0	0.00	0	0.000
740	A	0	0	0.00	0	0.000
741	A	0	0	0.00	0	0.000
742	A	0	0	0.00	0	0.000
743	A	0	0	0.00	0	0.000
744	A	0	0	0.00	0	0.000
745	A	0	0	0.00	0	0.000
746	A	0	0	0.00	0	0.000
747	A	0	0	0.00	0	0.000
748	A	0	0	0.00	0	0.000
749	A	0	0	0.00	0	0.000
750	A	0	0	0.00	0	0.000
751	A	0	0	0.00	0	0.000
752	A	0	0	0.00	0	0.000
753	A	0	0	0.00	0	0.000
754	A	0	0	0.00	0	0.000
755	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
756	A	0	0	0.00	0	0.000
757	A	0	0	0.00	0	0.000
758	A	0	0	0.00	0	0.000
759	A	0	0	0.00	0	0.000
760	A	0	0	0.00	0	0.000
761	A	0	0	0.00	0	0.000
762	A	0	0	0.00	0	0.000
763	A	0	0	0.00	0	0.000
764	A	0	0	0.00	0	0.000
765	A	0	0	0.00	0	0.000
766	A	0	0	0.00	0	0.000
767	A	0	0	0.00	0	0.000
768	A	0	0	0.00	0	0.000
769	A	0	0	0.00	0	0.000
770	A	0	0	0.00	0	0.000
771	A	0	0	0.00	0	0.000
772	A	0	0	0.00	0	0.000
773	A	0	0	0.00	0	0.000
774	A	0	0	0.00	0	0.000
775	A	0	0	0.00	0	0.000
776	A	0	0	0.00	0	0.000
777	A	7	5	1.00	23	0.217
778	A	6	4	1.00	23	0.174
779	A	5	3	1.00	21	0.143
780	A	6	4	1.00	23	0.174
781	A	9	4	1.00	23	0.174
782	A	0	0	0.00	0	0.000
783	A	0	0	0.00	0	0.000
784	A	0	0	0.00	0	0.000
785	A	0	0	0.00	0	0.000
786	A	0	0	0.00	0	0.000
787	A	0	0	0.00	0	0.000
788	A	8	5	1.00	21	0.238
789	A	7	4	1.00	21	0.190
790	A	3	3	1.00	19	0.158
791	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
792	A	0	0	0.00	0	0.000
793	A	0	0	0.00	0	0.000
794	A	8	5	1.00	21	0.238
795	A	7	5	1.00	21	0.238
796	A	6	5	1.00	21	0.238
797	A	5	5	1.00	21	0.238
798	A	4	4	1.00	21	0.190
799	A	5	5	1.00	21	0.238
800	A	6	5	1.00	21	0.238
801	A	7	5	1.00	21	0.238
802	A	10	7	1.00	23	0.304
803	A	9	7	1.00	23	0.304
804	A	8	7	1.00	23	0.304
805	A	7	6	1.00	23	0.261
806	A	7	6	1.00	23	0.261
807	A	8	7	1.00	23	0.304
808	A	9	7	1.00	23	0.304
809	A	10	7	1.00	23	0.304
810	A	10	8	1.00	23	0.348
811	A	9	8	1.00	23	0.348
812	A	8	7	1.00	23	0.304
813	A	8	7	1.00	23	0.304
814	A	8	7	1.00	23	0.304
815	A	9	8	1.00	23	0.348
816	A	10	8	1.00	23	0.348
817	A	11	10	1.00	23	0.435
818	A	10	9	1.00	23	0.391
819	A	9	8	1.00	23	0.348
820	A	5	5	1.00	23	0.217
821	A	3	3	1.00	23	0.130
822	A	7	7	1.00	23	0.304
823	A	11	10	1.00	23	0.435
824	A	11	10	1.00	23	0.435
825	A	10	9	1.00	23	0.391
826	A	10	9	1.00	23	0.391
827	A	10	9	1.00	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
828	A	10	9	1.00	23	0.391
829	A	11	10	1.00	23	0.435
830	A	12	11	1.00	23	0.478
831	A	11	10	1.00	23	0.435
832	A	11	10	1.00	23	0.435
833	A	11	10	1.00	23	0.435
834	A	11	10	1.00	23	0.435
835	A	11	10	1.00	23	0.435
836	A	12	11	1.00	23	0.478
837	A	10	10	1.00	25	0.400
838	A	9	9	1.00	25	0.360
839	A	4	4	1.00	25	0.160
840	A	8	8	1.00	25	0.320
841	A	13	13	1.00	25	0.520
842	A	11	10	1.00	25	0.400
843	A	10	10	1.00	25	0.400
844	A	9	9	1.00	25	0.360
845	A	12	12	1.00	25	0.480
846	A	13	13	1.00	25	0.520
847	A	14	14	1.00	25	0.560
848	A	12	10	1.00	25	0.400
849	A	11	10	1.00	25	0.400
850	A	10	10	1.00	25	0.400
851	A	13	13	1.00	25	0.520
852	A	13	13	1.00	25	0.520
853	A	14	14	1.00	25	0.560
854	A	15	14	1.00	25	0.560
855	A	10	10	1.00	25	0.400
856	A	9	9	1.00	25	0.360
857	A	8	8	1.00	25	0.320
858	A	4	4	1.00	25	0.160
859	A	4	4	1.00	25	0.160
860	A	13	13	1.00	25	0.520
861	A	14	14	1.00	25	0.560
862	A	11	10	1.00	25	0.400
863	A	10	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
864	A	9	9	1.00	25	0.360
865	A	9	9	1.00	25	0.360
866	A	6	6	1.00	25	0.240
867	A	10	10	1.00	25	0.400
868	A	14	14	1.00	25	0.560
869	A	11	11	1.00	25	0.440
870	A	10	10	1.00	25	0.400
871	A	10	10	1.00	25	0.400
872	A	10	10	1.00	25	0.400
873	A	10	10	1.00	25	0.400
874	A	14	14	1.00	25	0.560
875	A	8	6	1.00	23	0.261
876	A	7	5	1.00	23	0.217
877	A	5	4	1.00	21	0.190
878	A	7	5	1.00	23	0.217
879	A	10	5	1.00	23	0.217

Chapter 3

Listing of integrals

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1/
4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^4(c+dx)\,dx + a\int \sec^5(c+dx)\,dx\\ &= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x\right.\right.}{}\\ &= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.79, size = 99, normalized size = 1.16

$$\frac{9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^3+9\,a\cos\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.46, size = 110, normalized size = 1.29

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 +
31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

maple [A] time = 0.70, size = 92, normalized size = 1.08

$$\frac{2a\tan(dx+c)}{3d}+\frac{a\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{a\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.78, size = 95, normalized size = 1.12

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3a\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.94, size = 130, normalized size = 1.53

$$\frac{-\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+\frac{49a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12}-\frac{31a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12}+\frac{13a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{3a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] ((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(c
/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2
)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^4(c+dx)\,dx+\int\sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 3768, 3770, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^3(c+dx)\,dx+a\int \sec^4(c+dx)\,dx$$
$$=\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int\left(1+x^2\right)\,d\right.}{d}$$
$$=\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\,\text{ta}}{}$$

Mathematica [A] time = 0.16, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.72, size = 88, normalized size = 1.40

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+2\,a\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.50, size = 96, normalized size = 1.52

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.70, size = 72, normalized size = 1.14

$$\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2a\tan\left(dx+c\right)}{3d}+\frac{a\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*tan(d
*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.49, size = 70, normalized size = 1.11

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.51, size = 102, normalized size = 1.62

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
(d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(
c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^3\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3787, 3767, 8, 3768, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx+a\int \sec^3(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx-\frac{a\,\text{Subst}(\int 1\,dx,x,-\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

fricas [A] time = 0.61, size = 74, normalized size = 1.57

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2a\cos(dx+c)+a)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.43, size = 80, normalized size = 1.70

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.72, size = 51, normalized size = 1.09

$$\frac{a\tan(dx+c)}{d}+\frac{a\sec(dx+c)\tan(dx+c)}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+
c))
```

maxima [A] time = 0.32, size = 58, normalized size = 1.23

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4a\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

mupad [B] time = 1.06, size = 75, normalized size = 1.60

$$\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sec^2(c+dx)\,dx+\int\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**2, x) + Integral(sec(c + d*x)**3, x))
```

3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3787, 3770, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec(c+dx)\,dx+a\int \sec^2(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

fricas [B] time = 0.67, size = 60, normalized size = 2.50

$$\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.37, size = 63, normalized size = 2.62

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.52, size = 32, normalized size = 1.33

$$\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c)/d
```

maxima [A] time = 0.63, size = 29, normalized size = 1.21

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + a*tan(d*x + c))/d
```

mupad [B] time = 0.68, size = 47, normalized size = 1.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

sympy [**A**] time = 4.74, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+a\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a\sec(c)+a\right)\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + a*tan(c + d*x))/d, Ne(d, 0
)), (x*(a*sec(c) + a)*sec(c), True))
```

3.5 $\int (a + a\sec(c + dx))\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c + dx))}{d} + ax$$

```
[Out] a*x+a*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3770}

$$\frac{a\tanh^{-1}(\sin(c + dx))}{d} + ax$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (a + a\sec(c + dx))\,dx &= ax + a\int \sec(c + dx)\,dx \\ &= ax + \frac{a\tanh^{-1}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c + dx))}{d} + ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

fricas [B] time = 1.64, size = 36, normalized size = 2.25

$$\frac{2\,adx + a\log\left(\sin\left(dx + c\right) + 1\right) - a\log\left(-\sin\left(dx + c\right) + 1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 0.34, size = 49, normalized size = 3.06

$$ax + \frac{a\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) - 2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*a*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

maple [A] time = 0.03, size = 24, normalized size = 1.50

$$ax + \frac{a \ln (\sec (dx + c) + \tan (dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sec(d*x+c),x)
```

```
[Out] a*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.65, size = 23, normalized size = 1.44

$$ax + \frac{a \log (\sec (dx + c) + \tan (dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + a*log(sec(d*x + c) + tan(d*x + c))/d
```

mupad [B] time = 0.61, size = 20, normalized size = 1.25

$$a\,x + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a/cos(c + d*x),x)
```

```
[Out] a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [A] time = 2.02, size = 41, normalized size = 2.56

$$ax + a \left(\begin{cases} \frac{\log (\tan (c+dx)+\sec (c+dx))}{d} & \text{for}\ d \neq 0 \\ \frac{x\left(\tan (c) \sec (c)+\sec^2 (c)\right)}{\tan (c)+\sec (c)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=15

$$\frac{a\sin(c+dx)}{d}+ax$$

```
[Out] a*x+a*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3787, 2637, 8}

$$\frac{a\sin(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))\,dx &= a\int 1\,dx+a\int \cos(c+dx)\,dx\\ &= ax+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.69, size = 17, normalized size = 1.13

$$\frac{adx+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*sin(d*x + c))/d
```

giac [B] time = 0.39, size = 39, normalized size = 2.60

$$\frac{(dx+c)a + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

maple [A] time = 0.48, size = 21, normalized size = 1.40

$$\frac{a\,(dx+c)+a\sin\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(d*x+c)+a*sin(d*x+c))
```

maxima [A] time = 0.36, size = 20, normalized size = 1.33

$$\frac{(dx+c)a+a\sin\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a + a*sin(d*x + c))/d
```

mupad [B] time = 0.60, size = 15, normalized size = 1.00

$$a\,x+\frac{a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x + (a*sin(c + d*x))/d
```

sympy [A] time = 2.09, size = 15, normalized size = 1.00

$$ax+a\left(\begin{cases}\sin\,(c) & \text{for}\ d=0\\ \frac{\sin\,(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*x + a*Piecewise((sin(c), Eq(d, 0)), (sin(c + d*x)/d, True))
```

3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2635, 8, 2637}

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos(c+dx)\,dx+a\int \cos^2(c+dx)\,dx\\ &= \frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.84

$$\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 1.66, size = 29, normalized size = 0.76

$$\frac{adx + (a\cos(dx + c) + 2a)\sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 56, normalized size = 1.47

$$\frac{(dx + c)a + \frac{2\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.58, size = 38, normalized size = 1.00

$$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*sin(d*x+c))
```

maxima [A] time = 0.97, size = 34, normalized size = 0.89

$$\frac{(2dx + 2c + \sin(2dx + 2c))a + 4a\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

mupad [B] time = 1.06, size = 50, normalized size = 1.32

$$\frac{ax}{2} + \frac{a\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 3a\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cos^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**2, x)
)
```

3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx+a\int \cos^3(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx-\frac{a\,\text{Subst}\left(\int\left(1-x^2\right)dx,x,-\sin(\right.}{d}\\ &= \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.78, size = 42, normalized size = 0.78

$$\frac{3\,adx + \left(2\,a\cos\left(dx + c\right)^2 + 3\,a\cos\left(dx + c\right) + 4\,a\right)\sin\left(dx + c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

giac [A] time = 0.39, size = 72, normalized size = 1.33

$$\frac{3\,(dx + c)a + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 4\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 9\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 + 4*a*tan(1/2*d*x + 1/2*
c)^3 + 9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.82, size = 49, normalized size = 0.91

$$\frac{\frac{a\left(2 + \cos^2(dx + c)\right)\sin(dx + c)}{3} + a\left(\frac{\cos(dx + c)\sin(dx + c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

maxima [A] time = 0.32, size = 46, normalized size = 0.85

$$-\frac{4\left(\sin\left(dx + c\right)^3 - 3\,\sin\left(dx + c\right)\right)a - 3\left(2\,dx + 2\,c + \sin\left(2\,dx + 2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a)/d
```

mupad [B] time = 0.65, size = 55, normalized size = 1.02

$$\frac{a\,x}{2} + \frac{2\,a\,\sin\left(c + d\,x\right)}{3\,d} + \frac{a\,\cos\left(c + d\,x\right)\,\sin\left(c + d\,x\right)}{2\,d} + \frac{a\cos\left(c + d\,x\right)^2\,\sin\left(c + d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
(a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos^3(c+dx)\sec(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**3, x)
)
```

3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] 3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2635, 8, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos^3(c+dx)\,dx+a\int \cos^4(c+dx)\,dx\\
&=\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int (1\right.}{}\\
&=\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx}{}\\
&=\frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin}{4d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.65, size = 53, normalized size = 0.70

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

giac [A] time = 0.38, size = 86, normalized size = 1.13

$$\frac{9\left(dx+c\right)a+\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 + 49*a*tan(1/2*d*x + 1/
2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 + 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.96, size = 60, normalized size = 0.79

$$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.75, size = 57, normalized size = 0.75

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 4.17, size = 79, normalized size = 1.04

$$\frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12 +
 (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^4(c+dx)\sec(c+dx)\,dx+\int\cos^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**4, x)
)
```

3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tan^3(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^3(c+$$

```
[Out] 3/4*a^2*arctanh(sin(d*x+c))/d+9/5*a^2*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(d
*x+c)/d+1/2*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d
+3/5*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3788, 3768, 3770, 4046, 3767}

$$\frac{3a^2\tan^3(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) + (9*a^2*Tan[c + d*x])/(5*d) + (3*a^2*S
ec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) +
 (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (3*a^2*Tan[c + d*x]^3)/(5*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^5(c+dx)\,dx + \int \sec^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{2}\left(3a^2\right)$$
$$= \frac{3a^2\sec(c+dx)\tan(c+dx)}{4d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c}{}$$
$$= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{9a^2\tan(c+dx)}{5d} + \frac{3a^2\sec(c+dx)\tan(c+}{4d}$$

Mathematica [B] time = 1.72, size = 487, normalized size = 3.99

$$-\frac{a^2\sec(c)\sec^5(c+dx)\left(80\sin(2c+dx)-140\sin(c+2dx)-140\sin(3c+2dx)-240\sin(2c+3dx)-30\sin(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/640*(a^2*Sec[c]*Sec[c + d*x]^5*(75*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1
5*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 150*Cos[d*x]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) + 150*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 75*Cos[2*c + 3*d*x]*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] - 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] - 15*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 400
*Sin[d*x] + 80*Sin[2*c + d*x] - 140*Sin[c + 2*d*x] - 140*Sin[3*c + 2*d*x] -
 240*Sin[2*c + 3*d*x] - 30*Sin[3*c + 4*d*x] - 30*Sin[5*c + 4*d*x] - 48*Sin[
4*c + 5*d*x]))/d
```

fricas [A] time = 0.60, size = 124, normalized size = 1.02

$$\frac{15\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^2\cos\left(dx+c\right)^4+\right.}{40\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a^2*cos(d*x + c)^5*l
og(-sin(d*x + c) + 1) + 2*(24*a^2*cos(d*x + c)^4 + 15*a^2*cos(d*x + c)^3 +
12*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

giac [A] time = 0.47, size = 138, normalized size = 1.13

$$\frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\right.}{\left(\tan\right.}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 70*a^2*tan(1/2*d*x +
1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 - 90*a^2*tan(1/2*d*x + 1/2*c)^3 +
 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.91, size = 124, normalized size = 1.02

$$\frac{6a^2\tan(dx+c)}{5d}+\frac{3a^2\left(\sec^2(dx+c)\right)\tan(dx+c)}{5d}+\frac{a^2\left(\sec^3(dx+c)\right)\tan(dx+c)}{2d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{4d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 6/5*a^2*tan(d*x+c)/d+3/5*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)^3
*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a^2*ln(sec(d*x+c)+tan(d
*x+c))+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d
```

maxima [A] time = 0.34, size = 133, normalized size = 1.09

$$\frac{8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+40\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-15a^2\left(\frac{2\left(3\sin(\right.}{\sin(dx+}\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 15*a^2*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 5.70, size = 170, normalized size = 1.39

$$\frac{3a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}-\frac{\frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2}-7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{72a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}-9a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{13a^2\,\mathrm{t}}{}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((72*a^2*tan(c/2 + (d*x)/2)^5)/5
- 9*a^2*tan(c/2 + (d*x)/2)^3 - 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2
+ (d*x)/2)^9)/2 + (13*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2)^2
 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)
^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sec^4(c+dx)\,dx+\int2\sec^5(c+dx)\,dx+\int\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**4, x) + Integral(2*sec(c + d*x)**5, x) + Integ
ral(sec(c + d*x)**6, x))
```

3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec}{8d}$$

```
[Out] 7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*x
+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3767, 4046, 3768, 3770}

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(7a^2\right)\int \sec^3(c+dx)\,dx - \frac{\left(2a^2\right)\text{Subst}(}{}$$
$$= \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+dx)\tan(c+}{4d}$$
$$= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx}{8d}$$

Mathematica [B] time = 6.46, size = 877, normalized size = 9.14

$$-\frac{7\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d}+\frac{7\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-7*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(32*d) + (7*Cos[c + d*x]^2*Log[Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)
/(32*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*
d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2*(29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x
]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2
] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*x]^2*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*d*(Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin
[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d
*x])^2*(-29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.80, size = 111, normalized size = 1.16

$$\frac{21\,a^2\cos(dx+c)^4\log(\sin(dx+c)+1) - 21\,a^2\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\left(32\,a^2\cos(dx+c)^3 + 21\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2 +
16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.48, size = 122, normalized size = 1.27

$$\frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 83\,a^2\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x +
1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.87, size = 102, normalized size = 1.06

$$\frac{7a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{7a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{4a^2\tan(dx+c)}{3d}+\frac{2a^2\left(\sec^2(dx+c)\right)\tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 7/8*a^2*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a^2
*tan(d*x+c)/d+2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*
x+c)/d
```

maxima [A] time = 0.77, size = 145, normalized size = 1.51

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.96, size = 141, normalized size = 1.47

$$\frac{7a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}-\frac{\frac{7a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}-\frac{77a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12}+\frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12}-\frac{25a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/12
 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (25*
a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sec^3(c+dx)\,dx+\int 2\sec^4(c+dx)\,dx+\int\sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**3, x) + Integral(2*sec(c + d*x)**4, x) + Integ
ral(sec(c + d*x)**5, x))
```

3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] a^2*arctanh(sin(d*x+c))/d+5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+
1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 3768, 3770, 4046, 3767, 8}

$$\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d
*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^3(c+dx)\,dx + \int \sec^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d} + a^2\int \sec($$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx}{3d}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^2\tan(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan(c+dx}{d}$$

Mathematica [B] time = 0.65, size = 318, normalized size = 4.30

$$-\frac{a^2\sec(c)\sec^3(c+dx)\left(6\sin(2c+dx)-6\sin(c+2dx)-6\sin(3c+2dx)-10\sin(2c+3dx)+3\cos(2c+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/24*(a^2*Sec[c]*Sec[c + d*x]^3*(3*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 3*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 9*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + 9*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Cos[2*c + 3
*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[4*c + 3*d*x]*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] - 24*Sin[d*x] + 6*Sin[2*c + d*x] - 6*Sin[
c + 2*d*x] - 6*Sin[3*c + 2*d*x] - 10*Sin[2*c + 3*d*x]))/d
```

fricas [A] time = 0.65, size = 96, normalized size = 1.30

$$\frac{3a^2\cos(dx+c)^3\log(\sin(dx+c)+1) - 3a^2\cos(dx+c)^3\log(-\sin(dx+c)+1) + 2\left(5a^2\cos(dx+c)^2+3a\right.}{6d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.55, size = 106, normalized size = 1.43

$$\frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2\tan(\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.85, size = 78, normalized size = 1.05

$$\frac{5a^2\tan(dx+c)}{3d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\left(\sec^2(dx+c)\right)\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(
d*x+c))+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

maxima [A] time = 0.66, size = 85, normalized size = 1.15

$$\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6a^2\tan}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

mupad [B] time = 2.47, size = 112, normalized size = 1.51

$$\frac{2a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{16a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+6a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^2
*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sec^2(c+dx)\,dx+\int 2\sec^3(c+dx)\,dx+\int\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**2, x) + Integral(2*sec(c + d*x)**3, x) + Integ
ral(sec(c + d*x)**4, x))
```

3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=54

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*x
+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3788, 3767, 8, 4046, 3770}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \sec(c+dx)\,dx - \frac{\left(2a^2\right)\text{Subst}(\int 1$$
$$= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [B] time = 0.61, size = 219, normalized size = 4.06

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2)
+ (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*d)
```

fricas [A] time = 0.69, size = 83, normalized size = 1.54

$$\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a^2\cos\left(dx+c\right)+a^2\right)\sin}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

giac [A] time = 0.48, size = 90, normalized size = 1.67

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.71, size = 58, normalized size = 1.07

$$\frac{3a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2a^2\tan\left(dx+c\right)}{d}+\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 3/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*t
an(d*x+c)/d
```

maxima [A] time = 0.65, size = 81, normalized size = 1.50

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right) - 8\,a^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a^2*tan(d
*x + c))/d
```

mupad [B] time = 1.18, size = 83, normalized size = 1.54

$$\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*t
an(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sec\left(c+dx\right)dx + \int 2\sec^2\left(c+dx\right)dx + \int \sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x), x) + Integral(2*sec(c + d*x)**2, x) + Integral
(sec(c + d*x)**3, x))
```

3.14 $\int (a + a\sec(c + dx))^2\,dx$

Optimal. Leaf size=34

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+a^2x$$

```
[Out] a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\,dx &= a^2x+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\,dx\\ &= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.47, size = 171, normalized size = 5.03

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} -\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(d*x - 2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]
/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(4*d)
```

fricas [B] time = 0.89, size = 76, normalized size = 2.24

$$\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.38, size = 79, normalized size = 2.32

$$\frac{(dx+c)a^2+2a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-2a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

maple [A] time = 0.52, size = 50, normalized size = 1.47

$$a^2x+\frac{a^2\tan(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*x+a^2*tan(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*c
```

maxima [A] time = 0.48, size = 41, normalized size = 1.21

$$a^2x+\frac{2a^2\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a^2*log(sec(d*x + c) + tan(d*x + c))/d + a^2*tan(d*x + c)/d
```

mupad [B] time = 0.71, size = 56, normalized size = 1.65

$$a^2x+\frac{4a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(d
*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 1\,dx + \int 2\sec(c+dx)\,dx + \int \sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(2*sec(c + d*x), x) + Integral(sec(c + d*x)*
*2, x))
```

3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=34

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

```
[Out] 2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3788, 8, 4045, 3770}

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int 1\,dx+\int \cos(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx\\ &= 2a^2x+\frac{a^2\sin(c+dx)}{d}+a^2\int \sec(c+dx)\,dx\\ &= 2a^2x+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.38

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c)\cos(dx)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

fricas [A] time = 0.64, size = 53, normalized size = 1.56

$$\frac{4\,a^2dx + a^2\log\left(\sin\left(dx+c\right)+1\right) - a^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

giac [B] time = 0.41, size = 79, normalized size = 2.32

$$\frac{2\,(dx+c)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

maple [A] time = 0.54, size = 51, normalized size = 1.50

$$2a^2x + \frac{a^2\sin\left(dx+c\right)}{d} + \frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{2a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2*a^2*x+a^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*c
```

maxima [A] time = 0.83, size = 52, normalized size = 1.53

$$\frac{4\,(dx+c)a^2 + a^2\left(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.70, size = 33, normalized size = 0.97

$$2\,a^2\,x + \frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] 2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2 \cos (c + dx) \sec (c + dx)\, dx + \int \cos (c + dx) \sec^2 (c + dx)\, dx + \int \cos (c + dx)\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)*sec(
c + d*x)**2, x) + Integral(cos(c + d*x), x))
```

3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

```
[Out] 3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3788, 2637, 4045, 8}

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos(c+dx)\,dx+\int \cos^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx\\ &= \frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^2\right)\int 1\,dx\\ &= \frac{3a^2x}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.76

$$\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.73, size = 36, normalized size = 0.80

$$\frac{3\,a^2dx + \left(a^2\cos\left(dx+c\right) + 4\,a^2\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.47, size = 64, normalized size = 1.42

$$\frac{3\,(dx+c)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.52, size = 52, normalized size = 1.16

$$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2\sin\left(dx+c\right)+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*sin(d*x+c)+a^2*(d*
x+c))
```

maxima [A] time = 0.79, size = 48, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^2 + 4\,(dx+c)a^2 + 8\,a^2\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*a^2 + 8*a^2*sin(d*x
 + c))/d
```

mupad [B] time = 1.07, size = 57, normalized size = 1.27

$$\frac{3\,a^2\,x}{2} + \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(t
an(c/2 + (d*x)/2)^2 + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\sec^2(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2, x))
```

3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

```
[Out] a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2635, 8, 4044, 3013}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/d - (a^2*S
in[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \cos^2(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{d} + a^2\int 1\,dx + \int \cos(c+dx)\left(a^2+a^2\cos^2(c$$
$$= a^2x + \frac{a^2\cos(c+dx)\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(2a^2-a^2x^2\right)dx, x, -\sin(c}{d}$$
$$= a^2x + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{d} - \frac{a^2\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.72

$$\frac{a^2(21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx))+12dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(1
2*d)
```

fricas [A] time = 0.55, size = 49, normalized size = 0.86

$$\frac{3\,a^2dx + \left(a^2\cos\left(dx+c\right)^2 + 3\,a^2\cos\left(dx+c\right) + 5\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
+ c))/d
```

giac [A] time = 1.80, size = 80, normalized size = 1.40

$$\frac{3\left(dx+c\right)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.90, size = 64, normalized size = 1.12

$$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + 2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

maxima [A] time = 0.73, size = 61, normalized size = 1.07

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-6\,a^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.66, size = 61, normalized size = 1.07

$$a^{2}\,x+\frac{5\,a^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^{2}\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^{2}\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d
) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\cos^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3, x))
```

3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

```
[Out] 7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3788, 2633, 4045, 2635, 8}

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \cos^3(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(7a^2\right)\int \cos^2(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{Sub}}{}$$
$$= \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c}{4d}$$
$$= \frac{7a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+}{}$$

Mathematica [A] time = 0.14, size = 53, normalized size = 0.61

$$\frac{a^2(144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx))+84dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]
 + 3*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.72, size = 63, normalized size = 0.72

$$\frac{21\,a^2dx + \left(6\,a^2\cos\left(dx+c\right)^3 + 16\,a^2\cos\left(dx+c\right)^2 + 21\,a^2\cos\left(dx+c\right) + 32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*c
os(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

giac [A] time = 3.92, size = 96, normalized size = 1.10

$$\frac{21\,(dx+c)a^2 + \frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 83\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)*a^2 + 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*a^2*tan(1/2*
d*x + 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.98, size = 90, normalized size = 1.03

$$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{2a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.90, size = 83, normalized size = 0.95

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-24\left(2\,dx+2\,c+\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^2)/d
```

mupad [B] time = 4.19, size = 89, normalized size = 1.02

$$\frac{7\,a^2\,x}{8}+\frac{\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{77\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{25\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/2
)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\cos^{4}{\left(c+dx\right)}\sec{\left(c+dx\right)}\,dx+\int \cos^{4}{\left(c+dx\right)}\sec^{2}{\left(c+dx\right)}\,dx+\int \cos^{4}{\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4, x))
```

3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+$$

```
[Out] 3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d*
x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 2635, 8, 4044, 3013, 373}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos^4(c+dx)\,dx+\int \cos^5(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\ &= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx+\int \cos^3(c+a\\ &= \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{4}\left(3a^2\right)\int 1\\ &= \frac{3a^2x}{4}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}-\text{Sub}\\ &= \frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.14, size = 61, normalized size = 0.59

$$\frac{a^2(110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx))+60dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]
 + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

fricas [A] time = 0.54, size = 76, normalized size = 0.74

$$\frac{15\,a^2dx+\left(4\,a^2\cos\left(dx+c\right)^4+10\,a^2\cos\left(dx+c\right)^3+12\,a^2\cos\left(dx+c\right)^2+15\,a^2\cos\left(dx+c\right)+24\,a^2\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*c
os(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

giac [A] time = 1.22, size = 112, normalized size = 1.09

$$\frac{15\left(dx+c\right)a^2+\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*(d*x + c)*a^2 + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 70*a^2*tan(1/2*
d*x + 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x + 1/2*
c)^3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 1.25, size = 96, normalized size = 0.93

$$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

maxima [A] time = 0.66, size = 95, normalized size = 0.92

$$\frac{16\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a^{2}-80\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a^{2}+15\left(12\,dx+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 80
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

mupad [B] time = 4.42, size = 105, normalized size = 1.02

$$\frac{3\,a^2\,x}{4}+\frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}+7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{72\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{13\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/5
 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*ta
n(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=114

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13}{}$$

```
[Out] 13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*ta
n(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3791, 3768, 3770, 3767}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^5(c+dx)+a^3\sec^6(c$$
$$= a^3\int \sec^3(c+dx)\,dx + a^3\int \sec^6(c+dx)\,dx + \left(3a^3\right)\int \sec^4(c+dx)\,c$$
$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^3\int \mathrm{s}$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+}{8d}$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c}{8d}$$

Mathematica [B] time = 1.51, size = 487, normalized size = 4.27

$$-\frac{a^3\sec(c)\sec^5(c+dx)\left(1440\sin(2c+dx)-1500\sin(c+2dx)-1500\sin(3c+2dx)-3040\sin(2c+3dx)-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]

[Out] -1/3840*(a^3*Sec[c]*Sec[c + d*x]^5*(975*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1950*Cos
[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]) + 1950*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 975*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 975*Cos[4*c + 3*d*x]*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] - 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] - 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 4640*Sin[d*x] + 1440*Sin[2*c + d*x] - 1500*Sin[c + 2*d*x] - 1500*Si
n[3*c + 2*d*x] - 3040*Sin[2*c + 3*d*x] - 390*Sin[3*c + 4*d*x] - 390*Sin[5*c
 + 4*d*x] - 608*Sin[4*c + 5*d*x]))/d
```

fricas [A] time = 0.64, size = 124, normalized size = 1.09

$$\frac{195\,a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-195\,a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(304\,a^3\cos(dx+c\right.}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x + c)
^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

giac [A] time = 1.66, size = 138, normalized size = 1.21

$$\frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d
*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/
2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 1.06, size = 124, normalized size = 1.09

$$\frac{13a^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{13a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{38a^3\tan(dx+c)}{15d}+\frac{19a^3\tan(dx+c)\left(\sec^2(\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 13/8*a^3*sec(d*x+c)*tan(d*x+c)/d+13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+38/15
*a^3*tan(d*x+c)/d+19/15/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4*a^3*sec(d*x+c)^3*
tan(d*x+c)/d+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4
```

maxima [A] time = 0.82, size = 179, normalized size = 1.57

$$16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-45a^3\left(\frac{2(3\,\mathrm{s}}{\sin(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 5.48, size = 170, normalized size = 1.49

$$\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{91\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{133\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\tan}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)/
15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (
13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan
(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sec^3(c+dx)\,dx+\int3\sec^4(c+dx)\,dx+\int3\sec^5(c+dx)\,dx+\int\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**3, x) + Integral(3*sec(c + d*x)**4, x) + Integ
ral(3*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**6, x))
```

3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=93

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\,\mathrm{se}}{8d}$$

```
[Out] 15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3791, 3767, 8, 3768, 3770}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\,\mathrm{se}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^5(c+
$$

$$= a^3\int \sec^2(c+dx)\,dx + a^3\int \sec^5(c+dx)\,dx + \left(3a^3\right)\int \sec^3(c+dx)\,dx
$$

$$= \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(3a^3\right)\int s
$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+d
$$

$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+
$$

Mathematica [B] time = 6.42, size = 877, normalized size = 9.43

$$-\frac{15\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d}+\frac{15\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}\right.\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-15*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(64*d) + (15*Cos[c + d*x]^3*Log[Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3)/(64*d) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(1
28*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (3*Cos[c
+ d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*
x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(128*d*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Se
c[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec
[c + d*x])^3*(-19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*Cos[c + d*x]^3*Sec[c/2 + (d*
x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.82, size = 111, normalized size = 1.19

$$\frac{15\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^3\cos\left(dx+c\right)^3+15\right.}{16\,d\cos\left(dx+c\right)^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 +
8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.87, size = 122, normalized size = 1.31

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\cdots\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1
/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 1.02, size = 101, normalized size = 1.09

$$\frac{3a^3\tan(dx+c)}{d}+\frac{15a^3\sec(dx+c)\tan(dx+c)}{8d}+\frac{15a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{a^3\tan(dx+c)\left(\sec^2(d\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 3*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d*x+c)/d+15/8/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4*a^3*sec(d*x+c)^3*tan(d*x+
c)/d
```

maxima [A] time = 0.62, size = 156, normalized size = 1.68

$$\frac{16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^3-a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+\cdots\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c
))/d
```

mupad [B] time = 4.05, size = 141, normalized size = 1.52

$$\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/4
 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (49*
a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \sec^2(c+dx)\,dx + \int 3\sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**2, x) + Integral(3*sec(c + d*x)**3, x) + Integ
ral(3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=72

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*x
+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3791, 3770, 3767, 8, 3768}

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec(c+dx)+3a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^4(c+dx)\right.$$
$$= a^3\int \sec(c+dx)\,dx + a^3\int \sec^4(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c+dx)\,dx +$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(3a^3\right)\int \sec(c$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [B] time = 5.67, size = 154, normalized size = 2.14

$$-\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-4\tan(c)\cos(c+dx)-\sec(c)(-20\sin(2c+dx)+9\sin(c+2dx)+9\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/192*(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(60*Cos[c + d*x]^3*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - Sec[c]*(50*Sin[d*x] - 20*Sin[2*c + d*x] + 9*Sin[c + 2*d*x] + 9*Sin
[3*c + 2*d*x] + 22*Sin[2*c + 3*d*x]) - 4*Cos[c + d*x]*Tan[c]))/d
```

fricas [A] time = 0.71, size = 98, normalized size = 1.36

$$\frac{15\,a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-15\,a^3\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(22\,a^3\cos(dx+c)^2+9\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a
^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 3.91, size = 106, normalized size = 1.47

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1
/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.85, size = 80, normalized size = 1.11

$$\frac{5a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{11a^3\tan(dx+c)}{3d}+\frac{3a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\tan(dx+c)\left(\sec^2(dx+\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c
)*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.32, size = 104, normalized size = 1.44

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}-9\,a^{3}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*l
og(sec(d*x + c) + tan(d*x + c)) + 36*a^3*tan(d*x + c))/d
```

mupad [B] time = 2.57, size = 112, normalized size = 1.56

$$\frac{5\,a^{3}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-\frac{40\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+11\,a^{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^3
*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\sec\left(c+dx\right)dx+\int 3\sec^{2}\left(c+dx\right)dx+\int 3\sec^{3}\left(c+dx\right)dx+\int\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x), x) + Integral(3*sec(c + d*x)**2, x) + Integral
(3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.23 $\int (a + a\sec(c + dx))^3\, dx$

Optimal. Leaf size=66

$$\frac{5a^3\tan(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+a^3x$$

```
[Out] a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+5/2*a^3*tan(d*x+c)/d+1/2*(a^3+a^3*sec(d
*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3775, 3914, 3767, 8, 3770}

$$\frac{5a^3\tan(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*Tan[c + d*x])/(2*d) +
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\,dx = \frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d} + \frac{1}{2}a\int (a+a\sec(c+dx))(2a+5a\sec(c+dx))$$
$$= a^3x + \frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d} + \frac{1}{2}\left(5a^3\right)\int \sec^2(c+dx)\,dx + \frac{1}{2}\left(7a^3\right)\int$$
$$= a^3x + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d} - \frac{\left(5a^3\right)\text{Subst}(}{}$$
$$= a^3x + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^3\tan(c+dx)}{2d} + \frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+}{2d}$$

Mathematica [B] time = 0.93, size = 235, normalized size = 3.56

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(4*x - (14*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
+ 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2) + (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + S
in[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]))))/32
```

fricas [A] time = 0.83, size = 98, normalized size = 1.48

$$\frac{4\,a^3dx\cos(dx+c)^2 + 7\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1) - 7\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\,(}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 1.97, size = 100, normalized size = 1.52

$$\frac{2\,(dx+c)a^3 + 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.69, size = 71, normalized size = 1.08

$$a^3x + \frac{a^3c}{d} + \frac{7a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{3a^3\tan(dx+c)}{d} + \frac{a^3\sec(dx+c)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+7/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*
a^3*sec(d*x+c)*tan(d*x+c)/d
```

maxima [A] time = 0.41, size = 91, normalized size = 1.38

$$a^3x - \frac{a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{4d} + \frac{3a^3\log(\sec(dx+c)+\tan(dx+c))}{d} + \frac{3a^3 \mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^3*log(sec(d*x + c) + tan(d*x + c))/d + 3
*a^3*tan(d*x + c)/d
```

mupad [B] time = 0.74, size = 88, normalized size = 1.33

$$a^3x + \frac{7a^3\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{5a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3 -
 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^
2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 1\,dx + \int 3\sec(c+dx)\,dx + \int 3\sec^2(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(1, x) + Integral(3*sec(c + d*x), x) + Integral(3*sec(c + d*x
)**2, x) + Integral(sec(c + d*x)**3, x))
```

3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

```
[Out] 3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3791, 2637, 3770, 3767, 8}

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx=\int \left(3a^3+a^3\cos(c+dx)+3a^3\sec(c+dx)+a^3\sec^2(c+dx)\right)dx$$

$$=3a^3x+a^3\int \cos(c+dx)\,dx+a^3\int \sec^2(c+dx)\,dx+\left(3a^3\right)\int \sec(c+$$

$$=3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\,\text{Subst}(\int 1\,dx,x,}{d}$$

$$=3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

Mathematica [B] time = 0.86, size = 211, normalized size = 4.40

$$\frac{1}{8}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

fricas [A] time = 0.77, size = 91, normalized size = 1.90

$$\frac{6\,a^3dx\cos\left(dx+c\right)+3\,a^3\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,a^3\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^3\cos\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*
a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*
x + c))/(d*cos(d*x + c))
```

giac [A] time = 2.01, size = 80, normalized size = 1.67

$$\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^4 - 1))/d
```

maple [A] time = 0.65, size = 65, normalized size = 1.35

$$3a^3x+\frac{a^3\sin\left(dx+c\right)}{d}+\frac{a^3\tan\left(dx+c\right)}{d}+\frac{3a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3a^3c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 3*a^3*x+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))
+3/d*a^3*c
```

maxima [A] time = 0.74, size = 64, normalized size = 1.33

$$\frac{6\,(dx+c)a^3+3\,a^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+2\,a^3\sin\left(dx+c\right)+2\,a^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.70, size = 57, normalized size = 1.19

$$3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] 3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))/
(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)*se
c(c + d*x)**2, x) + Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(co
s(c + d*x), x))
```

3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=59

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

```
[Out] 7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*s
in(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 3770}

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(3a^3+3a^3\cos(c+dx)+a^3\cos^2(c+dx)+a^3\sec(c+dx)\right)dx$$
$$= 3a^3x+a^3\int \cos^2(c+dx)\,dx+a^3\int \sec(c+dx)\,dx+\left(3a^3\right)\int \cos(c$$
$$= 3a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin}{2d}$$
$$= \frac{7a^3x}{2}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin}{2d}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.37

$$\frac{a^3\left(12\sin(c+dx)+\sin(2(c+dx))-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.60, size = 65, normalized size = 1.10

$$\frac{7\,a^3dx+a^3\log\left(\sin\left(dx+c\right)+1\right)-a^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^3\cos\left(dx+c\right)+6\,a^3\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (
a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.49, size = 100, normalized size = 1.69

$$\frac{7\,(dx+c)a^3+2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.49, size = 72, normalized size = 1.22

$$\frac{a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{7a^3x}{2}+\frac{7a^3c}{2d}+\frac{3a^3\sin\left(dx+c\right)}{d}+\frac{a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+7/2*a^3*x+7/2/d*a^3*c+3*a^3*sin(d*x+c)/d+1/
d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.52, size = 74, normalized size = 1.25

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)a^3+2\,a^3\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\Big)+12\,a^3\sin(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 2*a^3*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.73, size = 88, normalized size = 1.49

$$\frac{7\,a^3\,x}{2}+\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)/
2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*
x)/2)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\cos^2(c+dx)\sec(c+dx)\,dx+\int 3\cos^2(c+dx)\sec^2(c+dx)\,dx+\int \cos^2(c+dx)\sec^3(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(cos(c + d*x)**2, x))
```

3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=63

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

```
[Out] 5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d*
x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+dx)+a^3\cos^3(c+dx)\right)dx$$
$$= a^3x + a^3\int \cos^3(c+dx)\,dx + \left(3a^3\right)\int \cos(c+dx)\,dx + \left(3a^3\right)\int \cos^2(c$$
$$= a^3x + \frac{3a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^3\right)\int 1\,dx -$$
$$= \frac{5a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{a^3\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.70

$$\frac{a^3(45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(12*d)
```

fricas [A] time = 0.43, size = 50, normalized size = 0.79

$$\frac{15\,a^3dx + \left(2\,a^3\cos\left(dx+c\right)^2 + 9\,a^3\cos\left(dx+c\right) + 22\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(
d*x + c))/d
```

giac [A] time = 0.48, size = 80, normalized size = 1.27

$$\frac{15\,(dx+c)a^3 + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^3*tan(1/2*d
*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3
)/d
```

maple [A] time = 0.73, size = 74, normalized size = 1.17

$$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + 3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + 3\sin\left(dx+c\right)a^3 + a^3\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+3*sin(d*x+c)*a^3+a^3*(d*x+c))
```

maxima [A] time = 0.58, size = 71, normalized size = 1.13

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{3}-12\left(dx+c\right)a^{3}-36\,a^{3}\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3 - 12*(d*x + c)*a^3 - 36*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.67, size = 63, normalized size = 1.00

$$\frac{5\,a^3\,x}{2}+\frac{11\,a^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^3\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*x
))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int 3\cos^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int \cos^{3}\left(c+dx\right)\sec^{3}\left(c+dx\right)\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)**3*sec(c + d*x), x) + Integral(3*cos(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(cos(c + d*x)**3, x))
```

3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=85

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

```
[Out] 15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^4(c\right.$$
$$= a^3\int \cos(c+dx)\,dx + a^3\int \cos^4(c+dx)\,dx + \left(3a^3\right)\int \cos^2(c+dx)\,d$$
$$= \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^3\cos^3(c+dx)\sin(c}{4d}$$
$$= \frac{3a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c+}{}$$
$$= \frac{15a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c}{}$$

Mathematica [A] time = 0.13, size = 51, normalized size = 0.60

$$\frac{a^3(104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx))+60dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
+ Sin[4*(c + d*x)]))/(32*d)
```

fricas [A] time = 0.58, size = 63, normalized size = 0.74

$$\frac{15\,a^3dx + \left(2\,a^3\cos(dx+c)^3 + 8\,a^3\cos(dx+c)^2 + 15\,a^3\cos(dx+c) + 24\,a^3\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*cos
(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.69, size = 96, normalized size = 1.13

$$\frac{15\,(dx+c)a^3 + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*a^3*tan(1/2*d
*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.79, size = 100, normalized size = 1.18

$$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(
2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
sin(d*x+c)*a^3)
```

maxima [A] time = 0.51, size = 94, normalized size = 1.11

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{3}-24\left(2\,dx+2\,c+s\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^
3 - 32*a^3*sin(d*x + c))/d
```

mupad [B] time = 4.17, size = 89, normalized size = 1.05

$$\frac{15\,a^3\,x}{8}+\frac{\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{55\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/2
)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=105

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx}{8d}$$

```
[Out] 13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos(
d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3791, 2635, 8, 2633}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^5(c+\right.$$
$$= a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^5(c+dx)\,dx + \left(3a^3\right)\int \cos^3(c+dx)\,dx$$
$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^3\int 1\,dx$$
$$= \frac{a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx}{4d}$$
$$= \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+}{}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 0.60

$$\frac{a^3(1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+dx))+780dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d
*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

fricas [A] time = 0.66, size = 76, normalized size = 0.72

$$\frac{195\,a^3dx + \left(24\,a^3\cos\left(dx+c\right)^4 + 90\,a^3\cos\left(dx+c\right)^3 + 152\,a^3\cos\left(dx+c\right)^2 + 195\,a^3\cos\left(dx+c\right) + 304\,a^3\right)\sin\left(dx\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a
^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.57, size = 112, normalized size = 1.07

$$\frac{195\,(dx+c)a^3 + \frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)*a^3 + 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*a^3*tan(
1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)
/d
```

maple [A] time = 1.27, size = 121, normalized size = 1.15

$$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^3\left(2+\cos^2\left(dx+c\right)\right)\sin\left(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.45, size = 117, normalized size = 1.11

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a^{3}-480\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}+45\,(12\,dx}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 48
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 4.38, size = 105, normalized size = 1.00

$$\frac{13\,a^3\,x}{8}+\frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x)
/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^9)
/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=129

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] 23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d*x
+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = $\{3791, 2633, 2635, 8\}$

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^5(c+dx)+a^3\cos^6(c\right.\\ &= a^3\int \cos^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,dx + \left(3a^3\right)\int \cos^4(c+dx)\\ &= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}\left(5a^3\right)\\ &= \frac{4a^3\sin(c+dx)}{d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{23a^3\cos^3(c+dx)\sin}{24d}\\ &= \frac{9a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{23a^3\cos^3(c}{}\\ &= \frac{23a^3x}{16}+\frac{4a^3\sin(c+dx)}{d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{23a^3\cos^3}{}\end{aligned}$$

Mathematica [A] time = 0.21, size = 73, normalized size = 0.57

$$\frac{a^3(2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c+dx))+5\sin}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

fricas [A] time = 0.64, size = 89, normalized size = 0.69

$$\frac{345\,a^3dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^3\cos\left(dx+c\right)^4+230\,a^3\cos\left(dx+c\right)^3+272\,a^3\cos\left(dx+c\right)^2+345\,a^3\cos\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230*
a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^
3)*sin(d*x + c))/d
```

giac [A] time = 2.03, size = 128, normalized size = 0.99

$$\frac{345\left(dx+c\right)a^3+\frac{2\left(345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5814\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3165\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(345*(d*x + c)*a^3 + 2*(345*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*a^3*ta
n(1/2*d*x + 1/2*c)^9 + 4554*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*a^3*tan(1/2*d
*x + 1/2*c)^5 + 3165*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*a^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 1.43, size = 143, normalized size = 1.11

$$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3a^3\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.53, size = 143, normalized size = 1.11

$$\frac{192\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3-5\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

mupad [B] time = 3.49, size = 121, normalized size = 0.94

$$\frac{23\,a^3\,x}{16}+\frac{\frac{23\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{759\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{969\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{211\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{105\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*x
)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)/2
)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2))/8
)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=136

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+$$

```
[Out] 49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*tan
(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+
c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3791, 3768, 3770, 3767}

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^6(c+\right.$$
$$= a^4\int \sec^3(c+dx)\,dx + a^4\int \sec^7(c+dx)\,dx + \left(4a^4\right)\int \sec^4(c+dx)\,dx$$
$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^4\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^5(c+}{}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{11a^4\sec(c+dx)\tan(c+dx}{4d}$$
$$= \frac{11a^4\tanh^{-1}(\sin(c+dx))}{4d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+}{16d}$$
$$= \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+}{16d}$$

Mathematica [A] time = 0.82, size = 211, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(23520\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(2352
0*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-11520*Sin[c] + 3750*Sin[d*x] + 3750*
Sin[2*c + d*x] + 15360*Sin[c + 2*d*x] - 1920*Sin[3*c + 2*d*x] + 3845*Sin[2*
c + 3*d*x] + 3845*Sin[4*c + 3*d*x] + 6912*Sin[3*c + 4*d*x] + 735*Sin[4*c +
5*d*x] + 735*Sin[6*c + 5*d*x] + 1152*Sin[5*c + 6*d*x])))/d
```

fricas [A] time = 1.32, size = 137, normalized size = 1.01

$$\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1) - 735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1) + 2\left(1152\,a^4\cos(dx+c)^5\right.}{480\,d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^
6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c
)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c
) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 3.64, size = 154, normalized size = 1.13

$$\frac{735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+\right.}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
```

```
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 1.19, size = 146, normalized size = 1.07

$$\frac{49a^4 \sec(dx+c)\tan(dx+c)}{16d} + \frac{49a^4 \ln(\sec(dx+c)+\tan(dx+c))}{16d} + \frac{24a^4 \tan(dx+c)}{5d} + \frac{12a^4 \tan(dx+c)\left(\sec\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 49/16*a^4*sec(d*x+c)*tan(d*x+c)/d+49/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+24/
5*a^4*tan(d*x+c)/d+12/5/d*a^4*tan(d*x+c)*sec(d*x+c)^2+41/24*a^4*sec(d*x+c)^
3*tan(d*x+c)/d+4/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+1/6*a^4*sec(d*x+c)^5*tan(d
*x+c)/d
```

maxima [B] time = 0.71, size = 270, normalized size = 1.99

$$128\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^4 + 640\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4 - 5a^4\left(\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)))/d
```

mupad [B] time = 4.65, size = 199, normalized size = 1.46

$$\frac{49\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d} - \frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} - \frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} - \frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{1}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3)
/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/
20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 -
 (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int 6\sec^5(c+dx)\,dx + \int 4\sec^6(c+dx)\,dx + \int \sec^7(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integ
ral(6*sec(c + d*x)**5, x) + Integral(4*sec(c + d*x)**6, x) + Integral(sec(c
 + d*x)**7, x))
```

3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=111

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7}{}$$

```
[Out] 7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x
+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*x+
c)^5/d
```

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3791, 3767, 8, 3768, 3770}

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^5(c+\right.$$
$$= a^4\int \sec^2(c+dx)\,dx + a^4\int \sec^6(c+dx)\,dx + \left(4a^4\right)\int \sec^3(c+dx)\,dx$$
$$= \frac{2a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \left(2a^4\right)\int \sec$$
$$= \frac{2a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx}{2d}$$
$$= \frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx}{2d}$$

Mathematica [B] time = 1.52, size = 498, normalized size = 4.49

$$-\frac{a^4\sec(c)\sec^5(c+dx)\left(960\sin(2c+dx)-660\sin(c+2dx)-660\sin(3c+2dx)-1600\sin(2c+3dx)+60\sin(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/960*(a^4*Sec[c]*Sec[c + d*x]^5*(525*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
+ 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1050*Cos[
d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) + 1050*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 525*Cos[2*c + 3*d*x]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 2360*Sin[d*x] + 960*Sin[2*c + d*x] - 660*Sin[c + 2*d*x] - 660*Sin[3*
c + 2*d*x] - 1600*Sin[2*c + 3*d*x] + 60*Sin[4*c + 3*d*x] - 210*Sin[3*c + 4*
d*x] - 210*Sin[5*c + 4*d*x] - 332*Sin[4*c + 5*d*x]))/d
```

fricas [A] time = 0.71, size = 124, normalized size = 1.12

$$\frac{105\,a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(166\,a^4\cos(dx+c)^4\right.}{60\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

giac [A] time = 0.97, size = 138, normalized size = 1.24

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+89\right.}{\left(\tan\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*
x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c
)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 1.39, size = 123, normalized size = 1.11

$$\frac{83a^4\tan(dx+c)}{15d}+\frac{7a^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{7a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{34a^4\tan(dx+c)\left(\sec^2\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 83/15*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x+c)/d+7/2/d*a^4*ln(sec(d*x
+c)+tan(d*x+c))+34/15/d*a^4*tan(d*x+c)*sec(d*x+c)^2+a^4*sec(d*x+c)^3*tan(d*
x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4
```

maxima [A] time = 0.69, size = 190, normalized size = 1.71

$$4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+120\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-15a^4\left(\frac{2(3}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d
```

mupad [B] time = 5.47, size = 170, normalized size = 1.53

$$\frac{7a^4\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{7a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\frac{98a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+\frac{896a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}-\frac{158a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+25a^4}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
(158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*
tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^
2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2
)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\sec^2(c+dx)\,dx+\int 4\sec^3(c+dx)\,dx+\int 6\sec^4(c+dx)\,dx+\int 4\sec^5(c+dx)\,dx+\int\sec^6(c+dx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integ
ral(6*sec(c + d*x)**4, x) + Integral(4*sec(c + d*x)**5, x) + Integral(sec(c
 + d*x)**6, x))
```

3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=96

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec}{8d}$$

```
[Out] 35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(d
*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3791, 3770, 3767, 8, 3768}

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^4(c+\right.$$

$$= a^4\int \sec(c+dx)\,dx + a^4\int \sec^5(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+dx)\,dx$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx}{4}$$

$$= \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+}{8d}$$

$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c}{8d}$$

Mathematica [B] time = 6.42, size = 877, normalized size = 9.14

$$-\frac{35\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^4\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d}+\frac{35\cos^4(c+dx)\log\left(\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-35*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(128*d) + (35*Cos[c + d*x]^4*Log[Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])
^4)/(128*d) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/
(256*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c
/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] - Sin[
c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (5*Cos[
c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c +
 d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(256*d*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*
Sec[c + d*x])^4*(-97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(Cos[c/2] + Sin[c/2])*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.53, size = 111, normalized size = 1.16

$$\frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(160\,a^4\cos(dx+c\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4
*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2
 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.78, size = 122, normalized size = 1.27

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+51}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*
x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 1.16, size = 102, normalized size = 1.06

$$\frac{35a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{20a^4\tan\left(dx+c\right)}{3d}+\frac{27a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{4a^4\tan\left(dx+c\right)\left(\sec^2\left(d\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 35/8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x
+c)*tan(d*x+c)/d+4/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4*a^4*sec(d*x+c)^3*tan
(d*x+c)/d
```

maxima [A] time = 0.54, size = 175, normalized size = 1.82

$$64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4-3\,a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log(sec(d
*x + c) + tan(d*x + c)) + 192*a^4*tan(d*x + c))/d
```

mupad [B] time = 3.99, size = 141, normalized size = 1.47

$$\frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)/
12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 -
(93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int \sec(c+dx)\,dx + \int 4\sec^2(c+dx)\,dx + \int 6\sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x), x) + Integral(4*sec(c + d*x)**2, x) + Integral
(6*sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integral(sec(c +
d*x)**5, x))
```

3.33 $\int (a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=91

$$\frac{5a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{3d}+a^4x+\frac{\tan(c+dx)\left(a^2\sec(c+dx)\right.}{3d}$$

```
[Out] a^4*x+6*a^4*arctanh(sin(d*x+c))/d+5*a^4*tan(d*x+c)/d+1/3*(a^2+a^2*sec(d*x+c
))^2*tan(d*x+c)/d+4/3*(a^4+a^4*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3775, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}+\frac{4\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4,x]
```

```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (5*a^4*Tan[c + d*x])/d + ((a^2 +
a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (4*(a^4 + a^4*Sec[c + d*x])*Tan[c
 + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
```

```
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4\,dx = \frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{1}{3}a\int (a+a\sec(c+dx))^2(3a+8a\sec(c+dx$$

$$=\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+\frac{1}{6}a\int ($$

$$=a^4x+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+($$

$$=a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4 s\right.}{}$$

$$=a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4\tan(c+dx)}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+}{3d}$$

Mathematica [B] time = 6.27, size = 773, normalized size = 8.49

$$\frac{1}{16}x\cos^4(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^4+\frac{5\sin\left(\frac{dx}{2}\right)\cos^4(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^4}{12d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/16 - (3*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
8*(a + a*Sec[c + d*x])^4)/(8*d) + (3*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2]
+ Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(8*d) +
(Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(
96*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (
Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(13*Cos[c/2] - 1
1*Sin[c/2]))/(192*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4
*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
(d*x)/2])) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Si
n[(d*x)/2])/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(-1
3*Cos[c/2] - 11*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*S
ec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 7.10, size = 110, normalized size = 1.21

$$\frac{3\,a^4dx\cos(dx+c)^3+9\,a^4\cos(dx+c)^3\log(\sin(dx+c)+1)-9\,a^4\cos(dx+c)^3\log(-\sin(dx+c)+1)+(2}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
- 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*
a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.41, size = 116, normalized size = 1.27

$$\frac{3\,(dx+c)a^4 + 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 38\,a^4\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38*a
^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^3)/d
```

maple [A] time = 0.85, size = 93, normalized size = 1.02

$$a^4x+\frac{a^4c}{d}+\frac{6a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{20a^4\tan\left(dx+c\right)}{3d}+\frac{2a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{d}+\frac{a^4\tan\left(dx+c\right)\left(\mathrm{s}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+6/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+2*a
^4*sec(d*x+c)*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.44, size = 116, normalized size = 1.27

$$a^4x+\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4}{3\,d}-\frac{a^4\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{d}+\frac{4\,a^4\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - a^4*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d + 4*
a^4*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^4*tan(d*x + c)/d
```

mupad [B] time = 0.88, size = 117, normalized size = 1.29

$$a^4\,x+\frac{12\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{10\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{76\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+18\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4,x)
```

```
[Out] a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^5
 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 1\,dx + \int 4\sec\left(c+dx\right)dx + \int 6\sec^2\left(c+dx\right)dx + \int 4\sec^3\left(c+dx\right)dx + \int \sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(4*sec(c + d*x), x) + Integral(6*sec(c + d*x
)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

```
[Out] 4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d+
1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3791, 2637, 3770, 3767, 8, 3768}

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(4a^4 + a^4\cos(c+dx) + 6a^4\sec(c+dx) + 4a^4\sec^2(c+dx) + a^4\sec$$
$$= 4a^4x + a^4\int \cos(c+dx)\,dx + a^4\int \sec^3(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c$$
$$= 4a^4x + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4\sin(c+dx)}{d} + \frac{a^4\sec(c+dx)\tan}{2d}$$
$$= 4a^4x + \frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^4\sin(c+dx)}{d} + \frac{4a^4\tan(c+dx)}{d} +$$

Mathematica [B] time = 1.38, size = 272, normalized size = 3.73

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)}{d} + \frac{4\cos(c)\sin(dx)}{d} + \frac{16\sin\left(\frac{dx}{2}\right.}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))))/64
```

fricas [A] time = 0.81, size = 111, normalized size = 1.52

$$\frac{16\,a^4dx\cos(dx+c)^2 + 13\,a^4\cos(dx+c)^2\log(\sin(dx+c)+1) - 13\,a^4\cos(dx+c)^2\log(-\sin(dx+c)+1) +}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2
+ 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.59, size = 129, normalized size = 1.77

$$\frac{8\,(dx+c)a^4 + 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} - \frac{2\left(7\,a^4\tan\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.83, size = 86, normalized size = 1.18

$$\frac{a^4\sin(dx+c)}{d}+4a^4x+\frac{4a^4c}{d}+\frac{13a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4a^4\tan(dx+c)}{d}+\frac{a^4\sec(dx+c)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^4*x+4/d*a^4*c+13/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a
^4*tan(d*x+c)/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

maxima [A] time = 0.63, size = 110, normalized size = 1.51

$$\frac{16(dx+c)a^4-a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12a^4\left(\log(\sin(dx+c)+1)-\mathrm{lo}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.91, size = 115, normalized size = 1.58

$$4a^4x+\frac{13a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{5a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-11a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^4,x)
```

```
[Out] 4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^
3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos(c+dx)\sec(c+dx)\,dx+\int 6\cos(c+dx)\sec^2(c+dx)\,dx+\int 4\cos(c+dx)\sec^3(c+dx)\,dx+\int \mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)*se
c(c + d*x)**2, x) + Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(
cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x), x))
```

3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

```
[Out] 13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+c
)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3791, 2637, 2635, 8, 3770, 3767}

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(6a^4+4a^4\cos(c+dx)+a^4\cos^2(c+dx)+4a^4\sec(c+dx)+a^4\sec^2(\right.$$
$$= 6a^4x+a^4\int \cos^2(c+dx)\,dx+a^4\int \sec^2(c+dx)\,dx+\left(4a^4\right)\int \cos(c+$$
$$= 6a^4x+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(}{2d}$$
$$= \frac{13a^4x}{2}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin}{2d}$$

Mathematica [B] time = 1.80, size = 241, normalized size = 3.30

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin(c)\cos(dx)}{d}+\frac{\sin(2c)\cos(2dx)}{d}+\frac{16\cos(c)\sin(dx)}{d}+\frac{\cos(2c)\sin(2}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/
d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

fricas [A] time = 0.90, size = 105, normalized size = 1.44

$$\frac{13\,a^4dx\cos(dx+c)+4\,a^4\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^4\cos(dx+c)\log(-\sin(dx+c)+1)+\left(a^4\cos\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4
*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(
d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.82, size = 129, normalized size = 1.77

$$\frac{13\,(dx+c)a^4+8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.66, size = 86, normalized size = 1.18

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13a^4x}{2}+\frac{13a^4c}{2d}+\frac{4a^4\sin(dx+c)}{d}+\frac{4a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+13/2*a^4*x+13/2/d*a^4*c+4*a^4*sin(d*x+c)/d+
4/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c)/d
```

maxima [A] time = 0.35, size = 85, normalized size = 1.16

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^4+8\,a^4\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+16\,a^4\,\mathrm{s}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```

mupad [B] time = 0.89, size = 117, normalized size = 1.60

$$\frac{13\,a^4\,x}{2}+\frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+11\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)
/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\cos^2(c+dx)\sec(c+dx)\,dx+\int 6\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 4\cos^2(c+dx)\sec^3(c+dx)\,dx\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)**2*sec(c + d*x), x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**2, x)
)
```

3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

```
[Out] 6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(d
*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3791, 2637, 2635, 8, 2633, 3770}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(4a^4+6a^4\cos(c+dx)+4a^4\cos^2(c+dx)+a^4\cos^3(c+dx)+a^4 s\right.$$
$$= 4a^4x + a^4\int \cos^3(c+dx)\,dx + a^4\int \sec(c+dx)\,dx + \left(4a^4\right)\int \cos^2(c$$
$$= 4a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{6a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\,s}{d}$$
$$= 6a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{7a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\,s}{d}$$

Mathematica [A] time = 0.11, size = 91, normalized size = 1.25

$$\frac{a^4\left(81\sin(c+dx)+12\sin(2(c+dx))+\sin(3(c+dx))-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin
[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.81, size = 80, normalized size = 1.10

$$\frac{36\,a^4dx+3\,a^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^4\cos\left(dx+c\right)^2+6\,a^4\cos\left(dx+c\right)+2\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1
) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.46, size = 116, normalized size = 1.59

$$\frac{18\left(dx+c\right)a^4+3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+38\,a^4\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

maple [A] time = 0.73, size = 94, normalized size = 1.29

$$\frac{\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)a^4}{3d}+\frac{20a^4\sin\left(dx+c\right)}{3d}+\frac{2a^4\cos\left(dx+c\right)\sin\left(dx+c\right)}{d}+6a^4x+\frac{6a^4c}{d}+\frac{a^4\ln\left(\sec\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a^4+20/3*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*si
n(d*x+c)/d+6*a^4*x+6/d*a^4*c+1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))

maxima [A] time = 0.93, size = 97, normalized size = 1.33

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{4}-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{4}-24\left(dx+c\right)a^{4}-3\,a^{4}\left(\log\left(\sin\left(dx+c\right)+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 24*(d*x + c)*a^4 - 3*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) - 36*a^4*sin(d*x + c))/d

mupad [B] time = 0.69, size = 93, normalized size = 1.27

$$6\,a^4\,x+\frac{20\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{2\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^4,x)

[Out] 6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*cos(
c + d*x)*sin(c + d*x))/d

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4,x)

[Out] Timed out

3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=87

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

```
[Out] 35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(
d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^4\right.$$
$$= a^4x + a^4\int \cos^4(c+dx)\,dx + \left(4a^4\right)\int \cos(c+dx)\,dx + \left(4a^4\right)\int \cos^3(c$$
$$= a^4x + \frac{4a^4\sin(c+dx)}{d} + \frac{3a^4\cos(c+dx)\sin(c+dx)}{d} + \frac{a^4\cos^3(c+dx)\,\mathrm{si}}{4d}$$
$$= 4a^4x + \frac{8a^4\sin(c+dx)}{d} + \frac{27a^4\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^4\cos^3(c+dx}{4d}$$
$$= \frac{35a^4x}{8} + \frac{8a^4\sin(c+dx)}{d} + \frac{27a^4\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^4\cos^3(c+}{4}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.64

$$\frac{a^4(672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx))+420c+420dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*
(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.90, size = 63, normalized size = 0.72

$$\frac{105\,a^4dx+\left(6\,a^4\cos\left(dx+c\right)^3+32\,a^4\cos\left(dx+c\right)^2+81\,a^4\cos\left(dx+c\right)+160\,a^4\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*
cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

giac [A] time = 1.23, size = 96, normalized size = 1.10

$$\frac{105\,(dx+c)a^4+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*a^4*tan(1
/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.90, size = 111, normalized size = 1.28

$$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+4*a^4*sin(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.75, size = 104, normalized size = 1.20

$$-\frac{128\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{4}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{4}-144\left(2\,dx+\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a^4 - 96*(d*x + c)*a^4 - 384*a^4*sin(d*x + c))/d
```

mupad [B] time = 4.12, size = 89, normalized size = 1.02

$$\frac{35\,a^4\,x}{8}+\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*x
)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2))/
4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a}{2}$$

```
[Out] 7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c)
^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d) +
(a^4*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos(c+dx)+4a^4\cos^2(c+dx)+6a^4\cos^3(c+dx)+4a^4\cos^4(c\right.$$
$$= a^4\int \cos(c+dx)\,dx + a^4\int \cos^5(c+dx)\,dx + \left(4a^4\right)\int \cos^2(c+dx)\,d$$
$$= \frac{a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} + \frac{a^4\cos^3(c+dx)\sin(c}{d}$$
$$= 2a^4x + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c+d}{}$$
$$= \frac{7a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c+}{}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 0.62

$$\frac{a^4(1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+dx))+840dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d
*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

fricas [A] time = 1.52, size = 76, normalized size = 0.75

$$\frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)\sin\left(d\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*
cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.62, size = 112, normalized size = 1.10

$$\frac{105\,(dx+c)a^4 + \frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*a^4*tan(1
/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*a^4*tan(1/2*d*x +
1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 1.22, size = 133, normalized size = 1.30

$$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right) + 2a^4\left(2+\cos^2\left(dx+c\right)\right)\sin\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+4*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(d*x+c))
```

maxima [A] time = 0.45, size = 128, normalized size = 1.25

$$\frac{8\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a^{4}-240\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{4}+15\left(12\,dx+12\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 240
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 1
20*a^4*sin(d*x + c))/d
```

mupad [B] time = 4.43, size = 105, normalized size = 1.03

$$\frac{7\,a^4\,x}{2}+\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{98\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{896\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\frac{158\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)/
2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 + 2
5*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=127

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+d}{24d}$$

```
[Out] 49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4*
cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x+c
)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

Rubi [A] time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3791, 2635, 8, 2633}

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+d}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^5(c+\right.$$
$$= a^4\int \cos^2(c+dx)\,dx + a^4\int \cos^6(c+dx)\,dx + \left(4a^4\right)\int \cos^3(c+dx)\,dx$$
$$= \frac{a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^4\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^5(c+}{}$$
$$= \frac{a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{41a^4\cos^3(c+d}{24}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+}{}$$
$$= \frac{49a^4x}{16} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c-}{}$$

Mathematica [A] time = 0.22, size = 73, normalized size = 0.57

$$\frac{a^4(5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5(c+dx))+5\sin(6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c +
 d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/
(960*d)
```

fricas [A] time = 0.78, size = 89, normalized size = 0.70

$$\frac{735\,a^4dx + \left(40\,a^4\cos(dx+c)^5 + 192\,a^4\cos(dx+c)^4 + 410\,a^4\cos(dx+c)^3 + 576\,a^4\cos(dx+c)^2 + 735\,a^4\cos(}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410*
a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152*a
^4)*sin(d*x + c))/d
```

giac [A] time = 1.24, size = 128, normalized size = 1.01

$$\frac{735\,(dx+c)a^4 + \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 9702\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 11802\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 7355\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(735*(d*x + c)*a^4 + 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*a^4*tan(1/2*
d*x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*a^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 1.36, size = 169, normalized size = 1.33

$$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+6a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos}{}\right.}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.72, size = 165, normalized size = 1.30

$$256\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a^{4}-5\left(4\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\sin\left(4\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4)/d
```

mupad [B] time = 3.42, size = 121, normalized size = 0.95

$$\frac{49\,a^4\,x}{16}+\frac{\frac{49\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{833\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{1967\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{207\,a^4\,\tan\left(\right.}{8}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 + (
d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (d*
x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x)/2
))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=147

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c}{}$$

```
[Out] 11/4*a^4*x+8*a^4*sin(d*x+c)/d+11/4*a^4*cos(d*x+c)*sin(d*x+c)/d+11/6*a^4*cos
(d*x+c)^3*sin(d*x+c)/d+2/3*a^4*cos(d*x+c)^5*sin(d*x+c)/d-16/3*a^4*sin(d*x+c
)^3/d+9/5*a^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*x+c)^7/d
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{3791, 2633, 2635, 8\}$

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])/
(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^5*
Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x]^5
)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^4\cos^4(c+dx)+6a^4\cos^5(c+dx)+4a^4\cos^6(\right.$$
$$= a^4\int\cos^3(c+dx)\,dx + a^4\int\cos^7(c+dx)\,dx + \left(4a^4\right)\int\cos^4(c+dx)$$
$$= \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + \frac{2a^4\cos^5(c+dx)\sin(c+dx)}{3d} + \left(3a^4\right)\int$$
$$= \frac{8a^4\sin(c+dx)}{d} + \frac{3a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{11a^4\cos^3(c+dx)\sin}{6d}$$
$$= \frac{3a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\cos^3(c}{}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\cos^3(}{}$$

Mathematica [A] time = 0.27, size = 83, normalized size = 0.56

$$\frac{a^4(33915\sin(c+dx)+13020\sin(2(c+dx))+5495\sin(3(c+dx))+2100\sin(4(c+dx))+651\sin(5(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(18480*d*x + 33915*Sin[c + d*x] + 13020*Sin[2*(c + d*x)] + 5495*Sin[3*
(c + d*x)] + 2100*Sin[4*(c + d*x)] + 651*Sin[5*(c + d*x)] + 140*Sin[6*(c +
d*x)] + 15*Sin[7*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.73, size = 102, normalized size = 0.69

$$\frac{1155\,a^4dx + \left(60\,a^4\cos\left(dx+c\right)^6 + 280\,a^4\cos\left(dx+c\right)^5 + 576\,a^4\cos\left(dx+c\right)^4 + 770\,a^4\cos\left(dx+c\right)^3 + 908\,a^4\,c\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(1155*a^4*d*x + (60*a^4*cos(d*x + c)^6 + 280*a^4*cos(d*x + c)^5 + 576
*a^4*cos(d*x + c)^4 + 770*a^4*cos(d*x + c)^3 + 908*a^4*cos(d*x + c)^2 + 115
5*a^4*cos(d*x + c) + 1816*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.64, size = 144, normalized size = 0.98

$$\frac{1155\,(dx+c)a^4 + \frac{2\left(1155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} + 7700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 21791\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 33792\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} + 31521\,a^4\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/420*(1155*(d*x + c)*a^4 + 2*(1155*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*a^4*
tan(1/2*d*x + 1/2*c)^11 + 21791*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*a^4*tan(
1/2*d*x + 1/2*c)^7 + 31521*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*a^4*tan(1/2*d
*x + 1/2*c)^3 + 5565*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^7)/d
```

maple [A] time = 1.58, size = 185, normalized size = 1.26

$$\frac{\frac{a^4\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos^4(dx+c))}{5}+\frac{8(\cos^2(dx+c))}{5}\right)\sin(dx+c)}{7}+4a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{6a^4\left(\frac{8}{3}+c\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c)+4*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+6/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.72, size = 187, normalized size = 1.27

$$-\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^4-672\left(3\sin(dx+c)^5-10\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*
sin(d*x + c))*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x
+ c))*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) -
 48*sin(2*d*x + 2*c))*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4)/d
```

mupad [B] time = 3.64, size = 137, normalized size = 0.93

$$\frac{11\,a^4\,x}{4}+\frac{\frac{11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{2}+\frac{110\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{3113\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{30}+\frac{5632\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{1501\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{10}+70\,a^4\tan\left(\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (11*a^4*x)/4 + (70*a^4*tan(c/2 + (d*x)/2)^3 + (1501*a^4*tan(c/2 + (d*x)/2)^
5)/10 + (5632*a^4*tan(c/2 + (d*x)/2)^7)/35 + (3113*a^4*tan(c/2 + (d*x)/2)^9
)/30 + (110*a^4*tan(c/2 + (d*x)/2)^11)/3 + (11*a^4*tan(c/2 + (d*x)/2)^13)/2
 + (53*a^4*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

Optimal. Leaf size=156

$$\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^5\tan}{}$$

```
[Out] 93/16*a^5*arctanh(sin(d*x+c))/d+16*a^5*tan(d*x+c)/d+93/16*a^5*sec(d*x+c)*ta
n(d*x+c)/d+85/24*a^5*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a^5*sec(d*x+c)^5*tan(d*x
+c)/d+28/3*a^5*tan(d*x+c)^3/d+13/5*a^5*tan(d*x+c)^5/d+1/7*a^5*tan(d*x+c)^7/
d
```

Rubi [A] time = 0.20, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3791, 3768, 3770, 3767}

$$\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^5\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24
*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3)/(
3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \int \left(a^5\sec^3(c+dx)+5a^5\sec^4(c+dx)+10a^5\sec^5(c+dx)+10a^5\sec^6(c$$
$$= a^5\int \sec^3(c+dx)\,dx + a^5\int \sec^8(c+dx)\,dx + \left(5a^5\right)\int \sec^4(c+dx)\,dx$$
$$= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d} + \frac{5a^5\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{5a^5\sec^5(c}{}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} + \frac{16a^5\tan(c+dx)}{d} + \frac{17a^5\sec(c+dx)\tan(c+d}{4d}$$
$$= \frac{17a^5\tanh^{-1}(\sin(c+dx))}{4d} + \frac{16a^5\tan(c+dx)}{d} + \frac{93a^5\sec(c+dx)\tan(c+}{16d}$$
$$= \frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d} + \frac{16a^5\tan(c+dx)}{d} + \frac{93a^5\sec(c+dx)\tan(c+}{16d}$$

Mathematica [A] time = 1.39, size = 229, normalized size = 1.47

$$-\frac{a^5(\cos(c+dx)+1)^5\sec^{10}\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(624960\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/3440640*(a^5*(1 + Cos[c + d*x])^5*Sec[(c + d*x)/2]^10*Sec[c + d*x]^7*(62
4960*Cos[c + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(374080*Sin[d*x] - 162400*Sin[2*c +
 d*x] + 118825*Sin[c + 2*d*x] + 118825*Sin[3*c + 2*d*x] + 305088*Sin[2*c +
3*d*x] - 16800*Sin[4*c + 3*d*x] + 62860*Sin[3*c + 4*d*x] + 62860*Sin[5*c +
4*d*x] + 107296*Sin[4*c + 5*d*x] + 9765*Sin[5*c + 6*d*x] + 9765*Sin[7*c + 6
*d*x] + 15328*Sin[6*c + 7*d*x]))))/d
```

fricas [A] time = 0.79, size = 150, normalized size = 0.96

$$\frac{9765\,a^5\cos(dx+c)^7\log(\sin(dx+c)+1) - 9765\,a^5\cos(dx+c)^7\log(-\sin(dx+c)+1) + 2\left(15328\,a^5\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(9765*a^5*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 9765*a^5*cos(d*x +
c)^7*log(-sin(d*x + c) + 1) + 2*(15328*a^5*cos(d*x + c)^6 + 9765*a^5*cos(d*
x + c)^5 + 7664*a^5*cos(d*x + c)^4 + 5950*a^5*cos(d*x + c)^3 + 3648*a^5*cos
(d*x + c)^2 + 1400*a^5*cos(d*x + c) + 240*a^5)*sin(d*x + c))/(d*cos(d*x + c
)^7)
```

giac [A] time = 2.24, size = 170, normalized size = 1.09

$$\frac{9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9765\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-65100\,a^5\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(9765*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9765*a^5*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) - 2*(9765*a^5*tan(1/2*d*x + 1/2*c)^13 - 65100*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 184233*a^5*tan(1/2*d*x + 1/2*c)^9 - 285696*a^5*tan(
1/2*d*x + 1/2*c)^7 + 260183*a^5*tan(1/2*d*x + 1/2*c)^5 - 132020*a^5*tan(1/2
*d*x + 1/2*c)^3 + 43995*a^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^7)/d
```

maple [A] time = 1.34, size = 168, normalized size = 1.08

$$\frac{93a^5\sec(dx+c)\tan(dx+c)}{16d}+\frac{93a^5\ln(\sec(dx+c)+\tan(dx+c))}{16d}+\frac{958a^5\tan(dx+c)}{105d}+\frac{479a^5\tan(dx+c)(}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x)
```

```
[Out] 93/16*a^5*sec(d*x+c)*tan(d*x+c)/d+93/16/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+958
/105*a^5*tan(d*x+c)/d+479/105/d*a^5*tan(d*x+c)*sec(d*x+c)^2+85/24*a^5*sec(d
*x+c)^3*tan(d*x+c)/d+76/35/d*a^5*tan(d*x+c)*sec(d*x+c)^4+5/6*a^5*sec(d*x+c)
^5*tan(d*x+c)/d+1/7/d*a^5*tan(d*x+c)*sec(d*x+c)^6
```

maxima [B] time = 0.74, size = 314, normalized size = 2.01

$$96\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^5+2240\left(3\tan(dx+c)^5+10\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^5 + 2240*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
+ c))*a^5 + 5600*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 - 175*a^5*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 2100*a^5*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.84, size = 228, normalized size = 1.46

$$\frac{93\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\frac{93\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{155\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2}+\frac{8773\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{40}-\frac{11904\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^5/cos(c + d*x)^3,x)
```

```
[Out] (93*a^5*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((37169*a^5*tan(c/2 + (d*x)/2)^5
)/120 - (943*a^5*tan(c/2 + (d*x)/2)^3)/6 - (11904*a^5*tan(c/2 + (d*x)/2)^7)
/35 + (8773*a^5*tan(c/2 + (d*x)/2)^9)/40 - (155*a^5*tan(c/2 + (d*x)/2)^11)/
2 + (93*a^5*tan(c/2 + (d*x)/2)^13)/8 + (419*a^5*tan(c/2 + (d*x)/2))/8)/(d*(
7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6
- 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)
^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^5\left(\int\sec^3(c+dx)\,dx+\int5\sec^4(c+dx)\,dx+\int10\sec^5(c+dx)\,dx+\int10\sec^6(c+dx)\,dx+\int5\sec^7(c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(sec(c + d*x)**3, x) + Integral(5*sec(c + d*x)**4, x) + Integ
ral(10*sec(c + d*x)**5, x) + Integral(10*sec(c + d*x)**6, x) + Integral(5*s
ec(c + d*x)**7, x) + Integral(sec(c + d*x)**8, x))
```

3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] -3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3818, 3787, 3768, 3770, 3767}

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^3(c+dx)(3a-4a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^3(c+dx)\,dx}{a} + \frac{4\int \sec^4(c+dx)\,dx}{a}$$
$$= -\frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec(c+dx)\,dx}{2a} - \frac{4\,\text{Subst}}{}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{4\tan(c+dx)}{ad} - \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^3(c+dx)\tan}{d(a+a\sec(c}$$

Mathematica [B] time = 3.48, size = 374, normalized size = 3.63

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-12\sin(2c+dx)-6\sin(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]
*Sec[c]*Sec[c + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27
*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d
*x] - 6*Sin[3*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Sec[c + d*
x]))
```

fricas [A] time = 0.63, size = 124, normalized size = 1.20

$$-\frac{9\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\log(-\sin(dx+c)+1}{12\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(d
*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3 +
 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a
*d*cos(d*x + c)^3)
```

giac [A] time = 0.93, size = 114, normalized size = 1.11

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16*t
```

```
an(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*a))/d
```

maple [A] time = 0.36, size = 183, normalized size = 1.78

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{3da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{2da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/3/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a/(tan(1/2*d*
x+1/2*c)-1)^2-5/2/d/a/(tan(1/2*d*x+1/2*c)-1)+3/2/d/a*ln(tan(1/2*d*x+1/2*c)-
1)-1/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3+1/d/a/(tan(1/2*d*x+1/2*c)+1)^2-5/2/d/a/
(tan(1/2*d*x+1/2*c)+1)-3/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.65, size = 205, normalized size = 1.99

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a-\frac{3a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
+ 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(d*x
 + c) + 1)))/d
```

mupad [B] time = 0.93, size = 96, normalized size = 0.93

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/2
 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*(t
an(c/2 + (d*x)/2)^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d-
sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
```

```
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^2(c+dx)(2a-3a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{2\int \sec^2(c+dx)\,dx}{a} + \frac{3\int \sec^3(c+dx)\,dx}{a}$$
$$= \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{3\int \sec(c+dx)\,dx}{2a} + \frac{2\,\mathrm{Subs}}{}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{2\tan(c+dx)}{ad} + \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\,\mathrm{ta}}{d(a+a\sec(}$$

Mathematica [B] time = 1.46, size = 250, normalized size = 2.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{c}\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]
*(-6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])))))/(2*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.75, size = 112, normalized size = 1.32

$$\frac{3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+\right.}{4\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x
 + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 + co
s(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [A] time = 0.85, size = 101, normalized size = 1.19

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

maple [A] time = 0.41, size = 143, normalized size = 1.68

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{1}{2da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2da}-\frac{1}{2da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2+3/2/d/a/(tan(1/2
*d*x+1/2*c)-1)-3/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a/(tan(1/2*d*x+1/2*c)
+1)^2+3/2/d/a/(tan(1/2*d*x+1/2*c)+1)+3/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.59, size = 162, normalized size = 1.91

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1)
))/d
```

mupad [B] time = 0.73, size = 95, normalized size = 1.12

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2 +
 (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*ta
n(c/2 + (d*x)/2)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{4}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)} \, dx$

Optimal. Leaf size=51

$$\frac{\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] -arctanh(sin(d*x+c))/a/d+tan(d*x+c)/a/d+tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3790, 3789, 3770, 3794}

$$\frac{\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\tan(c+dx)}{ad}-\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx\\
&= \frac{\tan(c+dx)}{ad}-\frac{\int \sec(c+dx)\,dx}{a}+\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.80, size = 194, normalized size = 3.80

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}\right.\right.}{ad(\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d
*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.67, size = 97, normalized size = 1.90

$$-\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)
^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x
+ c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [A] time = 3.71, size = 84, normalized size = 1.65

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

maple [A] time = 0.35, size = 99, normalized size = 1.94

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{1}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/d/a/(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d*x+
1/2*c)-1)-1/d/a/(tan(1/2*d*x+1/2*c)+1)-1/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.41, size = 119, normalized size = 2.33

$$-\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.68, size = 67, normalized size = 1.31

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
+ (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] arctanh(sin(d*x+c))/a/d-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3789, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\int \sec(c+dx)\,dx}{a}-\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.18, size = 109, normalized size = 2.87

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*S
in[(d*x)/2]))/(a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 1.71, size = 65, normalized size = 1.71

$$\frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1) - (\cos(dx+c)+1)\log(-\sin(dx+c)+1) - 2\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.99, size = 54, normalized size = 1.42

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a - tan(1/2*d*x + 1/2*c)/a)/d
```

maple [A] time = 0.33, size = 58, normalized size = 1.53

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d
*x+1/2*c)+1)
```

maxima [A] time = 0.67, size = 75, normalized size = 1.97

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.65, size = 31, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3794}

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] Tan[(c + d*x)/2]/(a*d)
```

fricas [A] time = 0.69, size = 22, normalized size = 1.00

$$\frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 3.89, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] tan(1/2*d*x + 1/2*c)/(a*d)
```

maple [A] time = 0.36, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] 1/a/d*tan(1/2*d*x+1/2*c)
```

maxima [A] time = 0.39, size = 23, normalized size = 1.05

$$\frac{\sin\left(dx+c\right)}{ad(\cos\left(dx+c\right)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

mupad [B] time = 0.59, size = 16, normalized size = 0.73

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))),x)
[Out] tan(c/2 + (d*x)/2)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] x/a-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\sec(c+dx)}\,dx &= -\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 2.00

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(c+\frac{dx}{2}\right)-2\sin\left(\frac{dx}{2}\right)+dx\cos\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(d*x*Cos[(d*x)/2] + d*x*Cos[c + (d*x)/2] - 2*Sin
[(d*x)/2]))/(2*a*d)
```

fricas [A] time = 0.65, size = 37, normalized size = 1.28

$$\frac{dx\cos(dx+c)+dx-\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 2.13, size = 28, normalized size = 0.97

$$\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

maple [A] time = 0.40, size = 37, normalized size = 1.28

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)+2/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.15, size = 49, normalized size = 1.69

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c
) + 1)))/d
```

mupad [B] time = 0.64, size = 23, normalized size = 0.79

$$\frac{x}{a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - tan(c/2 + (d*x)/2)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec{(c+dx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x) + 1), x)/a
```

3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x}{a}$$

```
[Out] -x/a+2*sin(d*x+c)/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3819, 3787, 2637, 8}

$$\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (2*Sin[c + d*x])/(a*d) - Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \cos(c+dx)(-2a+a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int 1\,dx}{a}+\frac{2\int \cos(c+dx)\,dx}{a}\\ &= -\frac{x}{a}+\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.23, size = 89, normalized size = 2.02

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{3dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)-2dx\,\mathrm{co}\right.}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

fricas [A] time = 0.51, size = 46, normalized size = 1.05

$$-\frac{dx\cos\left(dx+c\right)+dx-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

giac [A] time = 0.38, size = 58, normalized size = 1.32

$$-\frac{\frac{dx+c}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 + 1)*a))/d
```

maple [A] time = 0.55, size = 68, normalized size = 1.55

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+2/d/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-
2/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.61, size = 92, normalized size = 2.09

$$-\frac{\frac{2\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a}-\frac{2\,\sin\left(dx+c\right)}{\left(a+\frac{a\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)\left(\cos\left(dx+c\right)+1\right)}-\frac{\sin\left(dx+c\right)}{a\left(\cos\left(dx+c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos
(d*x + c) + 1)))/d
```

mupad [B] time = 0.66, size = 66, normalized size = 1.50

$$\frac{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(-c-d\,x\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sin(c+dx)}{ad}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3x}{2a}$$

```
[Out] 3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)*sin(d*x+c
)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2635, 8, 2637}

$$-\frac{2\sin(c+dx)}{ad}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^2(c+dx)(-3a+2a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{2\int \cos(c+dx)\,dx}{a} + \frac{3\int \cos^2(c+dx)\,dx}{a}$$
$$= -\frac{2\sin(c+dx)}{ad} + \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{3\int 1\,dx}{2a}$$
$$= \frac{3x}{2a} - \frac{2\sin(c+dx)}{ad} + \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.25, size = 117, normalized size = 1.58

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{5dx}{2}\right)\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] -
 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c +
(3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

fricas [A] time = 0.82, size = 57, normalized size = 0.77

$$\frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.33, size = 73, normalized size = 0.99

$$\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

maple [A] time = 0.51, size = 103, normalized size = 1.39

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*
c)^3-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+3/d/a*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [A] time = 0.69, size = 133, normalized size = 1.80

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.72, size = 89, normalized size = 1.20

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(c+d\,x)}{2}+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(a*
d*cos(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x}{2a}$$

```
[Out] -3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d*
x+c)/d/(a+a*sec(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^3(c+dx)(-4a+3a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \cos^2(c+dx)\,dx}{a} + \frac{4\int \cos^3(c+dx)\,dx}{a}$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int 1\,dx}{2a} - \frac{4\,\text{Subst}\left(\int \left(1-x^2\right)\right.}{}$$
$$= -\frac{3x}{2a} + \frac{4\sin(c+dx)}{ad} - \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{4\sin^3(c}{3ad}$$

Mathematica [A] time = 0.32, size = 143, normalized size = 1.52

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\frac{3dx}{2}\right)+18\sin\left(2c+\frac{3dx}{2}\right)-2\sin\left(2c+\frac{5dx}{2}\right)-2\sin\left(3c+\frac{5dx}{2}\right.\right.}{48ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*
c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

fricas [A] time = 0.48, size = 70, normalized size = 0.74

$$-\frac{9\,dx\cos(dx+c)+9\,dx-\left(2\cos(dx+c)^3-\cos(dx+c)^2+7\cos(dx+c)+16\right)\sin(dx+c)}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*c
os(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.93, size = 88, normalized size = 0.94

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)
^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^3*a))/d
```

maple [A] time = 0.58, size = 136, normalized size = 1.45

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^5+16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+3/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-3/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.07, size = 176, normalized size = 1.87

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.91, size = 70, normalized size = 0.74

$$\frac{\frac{15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}+\frac{3\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{4}-\frac{\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{12}+\frac{\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{3x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] ((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2 +
 (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) - (3
*x)/(2*a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

```
[Out] 15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*s
in(d*x+c)/a/d-cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))+4/3*sin(d*x+c)^3/a
/d
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2635, 8, 2633}

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^3*Sin[c + d*x
])/(d*(a + a*Sec[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^4(c+dx)(-5a+4a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{4\int \cos^3(c+dx)\,dx}{a} + \frac{5\int \cos^4(c+dx)\,dx}{a}$$
$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{15\int \cos^2(c+dx)\,dx}{4a} + \frac{4\,\mathrm{Su}}{}$$
$$= -\frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c-}{d(a+}$$
$$= \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \mathrm{co}$$

Mathematica [A] time = 0.32, size = 173, normalized size = 1.47

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(c+\frac{3dx}{2}\right)-120\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+40\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2]
 - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*S
in[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5
*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3
*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

fricas [A] time = 0.69, size = 79, normalized size = 0.67

$$\frac{45\,dx\cos(dx+c)+45\,dx+\left(6\cos(dx+c)^4-2\cos(dx+c)^3+13\cos(dx+c)^2-19\cos(dx+c)-64\right)\sin(}{24\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 +
 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c)
+ a*d)
```

giac [A] time = 3.92, size = 101, normalized size = 0.86

$$\frac{\frac{45\,(dx+c)}{a}-\frac{24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+109\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*
c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [A] time = 0.56, size = 171, normalized size = 1.45

$$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{115\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{109\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{7\tan\left(\frac{dx}{2}\right.}{4da\left(1+\tan^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-25/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^7-115/12/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-109/12/d
/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/d/a/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)+15/4/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.17, size = 217, normalized size = 1.84

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 2.45, size = 98, normalized size = 0.83

$$\frac{15x}{8a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}-\frac{\frac{25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12}+\frac{109\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{ad\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] (15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109*
tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d
*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{8\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] 7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x+
c)/a^2/d-8/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^3*
tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{8\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)(3a-5a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^2(c+dx)\left(16a^2-21a\right.}{3a^4}$$
$$= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{16\int \sec^2(c+dx)\,dx}{3a^2} + \frac{7}{}$$
$$= \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} +$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec^2(c+dx}{3a^2d(1+\mathrm{se}}$$

Mathematica [B] time = 1.90, size = 300, normalized size = 2.44

$$\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-2\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*
x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^(-2) - (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) -
2*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.71, size = 162, normalized size = 1.32

$$\frac{21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\right.}{12\left(a^2d\cos(dx+c)^4+2a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x +
 c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin
(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x + c)
- 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*c
os(d*x + c)^2)
```

giac [A] time = 1.95, size = 122, normalized size = 0.99

$$\frac{\frac{21 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{21 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.36, size = 162, normalized size = 1.32

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{1}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{5}{2a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-7/2/a^2/d*tan(1/2*d*x+1/2*c)+1/2/a^2/d/(tan
(1/2*d*x+1/2*c)-1)^2+5/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)-7/2/a^2/d*ln(tan(1/2*
d*x+1/2*c)-1)-1/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2+5/2/a^2/d/(tan(1/2*d*x+1/2
*c)+1)+7/2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.17, size = 190, normalized size = 1.54

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 0.74, size = 122, normalized size = 0.99

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (3
*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^4
- 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{4\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -2*arctanh(sin(d*x+c))/a^2/d+4/3*tan(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d/(1+sec
(d*x+c))-1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3816, 4008, 3787, 3770, 3767, 8}

$$\frac{4\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (4*Tan[c + d*x])/(3*a^2*d) + (2*Tan[c
+ d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a
+ a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2a-4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\left(-6a^2+4a^2\sec(c\right.}{3a^4}$$

$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{4\int \sec^2(c+dx)\,dx}{3a^2} - \frac{2\int \sec(c+}{a^2}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{4\,\mathrm{Subs}}{}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{4\tan(c+dx)}{3a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(}{3d(a+a\sec(c+}$$

Mathematica [B] time = 1.26, size = 247, normalized size = 2.78

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\sin(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x
]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/
2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.75, size = 146, normalized size = 1.64

$$-\frac{3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\mathrm{co}\right.}{3\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c)
 + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x +
 c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [A] time = 0.43, size = 106, normalized size = 1.19

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2} - \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.40, size = 120, normalized size = 1.35

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}-\frac{1}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{1}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-2\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+5/2/a^2/d*tan(1/2*d*x+1/2*c)-1/a^2/d/(tan(1/
2*d*x+1/2*c)-1)+2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)-1/a^2/d/(tan(1/2*d*x+1/2*c
)+1)-2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.49, size = 145, normalized size = 1.63

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.69, size = 92, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}-\frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2
*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 + (d
*x)/2))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] arctanh(sin(d*x+c))/a^2/d-5/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*tan(d*x+c
)/d/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3799, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (5*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x]
)) + Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3998 Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\,dx}{a^2} - \frac{5\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [B] time = 0.37, size = 160, normalized size = 2.42

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.\right.}{3a^2d(\sec(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[
c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d
*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 1.37, size = 114, normalized size = 1.73

$$\frac{3\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log(-\sin}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c
) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.83, size = 77, normalized size = 1.17

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

maple [A] time = 0.35, size = 77, normalized size = 1.17

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-3/2/a^2/d*tan(1/2*d*x+1/2*c)-1/a^2/d*ln(tan
(1/2*d*x+1/2*c)-1)+1/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.44, size = 98, normalized size = 1.48

$$-\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

mupad [B] time = 0.64, size = 43, normalized size = 0.65

$$-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)^
3)/(6*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+2/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3797, 3794}

$$\frac{2\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*d*(a^2 + a
^2*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 0.82

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[(c + d*x)/2]^3*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(12*a^2*d)
```

fricas [A] time = 0.59, size = 49, normalized size = 0.89

$$\frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

giac [A] time = 0.69, size = 31, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

maple [A] time = 0.41, size = 32, normalized size = 0.58

$$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.50, size = 46, normalized size = 0.84

$$\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

mupad [B] time = 0.60, size = 30, normalized size = 0.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] 1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*d*(a^2 + a^2*Se
c[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.14, size = 60, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\left(-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(\frac{dx}{2}\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[c
 + (3*d*x)/2]))/(12*a^2*d)
```

fricas [A] time = 0.79, size = 51, normalized size = 0.93

$$\frac{(2\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

giac [A] time = 0.50, size = 31, normalized size = 0.56

$$-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

maple [A] time = 0.33, size = 32, normalized size = 0.58

$$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.88, size = 47, normalized size = 0.85

$$\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

mupad [B] time = 0.60, size = 30, normalized size = 0.55

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$-\frac{4\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] x/a^2-4/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))
^2
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3777, 3919, 3794}

$$-\frac{4\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (4*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(3*d*(
a + a*Sec[c + d*x])^2)
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3a+a\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{4\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\
&= \frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{4\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 112, normalized size = 1.96

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12\sin\left(c+\frac{dx}{2}\right)-10\sin\left(c+\frac{3dx}{2}\right)+9dx\cos\left(c+\frac{dx}{2}\right)+3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c\right.\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] + 9*d*x*Cos[c + (d*x)/2] +
 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2] +
12*Sin[c + (d*x)/2] - 10*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

fricas [A] time = 1.37, size = 80, normalized size = 1.40

$$\frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.37, size = 50, normalized size = 0.88

$$\frac{\frac{6\,(dx+c)}{a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

maple [A] time = 0.46, size = 56, normalized size = 0.98

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-3/2/a^2/d*tan(1/2*d*x+1/2*c)+2/a^2/d*arctan(
tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.42, size = 72, normalized size = 1.26

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 0.63, size = 35, normalized size = 0.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -2*x/a^2+10/3*sin(d*x+c)/a^2/d-2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*
x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (10*Sin[c + d*x])/(3*a^2*d) - (2*Sin[c + d*x])/(a^2*d*(1 + Sec
[c + d*x])) - Sin[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(-4a+2a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos(c+dx)\left(-10a^2+6a^2\sec\right.}{3a^4}$$
$$= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{2\int 1\,dx}{a^2} + \frac{10\int \cos(c+dx)\,dx}{3a^2}$$
$$= -\frac{2x}{a^2} + \frac{10\sin(c+dx)}{3a^2d} - \frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [B] time = 0.53, size = 151, normalized size = 2.10

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+41\sin\left(c+\frac{3dx}{2}\right)+9\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c\right.\right.}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2
] - 12*d*x*Cos[c + (3*d*x)/2] - 12*d*x*Cos[2*c + (3*d*x)/2] + 66*Sin[(d*x)/
2] - 30*Sin[c + (d*x)/2] + 41*Sin[c + (3*d*x)/2] + 9*Sin[2*c + (3*d*x)/2] +
 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(48*a^2*d)
```

fricas [A] time = 0.82, size = 90, normalized size = 1.25

$$-\frac{6\,dx\cos\left(dx+c\right)^2+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^2+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)^
2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

giac [A] time = 0.34, size = 79, normalized size = 1.10

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 +
 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/
d
```

maple [A] time = 0.50, size = 88, normalized size = 1.22

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+5/2/a^2/d*tan(1/2*d*x+1/2*c)+2/a^2/d*tan(1/
2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.92, size = 118, normalized size = 1.64

$$\frac{\frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 + \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)
/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.70, size = 91, normalized size = 1.26

$$-\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 16\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 12\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 12\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (c + dx)}{6 a^2 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6
*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{8\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] 7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*x
+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{8\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(-5a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(-21a^2+16a\right.}{3a^4}$$

$$= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{16\int \cos(c+dx)\,dx}{3a^2} + \frac{7\int c}{}$$

$$= -\frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+d}{3d(a+a}$$

$$= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(}{3d(}$$

Mathematica [A] time = 0.39, size = 177, normalized size = 1.61

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(147\sin\left(c+\frac{dx}{2}\right)-239\sin\left(c+\frac{3dx}{2}\right)-63\sin\left(2c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{5dx}{2}\right)-15\sin\left(3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)/
2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d*x
)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d*x)
/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (7*d*
x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

fricas [A] time = 0.95, size = 99, normalized size = 0.90

$$\frac{21\,dx\cos(dx+c)^2+42\,dx\cos(dx+c)+21\,dx+\left(3\cos(dx+c)^3-6\cos(dx+c)^2-43\cos(dx+c)-32\right)\sin(}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)
^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.39, size = 95, normalized size = 0.86

$$\frac{\frac{21\,(dx+c)}{a^2} - \frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\,a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*
a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.60, size = 122, normalized size = 1.11

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-7/2/a^2/d*tan(1/2*d*x+1/2*c)-5/a^2/d/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)+7/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.13, size = 164, normalized size = 1.49

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2)/d
```

mupad [B] time = 0.73, size = 113, normalized size = 1.03

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-22\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x
)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)^
2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*
x+c))^2-4*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3817, 4020, 3787, 2633, 2635, 8}

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Cos[
c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(-6a+4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^3(c+dx)\left(-36a^2 \ldots\right.}{3 \ldots}$$

$$= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{10\int \cos^2(c+dx)\,dx}{a^2} + \ldots$$

$$= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+ \ldots}{3d(a+a\sec(c+dx) \ldots}$$

$$= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \ldots$$

Mathematica [A] time = 0.45, size = 199, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-156\sin\left(c+\frac{dx}{2}\right)+342\sin\left(c+\frac{3dx}{2}\right)+118\sin\left(2c+\frac{3dx}{2}\right)+30\sin\left(2c+\frac{5dx}{2}\right)+30\,\mathrm{s}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x)
/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin[(
d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c + (3*
d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c + (
7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*
x)/2]))/(192*a^2*d)
```

fricas [A] time = 1.40, size = 108, normalized size = 0.87

$$-\frac{15\,dx\cos\left(dx+c\right)^2+30\,dx\cos\left(dx+c\right)+15\,dx-\left(\cos\left(dx+c\right)^4-\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)^2+33\,\cos\left(\ldots\right.\right.}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)^
4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.91, size = 108, normalized size = 0.87

$$-\frac{\frac{30\,(dx+c)}{a^2}-\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1/
2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (a^
4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.60, size = 156, normalized size = 1.26

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^2d}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{40\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^2d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+9/2/a^2/d*tan(1/2*d*x+1/2*c)+10/a^2/d/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+40/3/a^2/d/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^3+6/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)-10/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.12, size = 207, normalized size = 1.67

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2)/d
```

mupad [B] time = 0.78, size = 135, normalized size = 1.09

$$-\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-60\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
```

```
x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/2)^
3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{76\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{\tan(c+dx)}{5d(a\sec(c}$$

```
[Out] 13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan(
d*x+c)/a^3/d-1/5*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*sec(d*x
+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-76/15*sec(d*x+c)^2*tan(d*x+c)/d/(a^
3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{76\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{\tan(c+dx)}{5d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - (11*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Se
c[c + d*x])^2) - (76*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c +
d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)(4a-7a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)\left(33a^2-43a^2\sec\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\tan(c}{15d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\tan(c}{15d\left(a^3+a^3\sec(c+\right.}$$

$$= \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+d}{15ad(a+a\sec(c+dx))}$$

$$= \frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c}{5d(a+}$$

Mathematica [B] time = 1.05, size = 351, normalized size = 2.17

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-4329\sin\left(c-\frac{dx}{2}\right)+1989\sin\left(c+\frac{dx}{2}\right)-3575\sin\left(2c+\frac{dx}{2}\right)-4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(24960*Cos[(c + d*x)/2]^5*(Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2
] - 4329*Sin[c - (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2]
 - 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d
*x)/2] + 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c
+ (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*S
in[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] +
```

```
 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)
/2])))/(a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.81, size = 206, normalized size = 1.27

$$\frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\left(\cos(dx+c)^5\right.}{60\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*c
os(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c
)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x +
 c)^3 + a^3*d*cos(d*x + c)^2)
```

giac [A] time = 3.53, size = 139, normalized size = 0.86

$$\frac{\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^{12}\tan\left(\frac{1}{2}dx-\right.}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 40
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.42, size = 181, normalized size = 1.12

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da^3}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da^3}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da^3}+\frac{1}{2da^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2da^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{13\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+7/2/d/a^3/(tan(1/2*d*
x+1/2*c)-1)-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^3/(tan(1/2*d*x+1/2*
c)+1)^2+7/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1
)
```

maxima [A] time = 0.62, size = 211, normalized size = 1.30

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

mupad [B] time = 0.68, size = 141, normalized size = 0.87

$$\frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^3\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
(2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 + (d
*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3))
 - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$\frac{9\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{3\tan(c+dx)\sec^2(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+9/5*tan(d*x+c)/a^3/d-1/5*sec(d*x+c)^3*tan(d*x+
c)/d/(a+a*sec(d*x+c))^3-3/5*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
3*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{9\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{3\tan(c+dx)\sec^2(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (9*Tan[c + d*x])/(5*a^3*d) - (Sec[c +
d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (3*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (3*Tan[c + d*x])/(d*(a^3 + a^3*Se
c[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(3a-6a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(18a^2-27a^2\sec(\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{9\tan(c+dx)}{5a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+}{5ad(a+a}$$

Mathematica [B] time = 1.34, size = 294, normalized size = 2.30

$$2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(8\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\,\mathrm{co}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)
/2]^2*Sec[c/2]*Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] +
 20*Cos[(c + d*x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos
[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan
[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.87, size = 190, normalized size = 1.48

$$-\frac{15\left(\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{4}+\right.}{10\left(a^{3}d\cos\left(dx+c\right)^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x +
 c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 +
57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [A] time = 1.99, size = 122, normalized size = 0.95

$$-\frac{\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}+\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{3}}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+85\,a^{12}\tan\left(\frac{1}{2}\right.}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^
12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.36, size = 139, normalized size = 1.09

$$\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^{3}}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{3}}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}-\frac{1}{d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{3}}-\frac{1}{d\,a^{3}\left(\tan\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.39, size = 165, normalized size = 1.29

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^{3}-\frac{a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3)/d
```

mupad [B] time = 0.68, size = 111, normalized size = 0.87

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atanh
(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d
*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{29\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

```
[Out] arctanh(sin(d*x+c))/a^3/d-1/5*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+
7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-29/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+
c))
```

Rubi [A] time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3816, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{29\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*S
ec[c + d*x])^3) + (7*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (29*Ta
n[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
```

```
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)(2a-5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-14a^2+15a^2\sec(c\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\,dx}{a^3} - \frac{29\int}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{}{15}$$

Mathematica [A] time = 0.51, size = 209, normalized size = 1.99

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(14\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*S
ec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos
[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Co
s[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.73, size = 158, normalized size = 1.50

$$\frac{15\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^3+3\cos(dx\right.}{30\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(d
*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

giac [A] time = 1.93, size = 94, normalized size = 0.90

$$\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+105a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d

maple [A] time = 0.41, size = 96, normalized size = 0.91

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x)

[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*tan(1/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)

maxima [A] time = 0.75, size = 119, normalized size = 1.13

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d

mupad [B] time = 0.69, size = 58, normalized size = 0.55

$$-\frac{105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)

[Out] -(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=83

$$\frac{7\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] 1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
7/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3799, 4000, 3794}

$$\frac{7\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (7*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\sec(c+dx)(-3a+5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\
&= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{7\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}\\
&= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{7\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 57, normalized size = 0.69

$$\frac{\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[(c + d*x)/2]^5*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*
(c + d*x))/2]))/(120*a^3*d)
```

fricas [A] time = 0.58, size = 75, normalized size = 0.90

$$\frac{\left(2\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.57, size = 46, normalized size = 0.55

$$\frac{3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

maple [A] time = 0.41, size = 45, normalized size = 0.54

$$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

maxima [A] time = 0.89, size = 67, normalized size = 0.81

$$\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

mupad [B] time = 0.62, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15)
)/(60*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(c+dx)}{5d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\tan(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/5*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
1/5*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{\tan(c+dx)}{5d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\tan(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + Tan[c + d*x]/(5*a*d*(a + a*Sec
[c + d*x])^2) + Tan[c + d*x]/(5*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{5a^2}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{5d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.16, size = 71, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5\sin\left(c+\frac{dx}{2}\right)+5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{80a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*Sin[(d*x)/2] - 5*Sin[c + (d*x)/2] + 5*Sin[c
 + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))/(80*a^3*d)
```

fricas [A] time = 0.57, size = 73, normalized size = 0.88

$$\frac{\left(\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.25, size = 31, normalized size = 0.37

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

maple [A] time = 0.36, size = 32, normalized size = 0.39

$$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.72, size = 47, normalized size = 0.57

$$\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)
^5)/(a^3*d)
```

mupad [B] time = 0.60, size = 30, normalized size = 0.36

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-5\right)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{2\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{2\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] 1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+2/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+
2/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{2\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{2\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}\\ &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}\\ &= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.22, size = 86, normalized size = 1.04

$$\frac{\sec\left(\frac{c}{2}\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+d\right.}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*Si
n[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(240*
a^3*d)
```

fricas [A] time = 0.58, size = 75, normalized size = 0.90

$$\frac{\left(7\,\cos\left(dx+c\right)^{2}+6\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.00, size = 46, normalized size = 0.55

$$\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

maple [A] time = 0.39, size = 45, normalized size = 0.54

$$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

maxima [A] time = 0.76, size = 67, normalized size = 0.81

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

mupad [B] time = 0.62, size = 45, normalized size = 0.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+15\right)}{60\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15)
)/(60*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

3.67 $\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{22\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] x/a^3-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^2-22/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{22\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (7*Tan[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - (22*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]
))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5a+2a\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2-7a^2\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^4}$$
$$= \frac{x}{a^3} - \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{22\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{x}{a^3} - \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{22\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx\right.}$$

Mathematica [A] time = 0.28, size = 162, normalized size = 1.84

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)+26\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\right.}{15a^3d(\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]*
Sin[(d*x)/2] + 26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c + d
*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c +
d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.56, size = 116, normalized size = 1.32

$$\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(32\cos(dx+c)^2+51\cos(dx+c)+2\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.51, size = 68, normalized size = 0.77

$$\frac{\frac{60\,(dx+c)}{a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.42, size = 75, normalized size = 0.85

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.96, size = 92, normalized size = 1.05

$$-\frac{\frac{\frac{105\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{20\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{120\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.69, size = 81, normalized size = 0.92

$$\frac{x}{a^3}-\frac{\frac{32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{15}-\frac{13\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{30}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 + (
d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{3}{\left(c+dx\right)}+3\sec^{2}{\left(c+dx\right)}+3\sec{\left(c+dx\right)}+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a
**3
```

3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{24\sin(c+dx)}{5a^3d}-\frac{3\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{3\sin(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -3*x/a^3+24/5*sin(d*x+c)/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-3/5*sin(
d*x+c)/a/d/(a+a*sec(d*x+c))^2-3*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{24\sin(c+dx)}{5a^3d}-\frac{3\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{3\sin(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 + (24*Sin[c + d*x])/(5*a^3*d) - Sin[c + d*x]/(5*d*(a + a*Sec[c +
 d*x])^3) - (3*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (3*Sin[c + d*
x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-6a+3a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-27a^2+18a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,d}{15a^4}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} - \int \cos$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} - \frac{3\int 1}{a^3}$$

$$= -\frac{3x}{a^3} + \frac{24\sin(c+dx)}{5a^3d} - \frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\,\mathrm{s}}{d\left(a^3+\right.}$$

Mathematica [A] time = 0.58, size = 169, normalized size = 1.64

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(20(\sin(c+dx)-3dx)\cos^5\left(\frac{1}{2}(c+dx)\right)-12\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\mathrm{co}\right.}{5a^3d(\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] - 12*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 96*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2]
+ 20*Cos[(c + d*x)/2]^5*(-3*d*x + Sin[c + d*x]) + Cos[(c + d*x)/2]*Tan[c/2]
 - 12*Cos[(c + d*x)/2]^3*Tan[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 1.30, size = 126, normalized size = 1.22

$$-\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(5\cos(dx+c)^3+39\cos(dx+c)^2+57\right.}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin
(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

giac [A] time = 4.36, size = 96, normalized size = 0.93

$$-\frac{\frac{60\,(dx+c)}{a^3} - \frac{40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^3} - \frac{a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10\,a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+85\,a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 +
 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.61, size = 107, normalized size = 1.04

$$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^3}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a
^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.21, size = 137, normalized size = 1.33

$$\frac{\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.73, size = 113, normalized size = 1.10

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x
)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=147

$$-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{76\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13x}{2a^3}-\frac{11\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c}{5d(a}$$

```
[Out] 13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*cos
(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*cos(d*x+c)*sin(d*x+c)/a/d/(a+
a*sec(d*x+c))^2-76/15*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{76\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13x}{2a^3}-\frac{11\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c}{5d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) -
 (11*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (76*Cos[c
 + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-7a+4a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-43a^2+33a^2\sec(\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c+d}{15d\left(a^3+a^3\sec(c+dx\right)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c+d}{15d\left(a^3+a^3\sec(c+dx\right)}$$

$$= -\frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos}{15ad}$$

$$= \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} -$$

Mathematica [A] time = 0.58, size = 181, normalized size = 1.23

$$2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(15(-12\sin(c+dx)+\sin(2(c+dx))+26dx)\cos^5\left(\frac{1}{2}(c+dx)\right)+46\tan\left(\frac{c}{2}\right)\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c +
d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)
/2] + 15*Cos[(c + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]) -
 3*Cos[(c + d*x)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(
1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.68, size = 135, normalized size = 0.92

$$\frac{195\,dx\cos(dx+c)^3+585\,dx\cos(dx+c)^2+585\,dx\cos(dx+c)+195\,dx+\left(15\cos(dx+c)^4-45\cos(dx+\right.}{30\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c
) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 -
 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.81, size = 113, normalized size = 0.77

$$\frac{\frac{390\,(dx+c)}{a^3}-\frac{60\left(7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.56, size = 141, normalized size = 0.96

$$-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20d\,a^3}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^3}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{13\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-
5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1
/2*d*x+1/2*c))
```

maxima [A] time = 1.02, size = 184, normalized size = 1.25

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 0.76, size = 137, normalized size = 0.93

$$-\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-46\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+508\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 +
```

```
(d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 + (d*
x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=193

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{43\tan(c+dx)\sec^3(c+dx)}{35a^4d(\sec(c+dx)+1)^2}-\frac{288\tan(c+dx)\sec^2(c+dx)}{35a^4d(\sec(c+dx)+1)}+\frac{21\tan(c+$$

```
[Out] 21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan(
d*x+c)/a^4/d-43/35*sec(d*x+c)^3*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*se
c(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^5*tan(d*x+c)/d/(a
+a*sec(d*x+c))^4-2/5*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A] time = 0.39, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{43\tan(c+dx)\sec^3(c+dx)}{35a^4d(\sec(c+dx)+1)^2}-\frac{288\tan(c+dx)\sec^2(c+dx)}{35a^4d(\sec(c+dx)+1)}+\frac{21\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]^3*Tan[c + d*x])/(35
*a^4*d*(1 + Sec[c + d*x])^2) - (288*Sec[c + d*x]^2*Tan[c + d*x])/(35*a^4*d*
(1 + Sec[c + d*x])) - (Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x
])^4) - (2*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^5(c+dx)(5a-9a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)\left(56a^2-73a^2\sec(\right.}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+}{5ad(a+a\sec(c+dx}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+}{5ad(a+a\sec(c+dx}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+}{5ad(a+a\sec(c+dx}$$

$$= \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+}{7d(a+a\sec(c+dx)}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3}{35a^4}$$

Mathematica [B] time = 1.56, size = 403, normalized size = 2.09

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-61054\sin\left(c-\frac{dx}{2}\right)+33614\sin\left(c+\frac{dx}{2}\right)-51842\sin\left(2c+\frac{dx}{2}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/2240*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(376320*Cos[(c + d*x)/2]^7*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(-24402*Sin[(d*x)/2] + 55556*Sin[(3*d*
x)/2] - 61054*Sin[c - (d*x)/2] + 33614*Sin[c + (d*x)/2] - 51842*Sin[2*c + (
d*x)/2] - 12460*Sin[c + (3*d*x)/2] + 33716*Sin[2*c + (3*d*x)/2] - 34300*Sin
[3*c + (3*d*x)/2] + 39788*Sin[c + (5*d*x)/2] - 2940*Sin[2*c + (5*d*x)/2] +
```

```
26068*Sin[3*c + (5*d*x)/2] - 16660*Sin[4*c + (5*d*x)/2] + 21351*Sin[2*c + (
7*d*x)/2] + 1295*Sin[3*c + (7*d*x)/2] + 14911*Sin[4*c + (7*d*x)/2] - 5145*S
in[5*c + (7*d*x)/2] + 7329*Sin[3*c + (9*d*x)/2] + 1225*Sin[4*c + (9*d*x)/2]
 + 5369*Sin[5*c + (9*d*x)/2] - 735*Sin[6*c + (9*d*x)/2] + 1152*Sin[4*c + (1
1*d*x)/2] + 280*Sin[5*c + (11*d*x)/2] + 872*Sin[6*c + (11*d*x)/2]))/(a^4*d
*(1 + Sec[c + d*x])^4)
```

fricas [A] time = 0.53, size = 250, normalized size = 1.30

$$\frac{735\left(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-7\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*
x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*
cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*co
s(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

giac [A] time = 0.50, size = 155, normalized size = 0.80

$$\frac{\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}-\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63a^{24}\tan\left(\frac{1}{2}\cdots\right.}{\cdots}}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7
+ 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a
^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.44, size = 200, normalized size = 1.04

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}-\frac{111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}+\frac{1}{2d\,a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\cdots}{2d\,a^4\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4
*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)+1/2/d/a^4/(tan(1/2*d*x
+1/2*c)-1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)-21/2/d/a^4*ln(tan(1/2*d*x+1/2
*c)-1)-1/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)+
21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.08, size = 231, normalized size = 1.20

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx-\cdots}{\cos(dx+\cdots}\right.}{a^4}}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^
4)/d
```

mupad [B] time = 0.75, size = 160, normalized size = 0.83

$$\frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4\,d}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^4\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\tan\left(\right.}{d\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*d
) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) -
 (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)/2)
^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a^4*d
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=159

$$\frac{244\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{88\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4\tan(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^4(c+}{7d(a\sec(c+dx)+}$$

```
[Out] -4*arctanh(sin(d*x+c))/a^4/d+244/105*tan(d*x+c)/a^4/d-88/105*sec(d*x+c)^2*t
an(d*x+c)/a^4/d/(1+sec(d*x+c))^2+4*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(
d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35*sec(d*x+c)^3*tan(d*x+c)/a/d/
(a+a*sec(d*x+c))^3
```

Rubi [A] time = 0.37, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{244\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{88\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4\tan(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^4(c+}{7d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (244*Tan[c + d*x])/(105*a^4*d) - (88*S
ec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*Tan[c + d
*x])/(a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a
*Sec[c + d*x])^4) - (12*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c +
 d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)(4a-8a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(36a^2-52a^2\sec\right.}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c}{35ad(a+a\sec(c+d}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c}{35ad(a+a\sec(c+d}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c}{35ad(a+a\sec(c+d}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{244\tan(c+dx)}{105a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(}{7d(a}$$

Mathematica [B] time = 1.29, size = 349, normalized size = 2.19

$$\cos\left(\tfrac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\tfrac{c}{2}\right)\sec(c)\left(-20524\sin\left(c-\tfrac{dx}{2}\right)+14644\sin\left(c+\tfrac{dx}{2}\right)-16660\sin\left(2c+\tfrac{dx}{2}\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]

[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^4*(107520*Cos[(c + d*x)/2]^7*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20
524*Sin[c - (d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] -
```

```
4690*Sin[c + (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*
x)/2] + 11668*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c
+ (5*d*x)/2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*
Sin[3*c + (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2]
 + 664*Sin[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d
*x)/2])))/(1680*a^4*d*(1 + Sec[c + d*x])^4)
```

fricas [A] time = 1.05, size = 234, normalized size = 1.47

$$-\frac{210\left(\cos(dx+c)^{5}+4\cos(dx+c)^{4}+6\cos(dx+c)^{3}+4\cos(dx+c)^{2}+\cos(dx+c)\right)\log(\sin(dx+c)+1)-}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d
*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*c
os(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-si
n(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x
+ c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a
^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4
*d*cos(d*x + c))
```

giac [A] time = 0.50, size = 139, normalized size = 0.87

$$-\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{4}}-\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{4}}+\frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a^{4}}-\frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+147a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+805}{a^{28}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c
)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

maple [A] time = 0.51, size = 158, normalized size = 0.99

$$\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}+\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{4}}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}-\frac{1}{d\,a^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4
*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4/(tan(1/2*d*x+1/
2*c)-1)+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)-4/d
/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 1.01, size = 186, normalized size = 1.17

$$\frac{\frac{1680\sin(dx+c)}{\left(a^{4}-\frac{a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{147\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{4}}+\frac{3360\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^{4}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

mupad [B] time = 0.70, size = 130, normalized size = 0.82

$$\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^4\,d}-\frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)}+\frac{49\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
+ tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c/2
+ (d*x)/2))/(8*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4} dx$

Optimal. Leaf size=136

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{43\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}+\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)s}{7ad(a\sec(c-}$$

```
[Out] arctanh(sin(d*x+c))/a^4/d+11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-43/21*tan
(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))
^4-2/7*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A] time = 0.32, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3816, 4019, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{43\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}+\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)s}{7ad(a\sec(c-}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*
x])^2) - (43*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Sec[c + d*x]^2*Tan[c + d*x]
)/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
```

```
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)(3a-7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(20a^2-35a^2\sec(\right.}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

$$= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\,\mathrm{se}}{7a}$$

Mathematica [A] time = 0.94, size = 193, normalized size = 1.42

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\frac{c}{2}\right)\left(-434\sin\left(c+\frac{dx}{2}\right)+525\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+203\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/84*(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(1344*Cos[(c + d*x)/2]^7*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
+ Sec[c/2]*(686*Sin[(d*x)/2] - 434*Sin[c + (d*x)/2] + 525*Sin[c + (3*d*x)/2
] - 147*Sin[2*c + (3*d*x)/2] + 203*Sin[2*c + (5*d*x)/2] - 21*Sin[3*c + (5*d
*x)/2] + 32*Sin[3*c + (7*d*x)/2])))/(a^4*d*(1 + Sec[c + d*x])^4)
```

fricas [A] time = 0.59, size = 202, normalized size = 1.49

$$\frac{21\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-21\left(\cos(dx\right.}{42\left(a^4d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
+ c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6
*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*
x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d
*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d
*cos(d*x + c) + a^4*d)
```

giac [A] time = 2.87, size = 110, normalized size = 0.81

$$\frac{\frac{168 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{168 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} - \frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315a^{24}\tan\left(\frac{1}{2}\right.}{a^{28}}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

maple [A] time = 0.40, size = 115, normalized size = 0.85

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^4}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4
*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4*ln(tan(1/2*d*x+
1/2*c)-1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.78, size = 139, normalized size = 1.02

$$-\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 +
168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

mupad [B] time = 0.67, size = 83, normalized size = 0.61

$$-\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56\,a^4}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(c
/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/2
+ (d*x)/2))/(8*a^4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{\tan(c+dx)}{5d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

```
[Out] 1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*s
ec(d*x+c))^3-8/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+1/5*tan(d*x+c)/d/(a^4
+a^4*sec(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3810, 3799, 4000, 3794}

$$\frac{\tan(c+dx)}{5d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x
])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[
c + d*x])^2) + Tan[c + d*x]/(5*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{3\int \frac{\sec(c+dx)(-3a+5a\sec(c+dx)}{(a+a\sec(c+dx))^2}}{35a^3}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}$$

Mathematica [A] time = 0.20, size = 69, normalized size = 0.58

$$\frac{\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)}{1120a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[(c + d*x)/2]^7*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[
(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(1120*a^4*d)
```

fricas [A] time = 1.43, size = 99, normalized size = 0.82

$$\frac{\left(2\cos(dx+c)^3+8\cos(dx+c)^2+13\cos(dx+c)+12\right)\sin(dx+c)}{35\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 2.87, size = 59, normalized size = 0.49

$$\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*
x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.41, size = 56, normalized size = 0.47

$$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/
2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.02, size = 87, normalized size = 0.72

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.67, size = 58, normalized size = 0.48

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+35\right)}{280\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5*
tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{13\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{13\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{11\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] 1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-11/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+13/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+13/105*tan(d*x+c)/d/(a^4+a^4*se
c(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3799, 4000, 3796, 3794}

$$\frac{13\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{13\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{11\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (11*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (13*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
 (13*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)(-4a+7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \cdots$$
$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \cdots$$

Mathematica [A] time = 0.24, size = 87, normalized size = 0.78

$$\frac{\sec\left(\frac{c}{2}\right)\left(-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35\sin\left(\frac{dx}{2}\right)\right)\sec^7\left(\frac{1}{2}(c+dx\cdots\right.}{1680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(21
*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(168
0*a^4*d)
```

fricas [A] time = 0.90, size = 99, normalized size = 0.88

$$\frac{\left(8\cos\left(dx+c\right)^3+32\cos\left(dx+c\right)^2+52\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.88, size = 59, normalized size = 0.53

$$-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.36, size = 58, normalized size = 0.52

$$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5+1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.03, size = 87, normalized size = 0.78

$$\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.66, size = 58, normalized size = 0.52

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+105\right)}{840\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 15
*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{8\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{8\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] -1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+4/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+8/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+8/105*tan(d*x+c)/d/(a^4+a^4*sec(
d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{8\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{8\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (8*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
(8*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [A] time = 0.27, size = 99, normalized size = 0.88

$$\frac{\sec\left(\frac{c}{2}\right)\left(-175\sin\left(c+\frac{dx}{2}\right)+168\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+91\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(3c+\frac{7dx}{2}\right)+2\right.}{6720a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(280*Sin[(d*x)/2] - 175*Sin[c + (d*x)/2] + 168
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 91*Sin[2*c + (5*d*x)/2] +
13*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

fricas [A] time = 0.81, size = 99, normalized size = 0.88

$$\frac{\left(13\cos\left(dx+c\right)^{3}+52\cos\left(dx+c\right)^{2}+32\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4a^{4}d\cos\left(dx+c\right)^{3}+6a^{4}d\cos\left(dx+c\right)^{2}+4a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.59, size = 59, normalized size = 0.53

$$\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d
*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.41, size = 58, normalized size = 0.52

$$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*
x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.79, size = 87, normalized size = 0.78

$$\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.67, size = 58, normalized size = 0.52

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-105\right)}{840\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2{(c+dx)}}{\sec^4{(c+dx)}+4\sec^3{(c+dx)}+6\sec^2{(c+dx)}+4\sec{(c+dx)}+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{2\tan(c+dx)}{35d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] 1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3+
2/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/35*tan(d*x+c)/d/(a^4+a^4*sec(d*x
+c))
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3796, 3794}

$$\frac{2\tan(c+dx)}{35d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x])/(35*a*d*(a + a
*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[c + d*x])^2) + (2
*Tan[c + d*x])/(35*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}\\
&= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{6\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}\\
&= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}\\
&= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 112, normalized size = 1.00

$$\frac{\sec\left(\frac{c}{2}\right)\left(-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{5dx}{2}\right)+12\,\mathrm{si}\right.}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 147
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2] -
35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

fricas [A] time = 0.65, size = 99, normalized size = 0.88

$$\frac{\left(12\,\cos\left(dx+c\right)^3+13\,\cos\left(dx+c\right)^2+8\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.50, size = 59, normalized size = 0.53

$$-\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.34, size = 58, normalized size = 0.52

$$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.64, size = 87, normalized size = 0.78

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

mupad [B] time = 0.67, size = 58, normalized size = 0.52

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\right)}{280\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)
**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.77 $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=111

$$-\frac{32\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}-\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}+\frac{x}{a^4}-\frac{2\tan(c+dx)}{7ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] x/a^4-11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-32/21*tan(d*x+c)/a^4/d/(1+sec
(d*x+c))-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-2/7*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^3
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{32\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}-\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}+\frac{x}{a^4}-\frac{2\tan(c+dx)}{7ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] x/a^4 - (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])^2) - (32*Tan[c + d*x
])/(21*a^4*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4
) - (2*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{-7a+3a\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} + \frac{\int \frac{35a^2-20a^2\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - $$

$$= \frac{x}{a^4} - \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

$$= \frac{x}{a^4} - \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

Mathematica [B] time = 0.39, size = 224, normalized size = 2.02

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(1652\sin\left(c+\frac{dx}{2}\right) - 1428\sin\left(c+\frac{3dx}{2}\right) + 756\sin\left(2c+\frac{3dx}{2}\right) - 560\sin\left(2c+\frac{5dx}{2}\right) + 16\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)/
2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*Co
s[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*d*x
)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (d*x)
/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c + (5
*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(2688*a^4*
d)
```

fricas [A] time = 1.04, size = 152, normalized size = 1.37

$$\frac{21\,dx\cos(dx+c)^4 + 84\,dx\cos(dx+c)^3 + 126\,dx\cos(dx+c)^2 + 84\,dx\cos(dx+c) + 21\,dx - \left(52\cos(dx+c\right.}{21\left(a^4d\cos(dx+c)^4 + 4\,a^4d\cos(dx+c)^3 + 6\,a^4d\cos(dx+c)^2 + 4\,a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)^
2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^2
+ 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.41, size = 83, normalized size = 0.75

$$\frac{\frac{168\,(dx+c)}{a^4} + \frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2
*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

maple [A] time = 0.47, size = 94, normalized size = 0.85

$$\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^{4}}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{4}}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*
tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d
*x+1/2*c))
```

maxima [A] time = 1.19, size = 112, normalized size = 1.01

$$-\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{3\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

mupad [B] time = 0.72, size = 102, normalized size = 0.92

$$\frac{x}{a^{4}}+\frac{-\frac{52\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{21}+\frac{16\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{21}-\frac{5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{28}+\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{56}}{a^{4}\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^4,x)
```

```
[Out] x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)
)/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d*x
)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^{4}(c+dx)+4\sec^{3}(c+dx)+6\sec^{2}(c+dx)+4\sec(c+dx)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec
(c + d*x) + 1), x)/a**4
```

3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=126

$$\frac{664\sin(c+dx)}{105a^4d}-\frac{4\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{88\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4x}{a^4}-\frac{12\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+}{7d(a\sec(c+}$$

```
[Out] -4*x/a^4+664/105*sin(d*x+c)/a^4/d-88/105*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-
4*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A] time = 0.30, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{664\sin(c+dx)}{105a^4d}-\frac{4\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{88\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4x}{a^4}-\frac{12\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+}{7d(a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 + (664*Sin[c + d*x])/(105*a^4*d) - (88*Sin[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) - (4*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - Sin[c
 + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (12*Sin[c + d*x])/(35*a*d*(a + a*Sec
[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(-8a+4a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-52a^2+36a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \int$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{}{d}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{}{d}$$

$$= -\frac{4x}{a^4} + \frac{664\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{}{35ad}$$

Mathematica [B] time = 0.46, size = 263, normalized size = 2.09

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(46130\sin\left(c+\frac{dx}{2}\right)-46116\sin\left(c+\frac{3dx}{2}\right)+18060\sin\left(2c+\frac{3dx}{2}\right)-19292\sin\left(2c+\frac{5dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/26880*(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*C
os[c + (d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d*x
)/2] + 5880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 840*
d*x*Cos[3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*x)/2
] + 46130*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c + (3*
d*x)/2] - 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 3791*Sin
[3*c + (7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)/2] - 1
05*Sin[5*c + (9*d*x)/2]))/(a^4*d)
```

fricas [A] time = 0.89, size = 162, normalized size = 1.29

$$-\frac{420\,dx\cos\left(dx+c\right)^4+1680\,dx\cos\left(dx+c\right)^3+2520\,dx\cos\left(dx+c\right)^2+1680\,dx\cos\left(dx+c\right)+420\,dx-\left(105\,\cos\right.}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*x
 + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*cos(
d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.50, size = 112, normalized size = 0.89

$$-\frac{\frac{3360\,(dx+c)}{a^4}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1
/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

maple [A] time = 0.60, size = 126, normalized size = 1.00

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}-\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^4}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{8\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5-23/24/d/a^
4*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*tan(1/2*d*x+1/
2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.80, size = 158, normalized size = 1.25

$$\frac{\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4)/d
```

mupad [B] time = 0.75, size = 137, normalized size = 1.09

$$-\frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-192\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\mathrm{s}}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^4,x)
```

```
[Out] -(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 114
4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin(c
/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(c/2
 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)
**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=176

$$-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{288\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)}-\frac{43\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)^2}+\frac{21x}{2a^4}-$$

```
[Out] 21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*c
os(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*cos(d*x+c)*sin(d*x+c)/a^
4/d/(1+sec(d*x+c))-1/7*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-2/5*cos(d
*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A] time = 0.39, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{288\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)}-\frac{43\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)^2}+\frac{21x}{2a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x]
)^2) - (288*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x])) - (Cos
[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Cos[c + d*x]*Sin[
c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(-9a+5a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-73a^2+56a^2\sec(c+dx\right.}{(a+a\sec(c+dx))^2}}{35a^4}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} -$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} -$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} -$$

$$= -\frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c}{7d(a}$$

$$= \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} -$$

Mathematica [A] time = 0.58, size = 289, normalized size = 1.64

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(128730\sin\left(c+\frac{dx}{2}\right) - 140826\sin\left(c+\frac{3dx}{2}\right) + 44310\sin\left(2c+\frac{3dx}{2}\right) - 60487\sin\left(2c+\frac{5d}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
(d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2] +
20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d*x*
Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)/2]
+ 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c + (3*
d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12001*Si
n[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*x)/2] -
 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c + (11*d*x
)/2]))/(35840*a^4*d)
```

fricas [A] time = 0.66, size = 171, normalized size = 0.97

$$\frac{735\,dx\cos(dx+c)^4 + 2940\,dx\cos(dx+c)^3 + 4410\,dx\cos(dx+c)^2 + 2940\,dx\cos(dx+c) + 735\,dx + \left(35\,\cos(\right.}{70\left(a^4d\cos(dx+c)^4 + 4\,a^4d\cos(dx+c)^3 + 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x +
 c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x
+ c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 11
52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.45, size = 128, normalized size = 0.73

$$\frac{\frac{2940\,(dx+c)}{a^4} - \frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^4} + \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3885\,a^{24}\tan}{a^{28}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c
)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.64, size = 160, normalized size = 0.91

$$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d\,a^4} - \frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4} + \frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4} - \frac{111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4} - \frac{9\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{7\,\mathrm{ta}}{d\,a^4\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*
tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.09, size = 204, normalized size = 1.16

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^4)/d
```

mupad [B] time = 0.81, size = 159, normalized size = 0.90

$$\frac{5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-78\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+596\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{280\,a^4\,d\cos\left(\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^4,x)
```

```
[Out] (5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2 +
 (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 + (d
*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(280*a^
4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=200

$$\frac{181\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{5\tan(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{67\tan(c+dx)\sec^2(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\tan(c+dx)}{63a^2d(a\sec(c$$

```
[Out] -5*arctanh(sin(d*x+c))/a^5/d+181/63*tan(d*x+c)/a^5/d-1/9*sec(d*x+c)^5*tan(d
*x+c)/d/(a+a*sec(d*x+c))^5-5/21*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c)
)^4-29/63*sec(d*x+c)^3*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-67/63*sec(d*x+c)
^2*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2+5*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.48, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{181\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{29\tan(c+dx)\sec^3(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{67\tan(c+dx)\sec^2(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}+\frac{5\tan(}{d\left(a^5\sec(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (181*Tan[c + d*x])/(63*a^5*d) - (Sec[c
 + d*x]^5*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) - (5*Sec[c + d*x]^4*Ta
n[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sec[c + d*x]^3*Tan[c + d*
x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sec[c + d*x]^2*Tan[c + d*x])/(6
3*a^3*d*(a + a*Sec[c + d*x])^2) + (5*Tan[c + d*x])/(d*(a^5 + a^5*Sec[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^5(c+dx)(5a-10a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)\left(60a^2-85a^2\sec(c+d\right.}{(a+a\sec(c+dx))^3}}{63a^4}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx}{63a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx}{63a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx}{63a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx}{63a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{2}{6}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{181\tan(c+dx)}{63a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+a}{21ad(a+a}$$

Mathematica [B] time = 1.97, size = 401, normalized size = 2.00

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-56952\sin\left(c-\frac{dx}{2}\right)+43722\sin\left(c+\frac{dx}{2}\right)-47208\sin\left(2c+\frac{dx}{2}\right)-18\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(322560*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 56
952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] -
18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3*
d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin[
3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2]
- 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (7
*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin[
5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] + 6
3*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2])))/(2016*a^5*d*(1 + Sec
[c + d*x])^5)
```

fricas [A] time = 1.31, size = 278, normalized size = 1.39

$$-\frac{315\left(\cos(dx+c)^6+5\cos(dx+c)^5+10\cos(dx+c)^4+10\cos(dx+c)^3+5\cos(dx+c)^2+\cos(dx+c)\right)1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos
(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 315*
(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(d*x
+ c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)^2 +
946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*cos(d*
x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5*d*cos(
d*x + c)^2 + a^5*d*cos(d*x + c))
```

giac [A] time = 0.98, size = 155, normalized size = 0.78

$$-\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}+\frac{2016\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^5}-\frac{7a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+72a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+37}{}}{1008d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2*c)
^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8
127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.36, size = 177, normalized size = 0.88

$$\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{14d\,a^5}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^5}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^5}+\frac{129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}-\frac{1}{d\,a^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5+3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
```

```
x+1/2*c)-1/d/a^5/(tan(1/2*d*x+1/2*c)-1)+5/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/
d/a^5/(tan(1/2*d*x+1/2*c)+1)-5/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.76, size = 206, normalized size = 1.03

$$\frac{\frac{2016\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{50}{}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

mupad [B] time = 0.72, size = 149, normalized size = 0.74

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a^5\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8\,a^5\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14\,a^5\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{144\,a^5\,d}-\frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^5\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) + t
an(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*ata
nh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2 +
(d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5} dx$

Optimal. Leaf size=177

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{661\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)\sec^2(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{\text{ta}}{9}$$

```
[Out] arctanh(sin(d*x+c))/a^5/d-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-
13/63*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4-34/105*sec(d*x+c)^2*ta
n(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+173/315*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))
^2-661/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.43, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3816, 4019, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{34\tan(c+dx)\sec^2(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{661\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{\text{ta}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*S
ec[c + d*x])^5) - (13*Sec[c + d*x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d
*x])^4) - (34*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^
3) + (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (661*Tan[c + d
*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^4(c+dx)(4a-9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)\left(39a^2-63a^2\sec(c+\right.}{(a+a\sec(c+dx))^3}}{63a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+d}{105a^2d(a+a\sec(c+dx)}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+d}{105a^2d(a+a\sec(c+dx)}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+d}{105a^2d(a+a\sec(c+dx)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\,s}{105}$$

Mathematica [A] time = 1.96, size = 219, normalized size = 1.24

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\sec\left(\frac{c}{2}\right)\left(-25515\sin\left(c+\frac{dx}{2}\right)+29757\sin\left(c+\frac{3dx}{2}\right)-11235\sin\left(2c+\frac{3dx}{2}\right)+1473\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/2520*(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(80640*Cos[(c + d*x)/2]^9*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) + Sec[c/2]*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c +
(3*d*x)/2] - 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835
*Sin[3*c + (5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2
] + 488*Sin[4*c + (9*d*x)/2])))/(a^5*d*(1 + Sec[c + d*x])^5)
```

fricas [A] time = 0.66, size = 246, normalized size = 1.39

$$\frac{315\left(\cos\left(dx+c\right)^{5}+5\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+10\,\cos\left(dx+c\right)^{2}+5\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(
d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^
5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x +
c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^
3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos
(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*c
os(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 0.88, size = 126, normalized size = 0.71

$$\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{5}}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{5}}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+27}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2
*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x
+ 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.42, size = 134, normalized size = 0.76

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^{5}}-\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d\,a^{5}}-\frac{13\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{5}}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}{d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)-1/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^5*ln(tan(1/2*d*x+1/2*c)+1
)
```

maxima [A] time = 0.69, size = 159, normalized size = 0.90

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1008\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{270\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{5}}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{5}}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{5}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 50
40*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(co
s(d*x + c) + 1) - 1)/a^5)/d
```

mupad [B] time = 0.69, size = 99, normalized size = 0.56

$$-\frac{\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5a^5}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144a^5}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5}+\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^5),x)
```

```
[Out] -((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*ta
n(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(ta
n(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5} dx$

Optimal. Leaf size=159

$$\frac{4\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}-\frac{32\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

```
[Out] 1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*sec(d*x+c)^3*tan(d*x+
c)/a/d/(a+a*sec(d*x+c))^4+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-32/315*
tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+4/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c)
)
```

Rubi [A] time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3810, 3799, 4000, 3794}

$$\frac{4\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}-\frac{32\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*Sec[c + d*x
]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a
^2*d*(a + a*Sec[c + d*x])^3) - (32*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c
+ d*x])^2) + (4*Tan[c + d*x])/(45*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$
$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$
$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}$$
$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}$$
$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}$$

Mathematica [A] time = 0.19, size = 97, normalized size = 0.61

$$\frac{\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin\left(\frac{9}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+d\right.}{315a^5d(\sec(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x
))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x)
)/2]))/(315*a^5*d*(1 + Sec[c + d*x])^5)
```

fricas [A] time = 1.70, size = 123, normalized size = 0.77

$$\frac{\left(8\cos\left(dx+c\right)^4+40\cos\left(dx+c\right)^3+84\cos\left(dx+c\right)^2+100\cos\left(dx+c\right)+83\right)\sin\left(dx+c\right)}{315\left(a^5d\cos\left(dx+c\right)^5+5a^5d\cos\left(dx+c\right)^4+10a^5d\cos\left(dx+c\right)^3+10a^5d\cos\left(dx+c\right)^2+5a^5d\cos\left(dx+c\right)+a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d
*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

giac [A] time = 1.46, size = 72, normalized size = 0.45

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315\tan\left(\frac{1}{2}dx+\right.}{5040a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

maple [A] time = 0.41, size = 71, normalized size = 0.45

$$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.71, size = 107, normalized size = 0.67

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{378\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{180\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.76, size = 127, normalized size = 0.80

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+180}{5040\,a^{5}\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 + 1
80*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a
^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}(c+dx)}{\sec^{5}(c+dx)+5\sec^{4}(c+dx)+10\sec^{3}(c+dx)+10\sec^{2}(c+dx)+5\sec(c+dx)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{\tan(c+dx)}{9d\left(a^5\sec(c+dx)+a^5\right)}-\frac{8\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{5\,\mathrm{ta}}{63}$$

```
[Out] -1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*sec(d*x+c)^3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-8/63*ta
n(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+1/9*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3811, 3810, 3799, 4000, 3794}

$$\frac{\tan(c+dx)}{9d\left(a^5\sec(c+dx)+a^5\right)}-\frac{8\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{5\,\mathrm{ta}}{63}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -(Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (5*Sec[c + d*
x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d
*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x]
)^2) + Tan[c + d*x]/(9*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$
$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{5\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$
$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx}$$
$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx}$$
$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx}$$

Mathematica [A] time = 0.21, size = 97, normalized size = 0.61

$$\frac{\sec\left(\frac{c}{2}\right)\left(-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+63\sin\left(\right.\right.}{8064a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*Si
n[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin[4
*c + (9*d*x)/2]))/(8064*a^5*d)
```

fricas [A] time = 0.67, size = 123, normalized size = 0.77

$$\frac{\left(2\cos\left(dx+c\right)^4+10\cos\left(dx+c\right)^3+21\cos\left(dx+c\right)^2+25\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{63\left(a^5d\cos\left(dx+c\right)^5+5a^5d\cos\left(dx+c\right)^4+10a^5d\cos\left(dx+c\right)^3+10a^5d\cos\left(dx+c\right)^2+5a^5d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x
 + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 1.43, size = 59, normalized size = 0.37

$$-\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.41, size = 58, normalized size = 0.36

$$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-2/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.60, size = 87, normalized size = 0.55

$$\frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{18\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{7\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{1008\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.72, size = 58, normalized size = 0.36

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+63\right)}{1008\,a^{5}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7*
tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{4}(c+dx)}{\sec^{5}(c+dx)+5\sec^{4}(c+dx)+10\sec^{3}(c+dx)+10\sec^{2}(c+dx)+5\sec(c+dx)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5} dx$

Optimal. Leaf size=139

$$\frac{2\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{45a^3d(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{15a^2d(a\sec(c+dx)+a)^3}-\frac{2\tan(c+dx)}{9ad(a\sec(c+dx)+a)^4}+\frac{}{9a}$$

```
[Out] 1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-2/9*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+1
/15*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/45*tan(d*x+c)/a^3/d/(a+a*sec(d*x+
c))^2+2/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3799, 4000, 3796, 3794}

$$\frac{2\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{45a^3d(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{15a^2d(a\sec(c+dx)+a)^3}-\frac{2\tan(c+dx)}{9ad(a\sec(c+dx)+a)^4}+\frac{}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (2*Tan[c + d*x])/(9*a*d*(a + a*
Sec[c + d*x])^4) + Tan[c + d*x]/(15*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Tan[
c + d*x])/(45*a^3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(45*d*(a^5 +
 a^5*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{\int \frac{\sec(c+dx)(-5a+9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{3a^2}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\int}{}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{}{45a}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{}{45a}$$

Mathematica [A] time = 0.25, size = 110, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\left(-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\right.\right.}{5760a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*Si
n[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Si
n[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

fricas [A] time = 0.57, size = 123, normalized size = 0.88

$$\frac{\left(2\cos(dx+c)^4+10\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2\right)\sin(dx+c)}{45\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c)+a^5d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 0.53, size = 46, normalized size = 0.33

$$\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{720a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*
x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.40, size = 45, normalized size = 0.32

$$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c))
```

maxima [A] time = 0.60, size = 67, normalized size = 0.48

$$\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.66, size = 45, normalized size = 0.32

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+45\right)}{720\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45)
)/(720*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{2\tan(c+dx)}{63d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}+\frac{5\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{}{9a}$$

```
[Out] -1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4
+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/63*tan(d*x+c)/a/d/(a^2+a^2*sec(
d*x+c))^2+2/63*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{2\tan(c+dx)}{63d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}+\frac{5\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (5*Tan[c + d*x])/(63*a*d*(a +
a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Ta
n[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(63*d*(a
^5 + a^5*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}$$

Mathematica [A] time = 0.25, size = 125, normalized size = 0.87

$$\frac{\sec\left(\frac{c}{2}\right)\left(-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\sin\left(2c+\frac{5dx}{2}\right)-63\sin\left(3c+\frac{5dx}{2}\right)+\right.}{16128a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 273
*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2] -
 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2]
))/(16128*a^5*d)
```

fricas [A] time = 0.84, size = 123, normalized size = 0.86

$$\frac{\left(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2\right)\sin(dx+c)}{63\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 3.30, size = 59, normalized size = 0.41

$$-\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

maple [A] time = 0.36, size = 58, normalized size = 0.41

$$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.50, size = 87, normalized size = 0.61

$$\frac{\frac{63 \sin(dx+c)}{\cos(dx+c)+1} - \frac{42 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\, a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.69, size = 58, normalized size = 0.41

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+42\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-63\right)}{1008\,a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^5),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 + 7
*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\, dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5} \, dx$

Optimal. Leaf size=143

$$\frac{8\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{8\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{4\tan(c+dx)}{63ad(a\sec(c+dx)+a}$$

```
[Out] 1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+
4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+8/315*tan(d*x+c)/a/d/(a^2+a^2*sec
(d*x+c))^2+8/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{8\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{8\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{4\tan(c+dx)}{63ad(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*Tan[c + d*x])/(63*a*d*(a + a
*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^3) + (
8*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (8*Tan[c + d*x])/(31
5*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}\\
&= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 138, normalized size = 0.97

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1728\sin\left(2c+\frac{5dx}{2}\right)-1260\sin\left(3c+\frac{5dx}{2}\right)\right.}{80640a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] + 3
612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*x)
/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c +
(7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

fricas [A] time = 0.66, size = 123, normalized size = 0.86

$$\frac{\left(83\cos\left(dx+c\right)^{4}+100\cos\left(dx+c\right)^{3}+84\cos\left(dx+c\right)^{2}+40\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{315\left(a^{5}d\cos\left(dx+c\right)^{5}+5a^{5}d\cos\left(dx+c\right)^{4}+10a^{5}d\cos\left(dx+c\right)^{3}+10a^{5}d\cos\left(dx+c\right)^{2}+5a^{5}d\cos\left(dx+c\right)+a^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(
d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

giac [A] time = 0.82, size = 72, normalized size = 0.50

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315\tan\left(\frac{1}{2}dx+\right.}{5040a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

maple [A] time = 0.39, size = 71, normalized size = 0.50

$$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.45, size = 107, normalized size = 0.75

$$\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{420\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{378\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{180\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

mupad [B] time = 0.76, size = 127, normalized size = 0.89

$$\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(315\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-420\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-180\,c}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^5),x)
```

```
[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 - 1
80*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a
^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x
)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=144

$$-\frac{488\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{x}{a^5}-\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{13\tan(c+dx)}{63ad(a\sec(c+dx)+$$

```
[Out] x/a^5-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-13/63*tan(d*x+c)/a/d/(a+a*sec(d*x
+c))^4-34/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-173/315*tan(d*x+c)/a^3/d/
(a+a*sec(d*x+c))^2-488/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{488\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{x}{a^5}-\frac{13\tan(c+dx)}{63ad(a\sec(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] x/a^5 - Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (13*Tan[c + d*x])/(63*a
*d*(a + a*Sec[c + d*x])^4) - (34*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*
x])^3) - (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (488*Tan[c
 + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{-9a+4a\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\int \frac{63a^2-39a^2\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}$$
$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}$$
$$= \frac{x}{a^5} - \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+d}$$
$$= \frac{x}{a^5} - \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+d}$$

Mathematica [A] time = 0.56, size = 280, normalized size = 1.94

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(100800\sin\left(c+\frac{dx}{2}\right)-88284\sin\left(c+\frac{3dx}{2}\right)+56700\sin\left(2c+\frac{3dx}{2}\right)-43236\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (d
*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] + 11
340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x*Co
s[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c + (9*d
*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*Sin[c
+ (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] - 43236*
Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c + (7*d*x)
/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(161280*a^5*d
)
```

fricas [A] time = 0.60, size = 188, normalized size = 1.31

$$\frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos(dx}{315\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*c
os(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x +
c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

giac [A] time = 1.01, size = 100, normalized size = 0.69

$$\frac{\frac{5040\,(dx+c)}{a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan
(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*
d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.47, size = 113, normalized size = 0.78

$$-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^5}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5d\,a^5}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^5}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.68, size = 132, normalized size = 0.92

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}-\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10
080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

mupad [B] time = 0.80, size = 125, normalized size = 0.87

$$\frac{x}{a^5}-\frac{\frac{863\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{315}-\frac{356\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{315}+\frac{169\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{420}-\frac{41\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{504}+\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{144}}{a^5\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^5,x)
```

```
[Out] x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2 +
(d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(1/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*s
ec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{496\sin(c+dx)}{63a^5d}-\frac{5\sin(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{67\sin(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\sin(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{5\,\mathrm{s}}{21ad(a\,\mathrm{s}}$$

```
[Out] -5*x/a^5+496/63*sin(d*x+c)/a^5/d-1/9*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*s
in(d*x+c)/a/d/(a+a*sec(d*x+c))^4-29/63*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-
67/63*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)/d/(a^5+a^5*sec(d*x+c
))
```

Rubi [A] time = 0.40, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{496\sin(c+dx)}{63a^5d}-\frac{5\sin(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{67\sin(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\sin(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{5x}{a^5}-\frac{5\,\mathrm{s}}{21ad(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (-5*x)/a^5 + (496*Sin[c + d*x])/(63*a^5*d) - Sin[c + d*x]/(9*d*(a + a*Sec[c
 + d*x])^5) - (5*Sin[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sin[c
+ d*x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sin[c + d*x])/(63*a^3*d*(a
+ a*Sec[c + d*x])^2) - (5*Sin[c + d*x])/(d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(-10a+5a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)\left(-85a^2+60a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^3}}{63a^4}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} -$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} -$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} -$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} -$$

$$= -\frac{5x}{a^5} + \frac{496\sin(c+dx)}{63a^5d} - \frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} -$$

Mathematica [B] time = 0.70, size = 319, normalized size = 2.01

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(143010\sin\left(c+\frac{dx}{2}\right)-138726\sin\left(c+\frac{3dx}{2}\right)+73290\sin\left(2c+\frac{3dx}{2}\right)-70389\sin\left(2c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -1/64512*(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*C
os[c + (d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d*x
)/2] + 22680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] + 56
70*d*x*Cos[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*Cos[4
*c + (9*d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] + 1430
10*Sin[c + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*d*x)/2
] - 70389*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*Sin[3*c
 + (7*d*x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/2] - 567
*Sin[5*c + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (11*d*x)/2]
))/(a^5*d)
```

fricas [A] time = 0.58, size = 198, normalized size = 1.25

$$-\frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos(dx+}{63\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*co
```

```
s(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)
^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d
*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5
*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 2.12, size = 129, normalized size = 0.81

$$-\frac{\frac{5040\,(dx+c)}{a^5} - \frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5} - \frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 812}{a^{45}}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/
2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

maple [A] time = 0.65, size = 145, normalized size = 0.91

$$\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5} - \frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{14d\,a^5} + \frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^5} - \frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^5} + \frac{129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5} + \frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^5\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5-3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)+2/d/a^5*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-10/d/a^5*arcta
n(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.06, size = 178, normalized size = 1.12

$$\frac{\frac{2016\,\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{8127\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{1512\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{72\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

mupad [B] time = 0.83, size = 159, normalized size = 1.00

$$\frac{7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 100\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 636\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 2512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\right.}{1008\,a^5\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^5,x)
```

```
[Out] (7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/2
+ (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/(100
8*a^5*d*cos(c/2 + (d*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x
)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=215

$$-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}-\frac{3832\sin(c+dx)\cos(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{31x}{2a^5}-\frac{577\sin(c+dx)\cos(c+d}{315a^3d(a\sec(c+dx)+a}$$

```
[Out] 31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*c
os(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-17/63*cos(d*x+c)*sin(d*x+c)/a/d/(
a+a*sec(d*x+c))^4-28/45*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-577/
315*cos(d*x+c)*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-3832/315*cos(d*x+c)*sin(
d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

Rubi [A] time = 0.51, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}-\frac{3832\sin(c+dx)\cos(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{577\sin(c+dx)\cos(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{28}{45}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]*Sin[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5)
 - (17*Cos[c + d*x]*Sin[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) - (28*Cos
[c + d*x]*Sin[c + d*x])/(45*a^2*d*(a + a*Sec[c + d*x])^3) - (577*Cos[c + d*
x]*Sin[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (3832*Cos[c + d*x]*Si
n[c + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(-11a+6a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)\left(-111a^2+85a^2\sec(c+\right.}{(a+a\sec(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3}$$

$$= -\frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(}{63ad(a}$$

$$= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17}{6}$$

Mathematica [A] time = 0.78, size = 345, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(7194600\sin\left(c+\frac{dx}{2}\right)-7472241\sin\left(c+\frac{3dx}{2}\right)+3432975\sin\left(2c+\frac{3dx}{2}\right)-3871989\sin\right(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
+ (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*x)
/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2] +
 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39060*
d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Sin[(d*
x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 3432975*Sin
[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c + (5*d*x)
/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 175184*Si
```

```
n[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (11*d*x)/2
] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*Sin[7*c +
(13*d*x)/2]))/(1290240*a^5*d)
```

fricas [A] time = 3.03, size = 207, normalized size = 0.96

$$\frac{9765\,dx\cos\left(dx+c\right)^{5}+48825\,dx\cos\left(dx+c\right)^{4}+97650\,dx\cos\left(dx+c\right)^{3}+97650\,dx\cos\left(dx+c\right)^{2}+48825\,dx\,\cdots}{630\left(a^{5}d\cos\left(dx+c\right)^{5}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d
*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x +
(315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*co
s(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x
+ c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c
)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

giac [A] time = 0.86, size = 145, normalized size = 0.67

$$\frac{\frac{78120\,(dx+c)}{a^5}-\frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1575\cdots}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x +
1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)
^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/
a^45)/d
```

maple [A] time = 0.66, size = 179, normalized size = 0.83

$$-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^5}+\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^5}-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d\,a^5}+\frac{25\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^5}-\frac{351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}-\frac{11\left(\tan^{3}\left(\cdots\right.\right.}{d\,a^5\left(1+\tan^{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)-11/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-9/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+31/d/a^5*arctan(tan(1/2*d*x+
1/2*c))
```

maxima [A] time = 1.13, size = 224, normalized size = 1.04

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^5+\frac{2a^5\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3024\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{450\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^5}-\frac{156240\arctan\left(\frac{\sin(dx\cdots}{\cos(dx+\cdots}\right.}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
- 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^5)/d
```

mupad [B] time = 0.95, size = 181, normalized size = 0.84

$$-\frac{35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-590\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-23288\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^5,x)
```

```
[Out] -(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 458
4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*cos
(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin(c/2
 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/2 + (
d*x)/2)^9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\, dx$

Optimal. Leaf size=122

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{8\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{35d}+\frac{4a\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 12/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+4/5*a*tan(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)+2/7*a*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-8/35*(a+a
*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3803, 3800, 4001, 3792}

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{8\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{35d}+\frac{4a\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sec[c + d*x]^3*Tan
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (8*Sqrt[a + a*Sec[c + d*x]]*Tan
[c + d*x])/(35*d) + (12*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{6}{7}\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{12(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35ad} + \frac{12}{}$$

$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{35d} + \frac{12(a+}{}$$

$$= \frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+}}{35}$$

Mathematica [A] time = 0.14, size = 58, normalized size = 0.48

$$\frac{2a\tan(c+dx)\left(5\sec^3(c+dx)+6\sec^2(c+dx)+8\sec(c+dx)+16\right)}{35d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(16 + 8*Sec[c + d*x] + 6*Sec[c + d*x]^2 + 5*Sec[c + d*x]^3)*Tan[c + d*
x])/(35*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.26, size = 82, normalized size = 0.67

$$\frac{2\left(16\cos(dx+c)^3+8\cos(dx+c)^2+6\cos(dx+c)+5\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{35\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x +
c)^3)
```

giac [A] time = 20.57, size = 120, normalized size = 0.98

$$-\frac{2\sqrt{2}\left(35a^4-\left(35a^4+\left(9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-49a^4\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\mathrm{sgn}(\cos(dx+c))\,\mathrm{ta}}{35\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*sqrt(2)*(35*a^4 - (35*a^4 + (9*a^4*tan(1/2*d*x + 1/2*c)^2 - 49*a^4)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*
x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

maple [A] time = 1.10, size = 82, normalized size = 0.67

$$-\frac{2\left(16\left(\cos^4(dx+c)\right)-8\left(\cos^3(dx+c)\right)-2\left(\cos^2(dx+c)\right)-\cos(dx+c)-5\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{35d\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.93, size = 331, normalized size = 2.71

$$-\frac{e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\;32i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}-\frac{\left(\frac{16i}{7\,d}+\frac{e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3}+\frac{\left(\frac{16i}{5\,d}+\frac{e^{c\,1i+d\,x\,1i}\,128i}{35\,d}\right)\sqrt{a}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] ((16i/(5*d) + (exp(c*1i + d*x*1i)*128i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d
*x*2i) + 1)^2) - ((16i/(7*d) + (exp(c*1i + d*x*1i)*16i)/(7*d))*(a + a/(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1
)*(exp(c*2i + d*x*2i) + 1)^3) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*32i)/(35*d*(exp(c*1i + d*x*1i) + 1))
 - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*16i)/(35*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**4, x)
```

3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{14a\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+14/15*a*tan(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)-4/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3800, 4001, 3792}

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{14a\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (14*a*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*Sqrt[a + a*Sec[c +
 d*x]]*Tan[c + d*x])/(15*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
5*a*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}+\frac{2\int \sec(c+dx)\left(\frac{3a}{2}-a\sec(c+dx)\right.}{5a}\\ &= -\frac{4\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}\\ &= \frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}-\frac{4\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2(a+a\sec}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 48, normalized size = 0.56

$$\frac{2a\tan(c+dx)\left(3\sec^2(c+dx)+4\sec(c+dx)+8\right)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(8 + 4*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1
+ Sec[c + d*x])])
```

fricas [A] time = 0.69, size = 72, normalized size = 0.84

$$\frac{2\left(8\cos\left(dx+c\right)^2+4\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 3.67, size = 101, normalized size = 1.17

$$\frac{2\sqrt{2}\left(15a^3+\left(7a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10a^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(2)*(15*a^3 + (7*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3)*tan(1/2*d*x
+ 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)
^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

maple [A] time = 1.01, size = 72, normalized size = 0.84

$$-\frac{2\left(8\left(\cos^3\left(dx+c\right)\right)-4\left(\cos^2\left(dx+c\right)\right)-\cos\left(dx+c\right)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 4.46, size = 115, normalized size = 1.34

$$\frac{8\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,2i+d\,x\,2i}\,5i-e^{c\,3i+d\,x\,3i}\,5i-e^{c\,5i+d\,x\,5i}\,2i+2i\right)}{15\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] (8*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2i
+ d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*2i + 2i))/(15*d*(
exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**3, x)
```

3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/3*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan(d*
x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3798, 3792}

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 36, normalized size = 0.64

$$\frac{2a\tan(c+dx)(\sec(c+dx)+2)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(2 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.65, size = 60, normalized size = 1.07

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x +
c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [A] time = 3.11, size = 82, normalized size = 1.46

$$\frac{2\sqrt{2}\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

maple [A] time = 0.93, size = 62, normalized size = 1.11

$$-\frac{2\left(2\left(\cos^2\left(dx+c\right)\right)-\cos\left(dx+c\right)-1\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/si
n(d*x+c)/cos(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
3/4)*sqrt(a)*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x
+ 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x +
 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2
*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4
*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + s
```

```
in(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*
x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)), x) + sqrt(a)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(3/4)*d)
```

mupad [B] time = 1.39, size = 108, normalized size = 1.93

$$\frac{4\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(3\,\sin\left(c+d\,x\right)+4\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{3\,d\,\left(12\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+4\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] (4*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(3*sin(c + d*x) + 4*sin(2*c
+ 2*d*x) + 3*sin(3*c + 3*d*x) + sin(4*c + 4*d*x)))/(3*d*(12*cos(c + d*x) +
8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**2, x)
```

3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3792}

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 1.12

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 0.75, size = 41, normalized size = 1.58

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [B] time = 7.08, size = 62, normalized size = 2.38

$$-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a\,\mathrm{sgn}\left(\cos(dx+c)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

maple [A] time = 0.90, size = 42, normalized size = 1.62

$$-\frac{2\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c), x)
```

mupad [B] time = 0.19, size = 41, normalized size = 1.58

$$\frac{2\sin(c+d\,x)\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] (2*sin(c + d*x)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(d*(cos(c + d*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x), x)
```

3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[c + d*x]],x]

[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\sec(c+dx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/
2]*Sqrt[a*(1 + Sec[c + d*x])])/d

fricas [A] time = 0.88, size = 133, normalized size = 3.59

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},-\frac{2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
 1))/d, -2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))/d]
```

giac [B] time = 9.02, size = 130, normalized size = 3.51

$$-\frac{\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*
sgn(cos(d*x + c))/(d*abs(a))
```

maple [B] time = 1.01, size = 89, normalized size = 2.41

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*2^(1/2)
```

maxima [B] time = 1.33, size = 146, normalized size = 3.95

$$\sqrt{a}\,\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(c + d*x) + a), x)
```

3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/d/
(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3805, 3774, 203}

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}\int\sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{a\,\mathrm{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.22, size = 62, normalized size = 1.00

$$\frac{a\tan(c+dx)\left(\cos(c+dx)+\frac{\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(Cos[c + d*x] + ArcTanh[Sqrt[1 - Sec[c + d*x]]]/Sqrt[1 - Sec[c + d*x]])*
Tan[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.76, size = 242, normalized size = 3.90

$$\left[\frac{\sqrt{-a}\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos\left(dx\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
 c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c))/(d*cos(d*x + c) + d), -(sqrt(a)*(cos(d*x + c) + 1)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c))) - sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 3.32, size = 282, normalized size = 4.55

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a\right|}\right)}{|a|}+\frac{8\left(3\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}\right.\right.}{\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4-6\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\right.\right.}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a - sqrt(-a)*a^2)/((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))*sgn(cos
(d*x + c))/d
```

maple [B] time = 1.10, size = 123, normalized size = 1.98

$$-\frac{\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\ \sin\left(dx+c\right)+2\left(\cos^{2}\left(dx+c\right)\right)-2\cos\left(dx+c\right)\right)\sqrt{\frac{a(}{}}}{2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*cos(
d*x+c)^2-2*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

maxima [B] time = 1.58, size = 791, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x), x)
```

3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{3a\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 3/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sin(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3805, 3774, 203}

$$\frac{3a\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{4}\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{8}\int \sqrt{a+a\sec(c+$$

$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\right.}{}$$

$$= \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(}{2d\sqrt{a+a\sec(c}$$

Mathematica [C] time = 0.11, size = 47, normalized size = 0.46

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 0.62, size = 270, normalized size = 2.65

$$\left[\frac{3\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\cos(dx+c)^2+3\right.}{8(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/
4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 + 3*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) +
 d)]
```

giac [B] time = 13.77, size = 378, normalized size = 3.71

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\left(5\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6\sqrt{-a}\,a+19\right.}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 19*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^2 - 17*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3
 + sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*a + a^2)^2)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.13, size = 221, normalized size = 2.17

$$\frac{\left(3\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos\left(dx+c\right)\sqrt{2}\,\sin\left(dx+c\right)+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c}{1+\cos(dx+}}}{2\cos}\right.\right.}{16d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin(
d*x+c)+3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos
(d*x+c)^4-4*cos(d*x+c)^3+12*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/
2)/cos(d*x+c)/sin(d*x+c)
```

maxima [B] time = 1.18, size = 1059, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**2, x)
```

3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{5a\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{5a\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 5/8*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+5/8*a*sin(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+5/12*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^(1/2)+1/3*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3805, 3774, 203}

$$\frac{5a\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{5a\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d) +
 (5*a*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (5*a*Cos[c + d*x]*Sin[
c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{6}\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{8}\int \cos(c+
$$
$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c}}$$
$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin}{3d\sqrt{a+a\sec(c}}$$
$$= \frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin}{12d\sqrt{a+a\sec(}}$$

Mathematica [C] time = 0.10, size = 47, normalized size = 0.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 1.59, size = 290, normalized size = 2.10

$$\left[\frac{15\sqrt{-a}\left(\cos(dx+c)+1\right)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\cos(dx+c)^3+\right.}{48\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*cos(d*x + c)^3 + 10*cos(d*x + c)^2
+ 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d), -1/24*(15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*cos(
d*x + c)^3 + 10*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 4.28, size = 475, normalized size = 3.44

$$-\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}\,\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\,|a|-6\,a\right|}\right)}{|a|}+\frac{8\left(63\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{10}\sqrt{-\cdots}}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) + 8*(63*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*a - 369*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^2 + 1638*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-
a)*a^3 - 1074*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*sqrt(-a)*a^4 + 171*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^5 - 13*sqrt(-a)*a^6)/((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)*sgn(c
os(d*x + c))/d
```

maple [B] time = 1.23, size = 310, normalized size = 2.25

$$-\frac{\left(15\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}+30\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(\cdots)}{1+\cos(\cdots)}}\cdots}{2\cdots}\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x+c
)^2*2^(1/2)+30*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x
+c)*2^(1/2)+15*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)
+64*cos(d*x+c)^6+16*cos(d*x+c)^5+40*cos(d*x+c)^4-120*cos(d*x+c)^3)*(a*(1+co
s(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

maxima [B] time = 2.08, size = 1921, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
```

```
 + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arct
an2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
```

```
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{35a\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+3$$

```
[Out] 35/64*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+35/64*a*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+35/96*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)+7/24*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*
a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3805, 3774, 203}

$$\frac{35a\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+3$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d)
 + (35*a*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (35*a*Cos[c + d*x]
*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*Cos[c + d*x]^2*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(
4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{7}{8}\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{35}{48}\int \cos$$

$$= \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c}{4d\sqrt{a}}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+}{24d\sqrt{a+}}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+}{24d\sqrt{a+}}$$

$$= \frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+}{96d\sqrt{a+}}$$

Mathematica [C] time = 0.10, size = 47, normalized size = 0.27

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 0.74, size = 310, normalized size = 1.78

$$\left[\frac{105\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\cos(dx+c\right.}{384(d\cos(dx+c)+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*cos(d*x + c)^4 + 56*cos(d*x + c)
^3 + 70*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(105*sqrt(a)*(cos(d*x +
c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) - (48*cos(d*x + c)^4 + 56*cos(d*x + c)^3 + 70*cos(d*x + c)
^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

giac [B] time = 4.46, size = 571, normalized size = 3.28

$$-\frac{\sqrt{2}\left(\frac{105\sqrt{2}\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\left(279\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^{14}\sqrt{-a}\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/768*sqrt(2)*(105*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*(279*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*sqrt(-a)*a + 285*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*sqrt(-a)*a^2 - 4605
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*s
qrt(-a)*a^3 + 37281*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^8*sqrt(-a)*a^4 - 35643*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^5 + 9175*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6 - 1311*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-
a)*a^7 + 43*sqrt(-a)*a^8)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)*sgn(cos(d*x + c))/d
```

maple [B] time = 1.26, size = 399, normalized size = 2.29

$$-\frac{\left(-105\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{2}-315\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3072/d*(-105*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(
d*x+c)*2^(1/2)-315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*2^(1/2)-315*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*
sin(d*x+c)*2^(1/2)-105*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*2^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*si
n(d*x+c)+768*cos(d*x+c)^8+128*cos(d*x+c)^7+224*cos(d*x+c)^6+560*cos(d*x+c)^
5-1680*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d
*x+c)
```

maxima [B] time = 2.83, size = 6638, normalized size = 38.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos(4*
d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c
) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2
*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 -
cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x
+ 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*
cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(
64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) +
7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
9*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
36*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*cos(4*d*x +
4*c)^3 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c
)^2 - 10*cos(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*
c)^2 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^
2 + 26*cos(4*d*x + 4*c)^2 + 25*cos(4*d*x + 4*c) + 8)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 - (32*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*
c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x
 + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*cos(3/4*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(
4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c)^2 - 8*cos(4*d*x + 4
*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x
+ 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*
x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*co
s(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((64*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 20*(sin(4*d*x + 4*c)^3 + (cos(4*d*
x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) + 8*(cos(4*d*x + 4*c)
^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 5*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 5*sin(4*d*x + 4*c)^3 + 4
```

```
*(5*sin(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c)^2 + 10*cos(4*d*x + 4*c) - 11)*
sin(4*d*x + 4*c) - 64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) + 40*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*
x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 10*(2*sin(4*d*x + 4*c)^3
+ 2*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (16*cos(4*d*x +
 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 17*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 5*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + 2*(32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d
*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*c
os(4*d*x + 4*c)^2 + 8*(4*cos(4*d*x + 4*c)^2 - sin(4*d*x + 4*c)^2 - 40*sin(4
*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*cos(4*
d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(cos(4
*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*s
in(4*d*x + 4*c)^2 - 85*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5
*(8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)) - (64*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*
cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
3 + 5*cos(4*d*x + 4*c)^3 + 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) -
8)*sin(4*d*x + 4*c)^2 - 18*cos(4*d*x + 4*c)^2 + 8*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 37*cos(4*d*x + 4*c) - 24)*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2
+ 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2 -
14*cos(4*d*x + 4*c)^2 + 16*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos
(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8
*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 43*cos(4*d*x + 4*c) - 24)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 24*cos(4*d*x + 4*c)
^2 + 2*(10*cos(4*d*x + 4*c)^3 + 10*(cos(4*d*x + 4*c) - 4)*sin(4*d*x + 4*c)^
2 - 50*cos(4*d*x + 4*c)^2 + (16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2
- 21*cos(4*d*x + 4*c) + 5)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 48*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + (8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - 5*cos(4*d*x + 4*c))*c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(128*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 8*(5*(cos(4*d*x
 + 4*c) - 4)*sin(4*d*x + 4*c) + 8*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 2*(5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c) + 21*cos(1/4*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 5*(cos(4*d*x + 4*c
) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)))*sqrt(a) - 105*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)
+ 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4
*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(
4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + s
in(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arct
an2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
```

```
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*s
in(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4
*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1))) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*
c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c)
 + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x
 + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/
```

```
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x
 + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sec{\left(c + dx \right)} + 1\right)} \cos^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**4, x)
```

3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{68a^2\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{68\tan(c+dx)(a\sec(c}{105d}$$

```
[Out] 68/105*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+68/45*a^2*tan(d*x+c)/d/(a+a*sec(
d*x+c))^(1/2)+34/63*a^2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/
9*a^2*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-136/315*a*(a+a*sec(d
*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.28, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3814, 21, 3803, 3800, 4001, 3792}

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{68a^2\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{68\tan(c+dx)(a\sec(c}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (68*a^2*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (34*a^2*Sec[c + d*x
]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sec[c + d*x]^4*T
an[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (136*a*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(315*d) + (68*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(10
5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{1}{9}(2a)\int \frac{\sec^4(c+dx)\left(\frac{17a}{2}+\frac{17}{2}a\sec(c\right.}{\sqrt{a+a\sec(c+dx)}}\\ &= \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{1}{9}(17a)\int \sec^4(c+dx)\sqrt{a+a\sec(c+d}\\ &= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{1}{21}(34a\\ &= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{68(a+}{}\\ &= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}-\frac{136a\sqrt{}}{}\\ &= \frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+}{9d\sqrt{a+}}\end{aligned}$$

Mathematica [A] time = 0.55, size = 70, normalized size = 0.43

$$\frac{2a^2\tan(c+dx)\left(35\sec^4(c+dx)+85\sec^3(c+dx)+102\sec^2(c+dx)+136\sec(c+dx)+272\right)}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(272 + 136*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 85*Sec[c + d*x]^3 + 3
5*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.65, size = 98, normalized size = 0.60

$$\frac{2\left(272\,a\cos\left(dx+c\right)^4+136\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+85\,a\cos\left(dx+c\right)+35\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(272*a*cos(d*x + c)^4 + 136*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 +
 85*a*cos(d*x + c) + 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [A] time = 8.34, size = 180, normalized size = 1.11

$$\frac{4\left(315\sqrt{2}\,a^6\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(525\sqrt{2}\,a^6\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(819\sqrt{2}\,a^6\text{sgn}\left(\cos\left(dx+c\right)\right)+47\left(2\sqrt{2}\,a^6\text{sg}\right.\right.\right.\right.}{315\left(a\tan\left(\frac{1}{2}dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*a^6*sgn(cos(d*x + c)) - (525*sqrt(2)*a^6*sgn(cos(d*x + c
)) - (819*sqrt(2)*a^6*sgn(cos(d*x + c)) + 47*(2*sqrt(2)*a^6*sgn(cos(d*x + c
))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d
)
```

maple [A] time = 0.99, size = 93, normalized size = 0.57

$$-\frac{2\left(272\left(\cos^5\left(dx+c\right)\right)-136\left(\cos^4\left(dx+c\right)\right)-34\left(\cos^3\left(dx+c\right)\right)-17\left(\cos^2\left(dx+c\right)\right)-50\cos\left(dx+c\right)-35\right)}{315d\cos\left(dx+c\right)^4\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/315/d*(272*cos(d*x+c)^5-136*cos(d*x+c)^4-34*cos(d*x+c)^3-17*cos(d*x+c)^2
-50*cos(d*x+c)-35)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x
+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 6.70, size = 429, normalized size = 2.65

$$\frac{\left(\frac{a\,32\mathrm{i}}{9\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}-\frac{\left(\frac{a\,80\mathrm{i}}{7\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,176\mathrm{i}}{63\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}+\frac{\left(\frac{a\,48\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+c}}{10}\right.}{\left(\mathrm{e}^{c\,1\mathrm{i}+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] (((a*32i)/(9*d) - (a*exp(c*1i + d*x*1i)*32i)/(9*d))*(a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i
 + d*x*2i) + 1)^4) - (((a*80i)/(7*d) - (a*exp(c*1i + d*x*1i)*176i)/(63*d))*
(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i +
 d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*48i)/(5*d) + (a*exp(c*1i +
```

```
 d*x*1i)*352i)/(105*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(
c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2
)*544i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (a*exp(c*1i + d*x*1i)*(a + a/(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*272i)/(315*d*(exp(c*1i
+ d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**4, x)
```

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{38a\tan(c+$$

```
[Out] -4/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d*
x+c)/a/d+152/105*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/105*a*(a+a*sec(
d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3800, 4001, 3793, 3792}

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{38a\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (152*a^2*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (38*a*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(3/2)*Tan[c +
 d*x])/(35*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int \sec(c+dx)\left(\frac{5a}{2}-a\sec(c+d\right.}{7a}$$
$$= -\frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+}{7ad}$$
$$= \frac{38a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{3/2}\tan(c+d}{35d}$$
$$= \frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d} - \frac{4(a+}{}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.52

$$\frac{2a^2\tan(c+dx)\left(15\sec^3(c+dx)+39\sec^2(c+dx)+52\sec(c+dx)+104\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(104 + 52*Sec[c + d*x] + 39*Sec[c + d*x]^2 + 15*Sec[c + d*x]^3)*Tan[
c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.68, size = 87, normalized size = 0.75

$$\frac{2\left(104\,a\cos(dx+c)^3+52\,a\cos(dx+c)^2+39\,a\cos(dx+c)+15\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 +
 d*cos(d*x + c)^3)
```

giac [A] time = 9.72, size = 151, normalized size = 1.30

$$-\frac{4\left(105\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))-\left(140\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))+19\left(2\sqrt{2}\,a^5\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.\right.\right.}{105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,d\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) - (140*sqrt(2)*a^5*sgn(cos(d*x +
c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2
)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*ta
n(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*d)
```

maple [A] time = 0.84, size = 83, normalized size = 0.72

$$-\frac{2\left(104\left(\cos^4(dx+c)\right)-52\left(\cos^3(dx+c)\right)-13\left(\cos^2(dx+c)\right)-24\cos(dx+c)-15\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\;a}{105d\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15
)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 5.08, size = 346, normalized size = 2.98

$$-\frac{\left(\frac{a\,16{}\mathrm{i}}{7\,d}+\frac{a\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,16{}\mathrm{i}}{7\,d}\right)\,\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}+\frac{\left(\frac{a\,8{}\mathrm{i}}{3\,d}-\frac{a\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,104{}\mathrm{i}}{105\,d}\right)\,\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}+\frac{\left(\frac{a\,8{}\mathrm{i}}{5\,d}+\frac{a\,{\mathrm{e}}^{c\,1{}\mathrm{i}+}}{3}\cdots\right)}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] (((a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*104i)/(105*d))*(a + a/(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*
2i + d*x*2i) + 1)) - (((a*16i)/(7*d) + (a*exp(c*1i + d*x*1i)*16i)/(7*d))*(a
 + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d
*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*8i)/(5*d) + (a*exp(c*1i + d*
x*1i)*184i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(c*1i
 + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*20
8i)/(105*d*(exp(c*1i + d*x*1i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}} \sec^{3}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**3, x)
```

3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+8/5*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)+2/5*a*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3798, 3793, 3792}

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(5*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(5*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx&=\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{3}{5}\int \sec(c+dx)(a+a\sec(c+dx\\&=\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{5d}+\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx\\&=\frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{5d}+\frac{2(a+a\,s\end{aligned}$$

Mathematica [A] time = 0.13, size = 48, normalized size = 0.56

$$\frac{2a^2\tan(c+dx)\left(\sec^2(c+dx)+3\sec(c+dx)+6\right)}{5d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(6 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

fricas [A] time = 0.52, size = 74, normalized size = 0.86

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 7.77, size = 121, normalized size = 1.41

$$\frac{4\left(5\,\sqrt{2}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+\left(2\,\sqrt{2}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-5\,\sqrt{2}\,a^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*tan
(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)*d)
```

maple [A] time = 0.84, size = 73, normalized size = 0.85

$$-\frac{2\left(6\left(\cos^{3}\left(dx+c\right)\right)-3\left(\cos^{2}\left(dx+c\right)\right)-2\cos\left(dx+c\right)-1\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\;a}{5d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 4.36, size = 116, normalized size = 1.35

$$\frac{4\,a\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\;\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,5\mathrm{i}-\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{5\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] (4*a*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2
i + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*3i + 3i))/(5*d*
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^{\frac{3}{2}} \sec^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**2, x)
```

3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] 8/3*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3793, 3792}

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}(4a)\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 38, normalized size = 0.64

$$\frac{2a^2\tan(c+dx)(\sec(c+dx)+5)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(5 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.58, size = 61, normalized size = 1.03

$$\frac{2\,(5\,a\cos\left(dx+c\right)+a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [A] time = 9.43, size = 93, normalized size = 1.58

$$\frac{4\left(2\sqrt{2}\,a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\sqrt{2}\,a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*a^3
*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

maple [A] time = 0.86, size = 63, normalized size = 1.07

$$-\frac{2\left(5\left(\cos^{2}\left(dx+c\right)\right)-4\cos\left(dx+c\right)-1\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\;a}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/
sin(d*x+c)/cos(d*x+c)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

mupad [B] time = 1.24, size = 111, normalized size = 1.88

$$\frac{2\,a\,\sqrt{\frac{a\,\left(\cos(c+d\,x)+1\right)}{\cos(c+d\,x)}}\,\left(12\,\sin\left(c+d\,x\right)+14\,\sin\left(2\,c+2\,d\,x\right)+12\,\sin\left(3\,c+3\,d\,x\right)+5\,\sin\left(4\,c+4\,d\,x\right)\right)}{3\,d\,\left(12\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+4\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

[Out] (2*a*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(12*sin(c + d*x) + 14*sin(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin(4*c + 4*d*x)))/(3*d*(12*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \sec(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x), x)

3.103 $\int (a + a\sec(c + dx))^{3/2}\, dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(2a)\int\frac{\frac{a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2a^2)\,\mathrm{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$=\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.21, size = 75, normalized size = 1.14

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d
```

fricas [A] time = 0.88, size = 235, normalized size = 3.56

$$\left[\frac{(a\cos(dx+c)+a)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{si}}{d\cos(dx+c)+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [((a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

giac [B] time = 16.03, size = 195, normalized size = 2.95

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+\frac{\sqrt{-a}\,a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\mathrm{co}}{|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn(cos(d*x + c))*tan(1
/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + sqrt(-a)*a^2*log(abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sq
rt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a))
/d
```

maple [B] time = 0.92, size = 180, normalized size = 2.73

$$-\frac{\left(\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos\left(dx+c\right)+\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos(d}\right.\right.}{d\left(1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)+2^(1/2)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-2*sin(d*x+c))*(a*(1+cos(d*x+c))/c
os(d*x+c))^(1/2)/(1+cos(d*x+c))*a
```

maxima [B] time = 1.00, size = 997, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
```

```
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(3/2), x)
```

3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 3*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3814, 21, 3805, 3774, 203}

$$\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^
2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-(2a)\int \frac{\cos(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(3a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}(3a)\int \sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{\left(3a^2\right)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 89, normalized size = 1.37

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2])
)/(2*d)
```

fricas [A] time = 0.61, size = 248, normalized size = 3.82

$$\left[\frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,(a\cos(dx+c)+a)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d\ldots}{\cos(dx+c)+1}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + 3*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
+ c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (a*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*(a*cos(d*x + c) + a)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))))/(d*cos(d*x + c) + d)]
```

giac [B] time = 7.16, size = 278, normalized size = 4.28

$$-\frac{\sqrt{2}\sqrt{-a}\,a^3\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a|a|}+\frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\cdots}\right.\right.}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\cdots\right.\right.}\cdots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(-a)*a^3*(3*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))*sgn(cos(d*x + c))
/d
```

maple [B] time = 0.99, size = 125, normalized size = 1.92

$$-\frac{\left(3\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)-2\cos(dx+c)\right)\sqrt{\frac{a(1+\cdots}{\cos\cdots}}\cdots}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*co
s(d*x+c)^2-2*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

maxima [B] time = 1.19, size = 803, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
```

```
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(c + d\,x)\left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{3}{2}} \cos(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 7/4*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3813, 21, 3805, 3774, 203}

$$\frac{7a^2\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*Si
n[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\cos(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c+dx\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}(7a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx}$$

$$= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(7a)\int \sqrt{a-}$$

$$= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\left(7a^2\right)\text{Subst}\Big(}{}$$

$$= \frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+d}{2d\sqrt{a+a\,s}}$$

Mathematica [A] time = 0.38, size = 108, normalized size = 1.02

$$\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left((7\sin(c+dx)+\sin(2(c+dx)))\sqrt{1-\sec(c+dx)}+7\tan(c+dx)\tanh^{-1}\left(\sqrt{\right.\right.}{4d(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(7*Sin[c
 + d*x] + Sin[2*(c + d*x)]) + 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x
]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

fricas [A] time = 0.64, size = 278, normalized size = 2.62

$$\left[\frac{7\,(a\cos(dx+c)+a)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,a\cos(dx+c\right.}{8\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a*cos(d*x + c)^2 + 7*a*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
), -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*
a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(d*x+c))]Warning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableWarning, assuming -2*a+a is positive. Hint: run assum
e to make assumptions on a variableWarning, assuming -2*a+a is positive. Hi
nt: run assume to make assumptions on a variableWarning, assuming -2*a+a is
 positive. Hint: run assume to make assumptions on a variableWarning, assum
ing -2*a+a is positive. Hint: run assume to make assumptions on a variableW
arning, assuming -2*a+a is positive. Hint: run assume to make assumptions o
n a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run as
sume to make assumptions on a variableEvaluation time: 0.94Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{[469762048,0]:[1,0,-2]%%},[14
]%%%},0]:[1,0,%%%{1,[1]%%%}]%%},[0,1]%%%} / %%%{%%%{%%{[67108864,0]:[1,0,-2
]%%},[12]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.04, size = 222, normalized size = 2.09

$$-\frac{\left(-7\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos\left(dx+c\right)\sqrt{2}\,\sin\left(dx+c\right)-7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c}{1+\cos(dx+c}}}{2\cos}\right.\right.}{16d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d*(-7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*si
n(d*x+c)-7*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*c
os(d*x+c)^4+20*cos(d*x+c)^3-28*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)/cos(d*x+c)/sin(d*x+c)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 11/8*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 21, 3805, 3774, 203}

$$\frac{11a^2\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}a\int \frac{\cos^2(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(c+\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(11a)\int \cos^2(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(11a$$

$$= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+}{3d\sqrt{a+}}$$

$$= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+}{3d\sqrt{a+}}$$

$$= \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c-}{12d\sqrt{a+}}$$

Mathematica [A] time = 0.55, size = 120, normalized size = 0.83

$$\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left((35\sin(c+dx)+11\sin(2(c+dx))+2\sin(3(c+dx)))\sqrt{1-\sec(c+dx)}+3\right.}{24d(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(35*Sin[
c + d*x] + 11*Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)]) + 33*ArcTanh[Sqrt[1 -
Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x
]])
```

fricas [A] time = 0.84, size = 300, normalized size = 2.08

$$\left[\frac{33\,(a\cos(dx+c)+a)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a\cos(dx+\right.}{48\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(33*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a*cos(d*x + c)^3 + 22*a*cos(d*x +
 c)^2 + 33*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c) + d), -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
```

```
 - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [**B**] time = 6.92, size = 535, normalized size = 3.72

$$-\frac{33\sqrt{-a}\,a\log\left(\left|\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\sqrt{2}+3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-33\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*(33*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*sqrt
(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(33*sqrt(2)*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*
a^2*sgn(cos(d*x + c)) - 303*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*sqrt
(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6
*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*
c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c))
+ 309*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*sqrt(2)*sqrt(-a)*a^7*sgn(cos
(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^3)/d
```

maple [**B**] time = 1.08, size = 311, normalized size = 2.16

$$-\frac{\left(33\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}+66\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/192/d*(33*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x+c
)^2*2^(1/2)+66*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x
+c)*2^(1/2)+33*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)
+64*cos(d*x+c)^6+112*cos(d*x+c)^5+88*cos(d*x+c)^4-264*cos(d*x+c)^3)*(a*(1+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^3 \left(a+\frac{a}{\cos(c+dx)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{46a^3\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{284a^3\tan(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sec^4(c+$$

```
[Out] 284/231*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+284/99*a^3*tan(d*x+c)/d/(a+a*
sec(d*x+c))^(1/2)+710/693*a^3*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1
/2)+46/99*a^3*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-568/693*a^2*
(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/11*a^2*sec(d*x+c)^4*(a+a*sec(d*x+c))^
(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.37, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3814, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}+\frac{46a^3\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (284*a^3*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) + (710*a^3*Sec[c + d
*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (46*a^3*Sec[c + d*x]
^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (568*a^2*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(693*d) + (2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(11*d) + (284*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(231*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{11d}+\frac{1}{11}(2a)\int \sec^4(c$$

$$=\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}}{11d}$$

$$=\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{2}{}$$

$$=\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{2}{}$$

$$=\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}-\frac{5}{}$$

$$=\frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\text{se}}{99d}$$

Mathematica [A] time = 0.21, size = 80, normalized size = 0.39

$$\frac{2a^3\tan(c+dx)\left(63\sec^5(c+dx)+224\sec^4(c+dx)+355\sec^3(c+dx)+426\sec^2(c+dx)+568\sec(c+dx)+\right.}{693d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(1136 + 568*Sec[c + d*x] + 426*Sec[c + d*x]^2 + 355*Sec[c + d*x]^3 +
 224*Sec[c + d*x]^4 + 63*Sec[c + d*x]^5)*Tan[c + d*x])/(693*d*Sqrt[a*(1 + S
ec[c + d*x])])
```

fricas [A] time = 0.60, size = 121, normalized size = 0.60

$$\frac{2\left(1136\,a^2\cos(dx+c)^5+568\,a^2\cos(dx+c)^4+426\,a^2\cos(dx+c)^3+355\,a^2\cos(dx+c)^2+224\,a^2\cos(dx+\right.}{693\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(1136*a^2*cos(d*x + c)^5 + 568*a^2*cos(d*x + c)^4 + 426*a^2*cos(d*x +
 c)^3 + 355*a^2*cos(d*x + c)^2 + 224*a^2*cos(d*x + c) + 63*a^2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c
)^5)
```

giac [A] time = 6.22, size = 209, normalized size = 1.03

$$-\frac{8\left(693\,\sqrt{2}\,a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(1617\,\sqrt{2}\,a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(3003\,\sqrt{2}\,a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-25\left(99\,\sqrt{2}\,a^8\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -8/693*(693*sqrt(2)*a^8*sgn(cos(d*x + c)) - (1617*sqrt(2)*a^8*sgn(cos(d*x +
 c)) - (3003*sqrt(2)*a^8*sgn(cos(d*x + c)) - 25*(99*sqrt(2)*a^8*sgn(cos(d*x
 + c)) + 4*(2*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 11*sqr
t(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

maple [A] time = 0.96, size = 105, normalized size = 0.52

$$-\frac{2\left(1136\left(\cos^6\left(dx+c\right)\right)-568\left(\cos^5\left(dx+c\right)\right)-142\left(\cos^4\left(dx+c\right)\right)-71\left(\cos^3\left(dx+c\right)\right)-131\left(\cos^2\left(dx+c\right)\right)-1\right.}{693d\cos\left(dx+c\right)^5\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/693/d*(1136*cos(d*x+c)^6-568*cos(d*x+c)^5-142*cos(d*x+c)^4-71*cos(d*x+c)
^3-131*cos(d*x+c)^2-161*cos(d*x+c)-63)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/
cos(d*x+c)^5/sin(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 9.32, size = 542, normalized size = 2.67

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{a^2\,64i}{11\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,64i}{11\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{a^2\,16i}{d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,640i}{231\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x}}{2}}}}{\left(e^{c\,1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(
11*d) + (a^2*exp(c*1i + d*x*1i)*64i)/(11*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)^5) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2))^(1/2)*((a^2*16i)/d + (a^2*exp(c*1i + d*x*1i)*640i)/(231*d)))/((exp
(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - ((a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(9*d) + (a^2*exp(c*1i + d
*x*1i)*2176i)/(99*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4
) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*80
i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*12688i)/(693*d)))/((exp(c*1i + d*x*1i) +
 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1
i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2272i)/(693*d*(exp(c*1i + d*x*
1i) + 1)) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*1136i)/(693*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d
*x*2i) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{832a^3\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}-\frac{4\tan(c+d}{}$$

```
[Out] 26/105*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d-4/63*(a+a*sec(d*x+c))^(5/2)*ta
n(d*x+c)/d+2/9*(a+a*sec(d*x+c))^(7/2)*tan(d*x+c)/a/d+832/315*a^3*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)+208/315*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3800, 4001, 3793, 3792}

$$\frac{208a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{832a^3\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}-\frac{4\tan(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (832*a^3*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (26*a*(a + a*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) +
 (2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad} + \frac{2\int \sec(c+dx)\left(\frac{7a}{2} - a\sec(c+ \right.}{9}$$
$$= -\frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} + \frac{2(a+a\sec(c+dx))^{7/2}\tan(c}{9ad}$$
$$= \frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{5/2}\tan(c}{63d}$$
$$= \frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{26a(a+a\sec(c+dx))^{3/2}\,\mathrm{ta}}{105d}$$
$$= \frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} +$$

Mathematica [A] time = 0.53, size = 70, normalized size = 0.48

$$\frac{2a^3\tan(c+dx)\left(35\sec^4(c+dx)+130\sec^3(c+dx)+219\sec^2(c+dx)+292\sec(c+dx)+584\right)}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(584 + 292*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 130*Sec[c + d*x]^3 +
35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.69, size = 108, normalized size = 0.74

$$\frac{2\left(584\,a^2\cos\left(dx+c\right)^4+292\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+130\,a^2\cos\left(dx+c\right)+35\,a^2\right)\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [A] time = 5.91, size = 180, normalized size = 1.23

$$\frac{8\left(315\sqrt{2}\,a^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(630\sqrt{2}\,a^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-13\left(63\sqrt{2}\,a^7\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+4\left(2\sqrt{2}\,a^7\mathrm{sg}\right.\right.\right.\right.}{315\left(a\tan\left(\frac{1}{2}dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) - (630*sqrt(2)*a^7*sgn(cos(d*x + c
)) - 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x +
c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*
d)
```

maple [A] time = 0.87, size = 95, normalized size = 0.65

$$-\frac{2\left(584\left(\cos^{5}\left(dx+c\right)\right)-292\left(\cos^{4}\left(dx+c\right)\right)-73\left(\cos^{3}\left(dx+c\right)\right)-89\left(\cos^{2}\left(dx+c\right)\right)-95\cos\left(dx+c\right)-35\right)\sqrt{}}{315d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2
-95*cos(d*x+c)-35)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x
+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 8.18, size = 456, normalized size = 3.12

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,32i}{9\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{9\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^4}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,96i}{7\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{63\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1}}{2}}}}{\left(e^{c\,1i+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*32i)/(
9*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*((a^2*96i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(63*d)))/((exp
(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*8i)/(3*d) - (a^2*exp(c*1i + d*
x*1i)*584i)/(315*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) +
 ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*56i)/
(5*d) + (a^2*exp(c*1i + d*x*1i)*904i)/(105*d)))/((exp(c*1i + d*x*1i) + 1)*(
exp(c*2i + d*x*2i) + 1)^2) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1168i)/(315*d*(exp(c*1i + d*x*1i) +
 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\tan(c+$$

```
[Out] 2/7*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d*
x+c)/d+64/21*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*(a+a*sec(d*x
+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3798, 3793, 3792}

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{5}{7}\int \sec(c+dx)(a+a\sec(c+\\ &= \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(a+a\sec(c+dx))^{5/2}\tan(c\\ &= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{21d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(\\ &= \frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{21d}+\frac{2a}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.52

$$\frac{2a^3\tan(c+dx)\left(3\sec^3(c+dx)+12\sec^2(c+dx)+23\sec(c+dx)+46\right)}{21d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(46 + 23*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 3*Sec[c + d*x]^3)*Tan[c
+ d*x])/(21*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.67, size = 95, normalized size = 0.82

$$\frac{2\left(46\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+12\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c
)^4 + d*cos(d*x + c)^3)
```

giac [A] time = 5.11, size = 151, normalized size = 1.30

$$-\frac{8\left(21\,\sqrt{2}\,a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(35\,\sqrt{2}\,a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+4\left(2\,\sqrt{2}\,a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\right.\right.\right.}{21\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) - (35*sqrt(2)*a^6*sgn(cos(d*x + c))
 + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*a^
6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/
2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)*d)
```

maple [A] time = 0.82, size = 85, normalized size = 0.73

$$-\frac{2\left(46\left(\cos^4\left(dx+c\right)\right)-23\left(\cos^3\left(dx+c\right)\right)-11\left(\cos^2\left(dx+c\right)\right)-9\cos\left(dx+c\right)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{21d\sin\left(dx+c\right)\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*(a
*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 4.58, size = 349, normalized size = 3.01

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{a^2\,20i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,4i}{21\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{a^2\,16i}{7\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}-\frac{a^2\,e^{c\,1i+d\,x\,1i}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*20i)/(
3*d) - (a^2*exp(c*1i + d*x*1i)*4i)/(21*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2)*((a^2*16i)/(7*d) + (a^2*exp(c*1i + d*x*1i)*16i)/(7*d)))/((exp(c*
1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a
+ a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*92i)/(21*d*(exp(
c*1i + d*x*1i) + 1)) + (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2))^(1/2)*48i)/(7*d*(exp(c*1i + d*x*1i) + 1)*(exp(c
*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x)**2, x)
```

3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+64/15*a^3*tan(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)+16/15*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3793, 3792}

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(5*d)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{1}{5}(8a)\int \sec(c+dx)(a+a\sec(c$$
$$=\frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+}{5d}$$
$$=\frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2a(a+}{}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.56

$$\frac{2a^3\tan(c+dx)\left(3\sec^2(c+dx)+14\sec(c+dx)+43\right)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

[Out] (2*a^3*(43 + 14*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])

fricas [A] time = 0.83, size = 82, normalized size = 0.92

$$\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)

giac [A] time = 6.12, size = 122, normalized size = 1.37

$$\frac{8\left(15\,\sqrt{2}\,a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+4\left(2\,\sqrt{2}\,a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-5\,\sqrt{2}\,a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

maple [A] time = 0.95, size = 75, normalized size = 0.84

$$-\frac{2\left(43\left(\cos^3\left(dx+c\right)\right)-29\left(\cos^2\left(dx+c\right)\right)-11\cos\left(dx+c\right)-3\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\;a^2}{15d\sin\left(dx+c\right)\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)

[Out] -2/15/d*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2*a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)

mupad [B] time = 4.51, size = 146, normalized size = 1.64

$$-\frac{2\,a^2\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\;\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,15\mathrm{i}-\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,70\mathrm{i}+\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,70\mathrm{i}-\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,15\mathrm{i}+\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,43\mathrm{i}-43}{15\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] -(2*a^2*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(
c*1i + d*x*1i)*15i - exp(c*2i + d*x*2i)*70i + exp(c*3i + d*x*3i)*70i - exp(
c*4i + d*x*4i)*15i + exp(c*5i + d*x*5i)*43i - 43i))/(15*d*(exp(c*1i + d*x*1
i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^{\frac{5}{2}} \sec (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x), x)
```

3.111 $\int (a + a\sec(c + dx))^{5/2}\, dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*tan(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14
*a^3*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}(2a)\int \sqrt{a+a\sec(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}a\sec(c+\right.$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + a^2\int\sqrt{a+a\sec(c+dx)}\,dx + \frac{1}{3}\left(7a^2\right)\int \sec($$

$$= \frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} - \frac{\left(2a^3\right)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\right.}{}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c}{3d}$$

Mathematica [C] time = 6.36, size = 360, normalized size = 3.67

$$\frac{\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\csc^3\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{1}{2}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[(1
 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(256*Cos[(c +
 d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]
*Sin[(c + d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(c + d*x)/
2]^2]*Sin[(c + d*x)/2]^6*(2 - 3*Sin[(c + d*x)/2]^2 + Sin[(c + d*x)/2]^4) +
(21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(c + d*x)/2]^2]]*(15 - 10*Sin[(c + d*x)
/2]^2 + 3*Sin[(c + d*x)/2]^4))/Sqrt[Sin[(c + d*x)/2]^2] - 14*Sqrt[1 - 2*Sin
[(c + d*x)/2]^2]*(45 + 30*Sin[(c + d*x)/2]^2 - 31*Sin[(c + d*x)/2]^4 + 12*S
in[(c + d*x)/2]^6)))/(672*d*Sec[c + d*x]^(5/2))
```

fricas [A] time = 2.81, size = 310, normalized size = 3.16

$$\left[\frac{3\left(a^2\cos(dx+c)^2 + a^2\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2\left(8\right.}{3\left(d\cos(dx+c)^2 + d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)
 + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^2 + d*cos(d*x + c)), -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [B] time = 8.77, size = 225, normalized size = 2.30

$$-\frac{\frac{3\sqrt{-a}\,a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(7\sqrt{2}a^4\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\sqrt{2}a^4\mathrm{sg}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\right.}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a)
 - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(7*sqrt(2)*a^4*sgn(cos(d*x + c))*tan(
1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.14, size = 214, normalized size = 2.18

$$-\frac{\left(3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^2(dx+c)\right)+3\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-}}{}\right.\right.}{3d\left(1+\cos\left(dx+c\right)\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2+3*
2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)-16*cos(d*x+c)*
sin(d*x+c)-2*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1+cos(d*x+c))
/cos(d*x+c)*a^2
```

maxima [B] time = 0.92, size = 1395, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
```

```
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(c+dx\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2),x)
[Out] Integral((a*sec(c + d*x) + a)**(5/2), x)
```

3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=94

$$\frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] 5*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-a^3*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3814, 4015, 3774, 203}

$$-\frac{a^3\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
3*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*
x]]*Sin[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + (2a)\int \cos(c+dx)\sqrt{a+a\sec(c+$$
$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}\left(5a^2\right)$$
$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{\left(5a^3\right)\mathrm{Su}}{}$$
$$= \frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+}}{d}$$

Mathematica [C] time = 2.51, size = 189, normalized size = 2.01

$$\frac{2\cos^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{5/2}\left(12\sin^2\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(\frac{3}{2},2,\frac{5}{2};1,\frac{9}{2};2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)*(12*HypergeometricPFQ[{3
/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2 + (Sec[(c +
 d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1
[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometri
c2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2))/8)*Tan[(c + d*x)
/2])/(105*d)
```

fricas [A] time = 0.75, size = 276, normalized size = 2.94

$$\left[\frac{5\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(a^2\cos(dx+c)\right.}{2\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a^2*cos(d*x + c) + 2*a^2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(5*(a
^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 7.65, size = 368, normalized size = 3.91

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+5\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 5*sqrt(-a)*a^2*log(abs
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 5*sqrt(-a)*a^2*log(abs((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2)
- 3)))*sgn(cos(d*x + c)) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x + c)) - sqrt(
2)*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

maple [A] time = 1.02, size = 128, normalized size = 1.36

$$-\frac{\left(5\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)+2\cos(dx+c)-4\right)}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/2/d*(5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*co
s(d*x+c)^2+2*cos(d*x+c)-4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
^2
```

maxima [B] time = 1.01, size = 1383, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
```

```
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] 19/4*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)*(a+a*sec(d*
x+c))^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3813, 4015, 3774, 203}

$$\frac{9a^3\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}a\int \cos(c+dx)\sqrt{a+\ldots}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d}\ldots$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d}\ldots$$

$$= \frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\ldots}{\ldots}$$

Mathematica [C] time = 0.56, size = 150, normalized size = 1.42

$$-\frac{a^2\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-32\tan(c+dx)\sqrt{1-\sec(c+dx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)+(\sin(c+d\ldots\right.}{4d(\cos(c+dx)+1)\sqrt{1-\sec(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(
Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[
c + d*x] - 32*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

fricas [A] time = 0.56, size = 294, normalized size = 2.77

$$\left[\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,a^2\cos(dx+\ldots\right.}{8\,(d\cos(dx+c)+d)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(19*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*
cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a^2*cos(d*x + c)^2 + 11*a^2*co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c) + d), -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a^2*co
s(d*x + c)^2 + 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [B] time = 8.78, size = 364, normalized size = 3.43

$$-\frac{\sqrt{2}\,\sqrt{-a}\,a^5\left(\frac{19\,\sqrt{2}\,\log\left(\left|\frac{2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}\,|a|-6\,a}{2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}\,|a|-6\,a}\right|\right)}{a^2|a|}+\frac{8\left(19\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^6 \cdots}{\left(\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\cdots\right.\right.}\right.}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(-a)*a^5*(19*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
+ 4*sqrt(2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2
*a^2))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.03, size = 224, normalized size = 2.11

$$\frac{\left(19\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sqrt{2}\,\sin(dx+c)+19\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(d\cdots}{1+\cos(\cdots}}}{2\cdots}\right.\right.}{16d\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(19*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin
(d*x+c)+19*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*c
os(d*x+c)^4-36*cos(d*x+c)^3+44*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)/sin(d*x+c)/cos(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^2(c+dx}{3d}$$

```
[Out] 25/8*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 4015, 3805, 3774, 203}

$$\frac{25a^3\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{13a^3\sin}{12d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (13*a^3*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}a\int \cos^2(c+dx)\sqrt{a}$$

$$=\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(}{3d}$$

$$=\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx}{}$$

$$=\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx}{}$$

$$=\frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+d}{12d\sqrt{a+a\,s}}$$

Mathematica [C] time = 0.82, size = 151, normalized size = 1.05

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(192\sqrt{1-\sec(c+dx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)+(159\cos(c+dx)+31\cos\right.}{72d(\cos(c+dx)+1)\sqrt{1-\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(165*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (31 + 159*Cos[c + d*x] + 31*Cos
[2*(c + d*x)] - 2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 192*Hypergeome
tric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 +
Sec[c + d*x])]*Sin[c + d*x])/(72*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]
])
```

fricas [A] time = 0.75, size = 320, normalized size = 2.22

$$\left[\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a^2\cos(dx+\right.}{48\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(75*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a
*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)^3 + 34*a^2*c
os(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(75*(a^2*cos(d*x + c) + a^2)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*a^2*cos
```

```
(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

giac [B] time = 7.32, size = 539, normalized size = 3.74

$$-\frac{75\sqrt{-a}\,a^2\log\left(\left|\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(75*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*sq
rt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*(75*sqrt(2)*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(
-a)*a^3*sgn(cos(d*x + c)) - 1125*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6174
*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^6*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4314*sqrt(2)*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6*sgn(cos(d*x +
 c)) + 807*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 49*sqrt(2)*sqrt(-a)*a^8*sg
n(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*a + a^2)^3)/d
```

maple [B] time = 1.07, size = 313, normalized size = 2.17

$$-\frac{\left(75\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)\sqrt{2}+150\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos}{1+\cos}}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d*(75*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x+c
)^2*2^(1/2)+150*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*
x+c)*2^(1/2)+75*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c
)+64*cos(d*x+c)^6+208*cos(d*x+c)^5+328*cos(d*x+c)^4-600*cos(d*x+c)^3)*(a*(1
+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$\frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos(}{96d\sqrt{a\sec(c+dx)}}$$

```
[Out] 163/64*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*a
^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*cos(d*x+c)*sin(d*x+c)/d/(
a+a*sec(d*x+c))^(1/2)+17/24*a^3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^
(1/2)+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 4015, 3805, 3774, 203}

$$\frac{163a^3\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^3(c+}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d
) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a^3*Cos[c + d*x
]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^3*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^3(c+dx)\sqrt{a}$$

$$= \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin}{4d}$$

$$= \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos}{}$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c}{24d\sqrt{a+}}$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c}{24d\sqrt{a+}}$$

$$= \frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c}{96d\sqrt{a+}}$$

Mathematica [C] time = 0.82, size = 161, normalized size = 0.88

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(512\sqrt{1-\sec(c+dx)}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)+(849\cos(c+dx)+233\,\mathrm{c}\right.}{320d(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(675*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (231 + 849*Cos[c + d*x] + 233*C
os[2*(c + d*x)] + 58*Cos[3*(c + d*x)] + 2*Cos[4*(c + d*x)])*Sqrt[1 - Sec[c
+ d*x]] + 512*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(320*d*(1 + Cos[c + d*
x])*Sqrt[1 - Sec[c + d*x]])
```

fricas [A] time = 0.53, size = 346, normalized size = 1.90

$$\left[\frac{489\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\,a^2\cos(dx\right.}{384\left(d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(489*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
```

```
a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*a^2*cos(d*x + c)^4 + 184*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*
(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 1
84*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(d*x+c))]Warning, assuming -2*a+a is positive. Hint: run assume to make ass
umptions on a variableWarning, assuming -2*a+a is positive. Hint: run assum
e to make assumptions on a variableWarning, assuming -2*a+a is positive. Hi
nt: run assume to make assumptions on a variableWarning, assuming -2*a+a is
 positive. Hint: run assume to make assumptions on a variableWarning, assum
ing -2*a+a is positive. Hint: run assume to make assumptions on a variableW
arning, assuming -2*a+a is positive. Hint: run assume to make assumptions o
n a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run as
sume to make assumptions on a variableEvaluation time: 1.43Unable to divide
, perhaps due to rounding error%%%{%%{[%%%{%%{[-2309237210123256509497344,0
]:[1,0,-2]%%},[35]%%%},0]:[1,0,%%%{1,[1]%%%}]%%},[0,1]%%%} / %%%{%%%{%%{[-1
4167099448608935641088,0]:[1,0,-2]%%},[32]%%%},[0,0]%%%} Error: Bad Argumen
t Value
```

maple [B] time = 1.14, size = 402, normalized size = 2.21

$$\frac{\left(489\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{2}+1467\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3072/d*(489*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*2^(1/2)+1467*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*si
n(d*x+c)*2^(1/2)+1467*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*
sin(d*x+c)*2^(1/2)+489*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*2^(1/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*si
n(d*x+c)-768*cos(d*x+c)^8-2176*cos(d*x+c)^7-2272*cos(d*x+c)^6-2608*cos(d*x+
c)^5+7824*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(
d*x+c)^3*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^4 \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] -2*a*tan(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3792}

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
[Out] (-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Mathematica [A] time = 0.12, size = 30, normalized size = 1.11

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
[Out] (2*Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]])/d
```

fricas [A] time = 0.96, size = 44, normalized size = 1.63

$$\frac{2\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\left(\cos(dx+c)+1\right)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*(cos(d*x + c) + 1)/(d*sin(d*x + c
))
```

giac [B] time = 1.44, size = 57, normalized size = 2.11

$$-\frac{2\sqrt{2}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(2)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x
 + c))/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

maple [A] time = 1.12, size = 42, normalized size = 1.56

$$-\frac{2\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{d\left(-1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
[Out] -2/d*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*sec(d*x + c) + a)*sec(d*x + c), x)
```

mupad [B] time = 0.79, size = 36, normalized size = 1.33

$$\frac{\sin\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos(c+d\,x)}}}{d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
[Out] (sin(c + d*x)*(a - a/cos(c + d*x))^(1/2))/(d*sin(c/2 + (d*x)/2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*sec(c + d*x), x)
```

3.117 $\int \sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sec(c+dx)}\,dx &= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.62, size = 188, normalized size = 4.95

$$-\frac{\sqrt{\cos(c)-i\sin(c)}\,\cos(c+dx)\left(\cot\left(\frac{1}{2}(c+dx)\right)+i\right)\sqrt{a-a\sec(c+dx)}\left(\tanh^{-1}\left(\frac{e^{idx}}{\sqrt{\cos(c)-i\sin(c)}\,\sqrt{e^{2idx}(\cos(c)+i\sin}}\right.\right.}{d\sqrt{i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*
(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*d*x)*(Co
s[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Cos[c + d*x]*(I + Co
t[(c + d*x)/2])*Sqrt[a - a*Sec[c + d*x]]*Sqrt[Cos[c] - I*Sin[c]])/(d*Sqrt[(
1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]]))
```

fricas [B] time = 0.88, size = 182, normalized size = 4.79

$$\left[\frac{\sqrt{-a}\log\left(-\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, -\frac{\sqrt{a}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log(-(4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, -sqrt(a)*arctan(2*(cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(
(2*a*cos(d*x + c) + a)*sin(d*x + c)))/d]
```

giac [B] time = 1.47, size = 65, normalized size = 1.71

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))*s
gn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c))/d
```

maple [B] time = 1.05, size = 91, normalized size = 2.39

$$-\frac{\sqrt{2}\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\sin\left(dx+c\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)}{d\left(-1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))/(-1+cos(d*x+c))
```

maxima [B] time = 1.68, size = 146, normalized size = 3.84

$$\frac{\sqrt{a}\arctan\left(\left(\cos\left(2dx+2c\right)^2+\sin\left(2dx+2c\right)^2+2\cos\left(2dx+2c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
```

```
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a - \frac{a}{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a - a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sec(c + dx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sec(c + d*x) + a), x)
```

3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

```
[Out] -arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/d
/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3805, 3774, 203}

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (a
*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx &= \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\sec(c+dx)}\,dx\\ &= \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.94, size = 260, normalized size = 4.00

$$\frac{\cos(c+dx)\sqrt{a-a\sec(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))}+\sqrt{\cos(c)-i\sin(c)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]]*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*
Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])]*(I +
Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*
d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I + Cot[(c +
 d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[Cos[c +
 d*x]*(Cos[d*x] + I*Sin[d*x])]))/(2*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(
-1 + E^((2*I)*d*x))*Sin[c]])
```

fricas [B] time = 0.90, size = 294, normalized size = 4.52

$$\left[\frac{\sqrt{-a}\log\left(\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-}{4d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a)*log((4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(cos(d*x +
 c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(d*sin(d*x +
 c)), 1/2*(sqrt(a)*arctan(2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a
*cos(d*x + c) - a)/cos(d*x + c))/((2*a*cos(d*x + c) + a)*sin(d*x + c)))*sin
(d*x + c) - 2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos
(d*x + c)))/(d*sin(d*x + c))]
```

giac [B] time = 0.79, size = 134, normalized size = 2.06

$$\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,a\,\mathrm{sgn}}{a\tan\left(\frac{1}{2}\right.}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)
^2 - a)/sqrt(a))*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)) - 2*sqr
t(a*tan(1/2*d*x + 1/2*c)^2 - a)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x
+ 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x + c))/d
```

maple [A] time = 1.28, size = 103, normalized size = 1.58

$$\frac{\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\sin(dx+c)\left(\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}+\cos(dx+c)\sqrt{2}\right)\sqrt{2}}{2d(-1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+c
os(d*x+c)*2^(1/2))/(-1+cos(d*x+c))*2^(1/2)
```

maxima [B] time = 1.28, size = 791, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*cos(c + d*x), x)
```

3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}+\frac{28\tan(c+dx}{15d\sqrt{a\sec(c+dx}}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)+28/15*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*sec(d*x+c)^2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)-2/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3822, 4010, 4001, 3795, 203}

$$\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}+\frac{28\tan(c+dx}{15d\sqrt{a\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (28*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Se
c[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[a + a*S
ec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)(4a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15ad} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{a^2}{2}+7a^2\right.}{\sqrt{a+a\sec(c+}}}{15a^2}$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+}{15ad}$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+}{15ad}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx}{5d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.22, size = 106, normalized size = 0.76

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\,\left(3\sec^2(c+dx)-\sec(c+dx)+13\right)-15\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{15d\sqrt{1-\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-15*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c +
d*x]]*(13 - Sec[c + d*x] + 3*Sec[c + d*x]^2))*Tan[c + d*x])/(15*d*Sqrt[1 -
Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.82, size = 347, normalized size = 2.48

$$\left[\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{30\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
```

```
*x + c) + 1)) + 4*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^
2), 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

giac [A] time = 9.90, size = 205, normalized size = 1.46

$$-\frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\left(\frac{17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{20a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{\cdots}{\mathrm{sgn}\left(\tan\left(\cdots\right.\right.}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\cdots}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(15*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*((17*a
^2*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 20*a^2/sgn(tan(
1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 15*a^2/sgn(tan(1/2*d*x +
1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

maple [B] time = 1.14, size = 314, normalized size = 2.24

$$-\frac{\left(15\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sin\left(dx+c\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^2\left(dx+c\right)\right)+30\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(\cdots}{\sin(d\cdots}\right.\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/60/d*(15*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+
1)/sin(d*x+c))*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2
+30*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)+15*ln(((-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+104*cos(d*x+c)^3-112*cos(d*x+c
)^2+32*cos(d*x+c)-24)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(
d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^4}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(
1/2)-4/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan
(d*x+c)/a/d
```

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3800, 4001, 3795, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d) - (4*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{a}{2}-a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{3a}$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} + \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}dx, x,\right.}{d}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\,\tan(c+}{3ad}$$

Mathematica [A] time = 0.16, size = 86, normalized size = 0.83

$$-\frac{\tan(c+dx)\left(\frac{2}{3}(1-\sec(c+dx))^{3/2}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((-(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]) + (2*(1 - Sec[c + d*
x])^(3/2))/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
*x])]))
```

fricas [A] time = 0.80, size = 316, normalized size = 3.04

$$\left[\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
 + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*s
in(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/3*(2*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)*(a*c
os(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)
^2 + a*d*cos(d*x + c))]
```

giac [A] time = 5.13, size = 136, normalized size = 1.31

$$\frac{\sqrt{2}\left(\frac{4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{3\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*log(a
bs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(
sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.11, size = 221, normalized size = 2.12

$$-\frac{\left(3\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-c}{\sin(dx+c)}\right.}{6d\sin\left(dx+c\right)\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d*(3*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)
/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+3*l
n(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-4*cos(d*x+c)^2+8*cos(d*x+
c)-4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{3}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)+2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3798, 3795, 203}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 1.14

$$-\frac{\tan(c+dx)\left(\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - 2*Sqrt[1 - Sec[c + d*
x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

fricas [A] time = 0.77, size = 262, normalized size = 3.59

$$\left[\frac{\sqrt{2}\left(a\cos\left(dx+c\right)+a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x
+ c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
, (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

giac [B] time = 6.77, size = 132, normalized size = 1.81

$$-\frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [A] time = 0.87, size = 121, normalized size = 1.66

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin\left(dx+c\right)+2\cos\left(dx+c\right)-2\right)}{d\sin\left(dx+c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(ln(((-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+2*cos(d*x+c)-2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c)^2}{\sqrt{a \sec(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c + d\,x)^2 \sqrt{a + \frac{a}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3795, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 1.39

$$\frac{\sqrt{2}\tan(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1 -
Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.83, size = 158, normalized size = 3.43

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx}{\sqrt{a}\sin(dx+c)}\right.}{\sqrt{a}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x +
c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
/(sqrt(a)*d)]
```

giac [A] time = 6.41, size = 65, normalized size = 1.41

$$\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}d\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [B] time = 0.81, size = 95, normalized size = 2.07

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a^
(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a} - \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 24.07, size = 5402, normalized size = 63.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + a*Sec[c + d*x]],x]
[Out] Result too large to show
```

fricas [A] time = 0.70, size = 294, normalized size = 3.46

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right) - 2\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\ldots}{\ldots}\right)}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x
+ c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*sqrt(-a)*log((2*a*cos(
d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d), (sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sec(d*x + c) + a), x)
```

maple [A] time = 0.90, size = 141, normalized size = 1.66

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))+ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c)))/a
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sec(c + d*x) + a), x)
```

3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctan(1/2*a^(
1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3823, 3904, 3887, 481, 203}

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx,\right.}{d}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 105, normalized size = 0.97

$$-\frac{\tan(c+dx)\left(-\cos(c+dx)\sqrt{1-\sec(c+dx)}+\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]
]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 -
 Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

fricas [A] time = 1.09, size = 417, normalized size = 3.86

$$\left[\frac{\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}\,(\cos(dx}{2\,(ad\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt
(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*
```

```
x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(
cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), (sqrt(a)*(cos(d*x + c) + 1)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)
)) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*
d)]
```

giac [B] time = 6.02, size = 365, normalized size = 3.38

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}- \cdots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt
(2)*abs(a) - 6*a))/(abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*log((sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 8*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a) - sqrt(-a)*a)/(((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.10, size = 201, normalized size = 1.86

$$\frac{\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\ \sin(dx+c)+2\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{\cdots}\right.}{2d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*ln(((
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c))*
(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 7/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*
a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3823, 4022, 3920, 3774, 203, 3795}

$$-\frac{\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d) - Sin[c + d*x]/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)(a-3a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{-\frac{7a^2}{2}+\frac{1}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{7\int \sqrt{a+a\sec(c+dx)}\,dx}{8a} - \int$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+}{\sqrt{a+a\sec(}}\right.}{4d}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\text{co}}{2}$$

Mathematica [A] time = 0.32, size = 118, normalized size = 0.80

$$\frac{\tan(c+dx)\left(\cos(c+dx)(2\cos(c+dx)-1)\sqrt{1-\sec(c+dx)} + 7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right) - 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\text{s}}}{}\right.\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*
x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])*T
an[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.92, size = 446, normalized size = 3.03

$$\left[\frac{4\sqrt{2}\,(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right) - 7\sqrt{-a}\,(\cos(d}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*s
qrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a
)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(7*sq
rt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(a*cos(d*x + c)
 + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

giac [B] time = 4.71, size = 462, normalized size = 3.14

$$\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{8\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{8\cdots}{}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(7*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/(abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 8*log((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-
a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 8*(17*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a) - 57*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a + 19*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a
^2 - 3*sqrt(-a)*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^2*a + a^2)^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.16, size = 380, normalized size = 2.59

$$\left(7\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sqrt{2}\,\sin(dx+c)+7\sqrt{2}\,\operatorname{arctanh}\left(\cdots\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin(
d*x+c)+7*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*ln(
((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
```

```
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d*x+c)+8*ln(((-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos(d*x+c)^4+12*cos(d*x+c)^3-4*cos(d
*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{13\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{9\tan(c+dx)\sec^2(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -15/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)-1/2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+31/5*tan(d*
x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+9/10*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d
*x+c))^(1/2)-13/10*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.42, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3816, 4021, 4010, 4001, 3795, 203}

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{13\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{9\tan(c+dx)\sec^2(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2)) + (31*Tan[c + d*x])/(5*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Sec[c +
d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - (13*Sqrt[a + a*Sec
[c + d*x]]*Tan[c + d*x])/(10*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{9}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sec^2(c+dx)\left(-9a^2+\frac{39}{4}a^2\sec(c\right.}{\sqrt{a+a\sec(c+dx)}}}{5a^3}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{a+a\sec(c+dx)}\ \mathrm{ta}}{10a^2d}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 0.68

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(4\sec^3(c+dx)-4\sec^2(c+dx)+36\sec(c+dx)+49\right)-75\sqrt{2}(\sec(c+dx)+1)\right.}{20d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-75*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(49 + 36*Sec[c + d*x] - 4*Sec[c + d*x]^2 + 4*Sec[c
```

```
 + d*x]^3))*Tan[c + d*x]])/(20*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(3/2))
```

fricas [A] time = 0.90, size = 414, normalized size = 2.26

$$\left[-\frac{75\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1\cdots}\right.}{40\left(a^2d\cos(dx+c)^4+2a^2d\,c\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqr
t(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 -
 4*cos(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/2
0*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) + 2*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 - 4*cos(d*x + c
) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

giac [A] time = 8.53, size = 245, normalized size = 1.34

$$\frac{\frac{\left(\left(\left(\frac{5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{127\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{175\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{85\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\cdots}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
 - 127*sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 +
175*sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 - 85*
sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2
*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 75*sqrt(2)*lo
g(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.03, size = 417, normalized size = 2.28

$$\frac{\left(75\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^4(dx+c)\right)+150\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cdots}{1\cdots}}}{\cdots}\right.\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/80/d*(75*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^4+
```

```
150*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3-150*ln(
((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-75*ln(((-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+392*cos(d*x+c)^5-496*cos(d*x+c)^4-216
*cos(d*x+c)^3+384*cos(d*x+c)^2-96*cos(d*x+c)+32)*(a*(1+cos(d*x+c))/cos(d*x+
c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+}}$$

```
[Out] 11/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)-1/2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^(1/2)+7/6*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A] time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3816, 4010, 4001, 3795, 203}

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (13*Tan[c + d*x])/(3*a*d*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{7}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{6a^2d} - \frac{\int \frac{\sec(c+dx)\left(-\frac{7a^2}{4}+\frac{13}{2}\right.}{\sqrt{a+a\sec(c}}}{3a^3}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\,\tan(c}{6a^2d}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\,\tan(c}{6a^2d}$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A] time = 0.36, size = 114, normalized size = 0.79

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(4\sec^2(c+dx)-12\sec(c+dx)-19\right)+33\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+}}{\sqrt{2}}\right.\right.}{12d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((33*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(-19 - 12*Sec[c + d*x] + 4*Sec[c + d*x]^2))*Tan[c +
 d*x])/(12*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 3.57, size = 387, normalized size = 2.67

$$\left[-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{24\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(
-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) + 4*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*s
```

```
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(33*sqrt(2)*(cos(d*x
+ c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(19*
cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos
(d*x + c))]
```

giac [A] time = 5.77, size = 207, normalized size = 1.43

$$-\frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}-\frac{46\sqrt{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\frac{27\sqrt{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}-\frac{33\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{\ldots}\right.\right.}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\ldots\right.\right.}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) -
 46*sqrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 27*sq
rt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
 + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 33*sqrt(2)*log(abs(
-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqr
t(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.03, size = 322, normalized size = 2.22

$$\frac{\left(-1+\cos\left(dx+c\right)\right)\left(33\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+66\ln\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))*(33*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+
c)^2*sin(d*x+c)+66*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(
d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(d
*x+c)+33*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/
sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-76*cos(d*x+c)^3
+28*cos(d*x+c)^2+64*cos(d*x+c)-16)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(
d*x+c)^3/cos(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{4}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\left(a\left(\sec(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -7/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*tan(d*x+c)/a/d/(a+a*sec
(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3799, 4001, 3795, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[
c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{2a^2}$$
$$= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{7\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}dx}{4a}$$
$$= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{7\,\text{Subst}\left(\int \frac{1}{2a+x^2}dx, x, -\frac{a\tan(}{\sqrt{a+a\,\text{se}}}\right.}{2ad}$$
$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.39, size = 104, normalized size = 0.99

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}(4\sec(c+dx)+5) - 7\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-7*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(5 + 4*Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - S
ec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 2.26, size = 336, normalized size = 3.20

$$\left[-\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/
4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 4)*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 20.50, size = 159, normalized size = 1.51

$$\frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\text{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{9\sqrt{2}}{\text{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} - \frac{7\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\text{asgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
9*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a) - 7*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/
2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))/d
```

maple [B] time = 0.99, size = 225, normalized size = 2.14

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(7\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)-7\ln\left(\sqrt{-\frac{2}{1}}\right.\right.}{4d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(7*ln(((-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-7*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+10*cos(d*x+c)^3-12*cos(d*x+c)^2-6*cos(d*x+c)+8)/sin(d*x+c)^3/a^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 3/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)-1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3797, 3795, 203}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\
&= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 94, normalized size = 1.22

$$\frac{\tan(c+dx)\left(3\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Sqrt[1 - Sec[c + d*x]] + 3*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt
[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
Sec[c + d*x]))^(3/2))
```

fricas [B] time = 0.71, size = 329, normalized size = 4.27

$$\left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.\cdots}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(3*sqr
t(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 8.56, size = 122, normalized size = 1.58

$$\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 0.82, size = 222, normalized size = 2.88

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\cos(dx+c)+3\ln\left(\frac{\sqrt{-\frac{2\cos\cdots}{1+\cos\cdots}}}{\cdots}\right.\cdots\right.}{4d(1+\cos(dx+c))\sin(dx\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))*cos(d*x+c)+3*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3796, 3795, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2
]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\
&= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 93, normalized size = 1.21

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{4d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]
]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec
[c + d*x]))^(3/2))
```

fricas [B] time = 0.63, size = 327, normalized size = 4.25

$$\left[-\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 8.76, size = 122, normalized size = 1.58

$$\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))/d
```

maple [B] time = 0.79, size = 222, normalized size = 2.88

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\cos(dx+c)-\ln\left(\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\right.\right.}{4d(1+\cos(dx+c))\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-sin(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))*cos(d*x+c)-ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}} dx$

Optimal. Leaf size=114

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*
tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqr
t[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$
$$= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$
$$= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} + \frac{5\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [C] time = 24.84, size = 5524, normalized size = 48.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 1.15, size = 491, normalized size = 4.31

$$\left[-\frac{5\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - 3a\cos(dx+c)^2 - 2a\cos(d\right.}{\cos(dx+c)^2+2\cos(dx+c)+1}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) + 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [A] time = 3.00, size = 54, normalized size = 0.47

$$\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{4a^2d\,\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*d
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

maple [B] time = 0.90, size = 370, normalized size = 3.25

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4\sqrt{2}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)+4\sqrt{-\frac{2\cos(dx}{1+\cos(d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*2^(1/2)*sin(d*x+c)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)+4*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))*2^(1/2)*sin(d*x+c)+5*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*cos(d*x+c)+5*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*
cos(d*x+c)^2+2*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-3/2), x)
```

3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -3*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*
sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A] time = 0.26, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4022, 3920, 3774, 203, 3795}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c
 + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-3a+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{3a^2-\frac{3}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^3}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int \sqrt{a+a\sec(c+dx)}\,dx}{2a^2} +$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\,\text{ta}}{\sqrt{a+a}}\right.}{ad}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} +$$

Mathematica [A] time = 0.98, size = 129, normalized size = 0.90

$$\frac{\tan(c+dx)\left(2(2\cos(c+dx)+3)\sqrt{1-\sec(c+dx)} - 12(\sec(c+dx)+1)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right) + 9\sqrt{2}\,(\sec(c\right.}{4d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*(3 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 12*ArcTanh[Sqrt[1 - Sec[c
 + d*x]]]*(1 + Sec[c + d*x]) + 9*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
 Sec[c + d*x]))^(3/2))
```

fricas [A] time = 1.08, size = 518, normalized size = 3.60

$$\left[-\frac{9\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{-a}\,\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + 3a\cos(dx+c)^2 + 2a\cos(dx+c) -}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 12*(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*cos(d*x + c)^2 + 3*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [B] time = 4.29, size = 415, normalized size = 2.88

$$-\frac{16\,\sqrt{2}\left(3\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\right)}{\left(\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{4}-6\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}a+a^{2}\right)\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\,9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(16*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
9*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*sqrt(2)*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)) - 12*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) -
 6*a))/(sqrt(-a)*abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.07, size = 384, normalized size = 2.67

$$-\frac{\left(6\sqrt{2}\,\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+9\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(d}{\sin(dx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(6*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))+9*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-
6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-4*cos(d*x+c)
```

```
^4-9*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(
d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^3+8*co
s(d*x+c)^2-6*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/a
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c}{2d(a}$$

```
[Out] 19/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*cos(d*
x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/4*arctan(1/2*a^(1/2)*tan(d*x+c)
*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-7/4*sin(d*x+c)/a/d/(a+a*
sec(d*x+c))^(1/2)+cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3817, 4022, 3920, 3774, 203, 3795}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{7\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a^(3/2)*d)
- (13*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (7*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(-4a+\frac{5}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d\!\!\!\!}{4a^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \int$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + 19$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - 19$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} -$$

Mathematica [C] time = 3.37, size = 197, normalized size = 1.06

$$-\frac{\sin(2(c+dx)) - \frac{(\cos(c+dx)+1)\tan(c+dx)\sec(c+dx)\left(13\left(2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}-\cos(c+dx)\sqrt{1-\sec(c+dx)}+7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\right.\right.}{4\sqrt{1-\sec(c+dx)}}}{4d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(Sin[2*(c + d*x)] - ((1 + Cos[c + d*x])*(13*(7*ArcTanh[Sqrt[1 - Sec[c
+ d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]
*Sqrt[1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]) - 40*Hyp
ergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sec[c
 + d*x]*Tan[c + d*x])/(4*Sqrt[1 - Sec[c + d*x]]))/(d*(a*(1 + Sec[c + d*x]))
^(3/2))
```

fricas [A] time = 0.87, size = 536, normalized size = 2.90

$$\left[-\frac{13\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(\ldots}{\cos(dx+c)^2+2\cos(dx+c)+1}\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d), 1/4*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) - 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (2*c
os(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
+ a^2*d)]
```

giac [B] time = 6.46, size = 501, normalized size = 2.71

$$-\frac{\frac{13\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{19\log\left(\frac{\left|-8\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\ldots\right.\right|}{\left|-8\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\ldots\right.\right|}\right)}{\sqrt{-a}\,|a|\mathrm{sgn}\ldots}\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(13*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*sqrt(2)
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) - 19*log(abs(-8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 16*sqrt(2)*abs(a) + 24*a)/abs(-8*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 16*sqrt
(2)*abs(a) + 24*a))/(sqrt(-a)*abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 4*s
qrt(2)*(29*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^6 - 133*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4*a + 55*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*a^2 - 7*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)))/d
```

maple [B] time = 1.13, size = 560, normalized size = 3.03

$$-\frac{(-1+\cos(dx+c))\left(19\sin(dx+c)\left(\cos^2(dx+c)\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}+26\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))*(19*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))*2^(1/2)+26*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d
*x+c)^2*sin(d*x+c)+38*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*
2^(1/2)*sin(d*x+c)+52*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*si
n(d*x+c)+19*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+26
*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)-8*cos(d*x+c)^5+20*cos(d
*x+c)^4+16*cos(d*x+c)^3-28*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2
)/sin(d*x+c)^3/cos(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{95\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{197\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^3(}{4d(a\sec(c+dx)}$$

```
[Out] 163/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)-1/4*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-17/16*sec(
d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-197/24*tan(d*x+c)/a^2/d/(a+a
*sec(d*x+c))^(1/2)+95/48*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^3/d
```

Rubi [A] time = 0.42, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4019, 4010, 4001, 3795, 203}

$$\frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{95\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{197\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^3(}{4d(a\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) - (17*Sec[c + d*x]^2*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(
3/2)) - (197*Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + (95*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] && !LtQ[m, -1]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(17a^2-\frac{95}{4}a^2\sec(\right.}{\sqrt{a+a\sec(c+dx)}}}{8a^4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{a+a\sec(c+dx)}\,t}{48a^3d}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+}{16ad(a+a\sec(c+dx))}$$

Mathematica [A] time = 1.55, size = 135, normalized size = 0.74

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32\sec^3(c+dx)-160\sec^2(c+dx)-503\sec(c+dx)-299\right)+978\sqrt{2}\cos^4\left(\frac{1}{2}(c+\right.\right.}{48d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((978*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-299 - 503*Sec[c + d*x] - 160*Sec[c
```

```
+ d*x]^2 + 32*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.55, size = 455, normalized size = 2.49

$$\left[-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d}{\cos(d}\right.}{192\left(a^3d\cos(dx+c)^4+3a^3d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x
+ c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(299*cos(d*x + c)^3 +
503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(489*sqrt(2)*(cos(d*x + c)
^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)
^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

giac [A] time = 5.42, size = 254, normalized size = 1.39

$$-\frac{\frac{\left(\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{23\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{465\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)) + 23*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c
)^2 - 668*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)
^2 + 465*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 489*
sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.03, size = 417, normalized size = 2.28

$$\frac{(-1+\cos(dx+c))^2\left(-489\left(\cos^3(dx+c)\right)\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^2*(-489*cos(d*x+c)^3*sin(d*x+c)*ln(((-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)-1467*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)^2*sin(d*x+c)-1467*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*sin(
d*x+c)-489*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+1196*cos(d*x+
c)^4+816*cos(d*x+c)^3-1372*cos(d*x+c)^2-768*cos(d*x+c)+128)*(a*(1+cos(d*x+c
))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}{\left(c + dx \right)}}{\left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -75/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)-1/4*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+13/16*tan(
d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+9/4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A] time = 0.31, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3816, 4008, 4001, 3795, 203}

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) + (13*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (9*Tan[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{39a^2}{4}+9a^2\sec(c+\right.}{\sqrt{a+a\sec(c+dx)}}}{8a^4}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.85, size = 125, normalized size = 0.86

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32\sec^2(c+dx)+85\sec(c+dx)+49\right)-150\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tan\right.}{16d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-150*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(49 + 85*Sec[c + d*x] + 32*Sec[c + d
*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(
5/2))
```

fricas [A] time = 0.76, size = 404, normalized size = 2.79

$$\left[-\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
```

```
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^2 + 85*cos(d*x + c)
 + 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(75*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) + 2*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 10.27, size = 216, normalized size = 1.49

$$-\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{17\,\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{83\,\sqrt{2}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\frac{75\,\sqrt{2}\cdots}{\cdots}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 17*sqrt(2)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 75*
sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 0.98, size = 316, normalized size = 2.18

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(75\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(75*ln(((-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+150*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+75*ln(((-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)+98*cos(d*x+c)^3+72*cos(d*x+c)^2-106*cos(d*x+c)-64)
/sin(d*x+c)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] 19/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)
/d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-13/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3799, 4000, 3795, 203}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Ta
n[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{5a}{2}+4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{19\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{}{\sqrt{}}\right.}{16a^2d}$$
$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.78, size = 116, normalized size = 1.08

$$\frac{\tan(c+dx)\left(76\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right) - 2\sqrt{1-\sec(c+dx)}\,(13\sec(c+dx)+9)\right)}{32d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((76*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 - 2*Sqrt[1 - Sec[c + d*x]]*(9 + 13*Sec[c + d*x]))*Tan[c + d*x])
/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [B] time = 1.20, size = 403, normalized size = 3.77

$$\left[-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(19*sqrt(2
)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) + 2*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 7.54, size = 160, normalized size = 1.50

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} + \frac{11\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + \frac{19\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}d\right.\right.\right.}{\sqrt{-a}\,a^2\mathrm{sgn}(}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 11*sqrt(2)/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 19*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2
*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.02, size = 323, normalized size = 3.02

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(19\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\text{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(19*ln(((-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+38*sin(d*x+c)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+18*cos(d*x+c)^3+19*ln(((-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*cos(d*x+c)^2-26*cos(d*x+c))/(1+cos(
d*x+c))/sin(d*x+c)^3/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] 5/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)-1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3797, 3796, 3795, 203}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$
$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$
$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \ldots\right)}{16a^2d\ldots}$$
$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3\ldots}}$$

Mathematica [A] time = 0.74, size = 115, normalized size = 1.07

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\,(5\sec(c+dx)+1)+10\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{16d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((10*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(1 + 5*Sec[c + d*x]))*Tan[c + d*x])/(1
6*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.74, size = 399, normalized size = 3.73

$$\left[-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\ldots}{\cos(dx+c)^2+2\cos(dx+c)\ldots}\right)\ldots}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\ldots\right)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) - 2*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 10.30, size = 160, normalized size = 1.50

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{3\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{5\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\ldots\right.\right)}{\sqrt{-a}\,a^2\ldots}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*sqrt(2)/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 5*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*
d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 0.85, size = 315, normalized size = 2.94

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)+10\sin(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*ln(((-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+10*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))*cos(d*x+c)+5*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-2*cos(d*x+c)^3-8*cos(d*x+c)^2+10*cos(d*x+c))/(1+cos(d*x+c))^2/sin(d*x+c)
/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=107

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] 3/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3796, 3795, 203}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{}{\sqrt{a}}\right.}{16a^2d}$$
$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

Mathematica [C] time = 0.07, size = 52, normalized size = 0.49

$$\frac{\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{4a^2d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x])/(4*a^2*
d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [B] time = 0.69, size = 403, normalized size = 3.77

$$\left[-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*sqrt(2)*(
cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - 2*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3
*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 22.80, size = 160, normalized size = 1.50

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} - \frac{5\sqrt{2}}{a^3\text{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + \frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx\right.\right.\right.}{\sqrt{-a}a^2\text{sgn}(t}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2 /(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 5*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1 /2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d *x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1 /2*d*x + 1/2*c)^2 - 1)))/d

maple [B] time = 0.78, size = 315, normalized size = 2.94

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)+6\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)

[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*ln(((-2*cos(d*x+c)/(1+cos(d*x +c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+ c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1) /sin(d*x+c))*cos(d*x+c)+3*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+ c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-14*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c))/(1+cos(d*x+c))^2/sin(d*x+c)/ a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)

[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)

3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-43/32*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/
4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3777, 3922, 3920, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Tan
[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
```

```
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4a+\frac{3}{2}a\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2-\frac{11}{4}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\sec(c+dx)}\,d}{a^3}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \right.}{a^2d}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 25.20, size = 5564, normalized size = 38.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] Result too large to show
```

fricas [B] time = 1.01, size = 585, normalized size = 4.06

$$\left[-\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)^2+2\cos(d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
```

```
+ 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) + 4*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d
*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c))) - 2*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [A] time = 2.69, size = 92, normalized size = 0.64

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{13\,\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 13*sqrt(2)/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/d
```

maple [B] time = 1.02, size = 550, normalized size = 3.82

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(32\sqrt{2}\,\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(d}{2\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(32*2^(1/2)*sin(
d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+64*2^(1/2)*si
n(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)+43*ln(((-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+32*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+86*sin(d*x+c)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(
d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+43*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-30*cos(d*x+c)^3+8*cos(d*x+c)^2+22*cos(d*x+c))/(1+cos(d*x+c))/si
n(d*x+c)^3/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-5/2), x)
```

3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{15\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{4}$$

```
[Out] -5*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+115
/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d
*2^(1/2)+35/16*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3817, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{35\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{15\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*S
in[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(-5a+\frac{5}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{35a^2}{2}+\frac{45}{4}a^2\,\mathrm{se}\right.}{\sqrt{a+a\sec(c+dx)}}}{8a^4}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+a}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+a}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+a}}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\sec(c+dx)}$$

Mathematica [A] time = 2.23, size = 169, normalized size = 0.97

$$\frac{\sqrt{1-\sec(c+dx)}\,(16\sin(c+dx)+5\tan(c+dx)(7\sec(c+dx)+11))-80\tan(c+dx)(\sec(c+dx)+1)^2\tanh^{-}}{16d\sqrt{1-\sec(c+dx)}\,(a(\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (460*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Sec
[c + d*x]^3*Sin[(c + d*x)/2] - 80*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[
c + d*x])^2*Tan[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(16*Sin[c + d*x] + 5*(11
+ 7*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c
 + d*x]))^(5/2))
```

fricas [A] time = 0.98, size = 606, normalized size = 3.48

$$\left[-\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cdots}{\cos(dx+c)^2+2\cos(dx+c)\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) - 4*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1
/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 3
5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [B] time = 22.08, size = 459, normalized size = 2.64

$$\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{21\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\cdots}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\cdots\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 21*sqrt(2)/(a^3*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 128*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)
*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 115*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)) + 160*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
```

```
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/(sqrt(-a)*a*abs(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

maple [B] time = 1.10, size = 552, normalized size = 3.17

$$-\frac{(-1+\cos(dx+c))^2\left(-80\sqrt{2}\,\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))^2*(-80*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))-115*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)-160*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*cos(d*x+c)+32*cos(d*x+c)^4-230*sin(d*x+c)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))*cos(d*x+c)-80*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*2^(1/2)*sin(d*x+c)+78*cos(d*x+c)^3-115*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)-40*cos(d*x+c)^2-70*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c)
)^(1/2)/sin(d*x+c)^5/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3795, 203}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(Sqrt[a]*d))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [C] time = 0.46, size = 94, normalized size = 1.96

$$\frac{i\sqrt{2}\left(-1+e^{i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*(-1 + E^(I*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*
Sec[c + d*x]])
```

fricas [A] time = 0.83, size = 161, normalized size = 3.35

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqr
t((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(
d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))/d, sqrt(2)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/(s
qrt(a)*d)]
```

giac [C] time = 2.20, size = 81, normalized size = 1.69

$$-\frac{\sqrt{2}\left(\frac{\arctan(-i)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(arctan(-I)*sgn(tan(1/2*d*x + 1/2*c))/sqrt(a) + arctan(sqrt(a*tan(
1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(tan(1/2*d*x + 1/2*c))))/d
```

maple [B] time = 1.02, size = 83, normalized size = 1.73

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\,\sin\left(dx+c\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(a*(-1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)}{\sqrt{-a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{-a\left(\sec\left(c+dx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)/sqrt(-a*(sec(c + d*x) - 1)), x)
```

3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a^
(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{\int \sqrt{a-a\sec(c+dx)}\,dx}{a} + \int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] time = 0.54, size = 127, normalized size = 1.46

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1
 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Se
c[c + d*x]])
```

fricas [A] time = 1.25, size = 301, normalized size = 3.46

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-2\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c\right.}{}\right.}{2\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*s
in(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - 2*sqrt(-a)*log((2*(cos(d*
x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) -
 (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), (sqrt(2)*sqrt(a
)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sec(d*x + c) + a), x)
```

maple [A] time = 1.07, size = 119, normalized size = 1.37

$$-\frac{\left(-1+\cos \left(dx+c\right)\right)\left(\sqrt{2}\ \arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+2\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sqrt{2}}{2}\right)\right)\sqrt{2}}{d\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\ \sin\left(dx+c\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)+2*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(a*(-1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sec\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x) + a), x)
```

3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=383

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40d}+\frac{57\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{80d(\sec(c+dx)+1)}-\frac{19}{}$$

```
[Out] -9/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+57/80*(a+a*sec(d*x+c))^(2/3)*tan(
d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/a/d-19/160*3^
(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/
3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c
))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3800, 4001, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40d}+\frac{57\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{80d(\sec(c+dx)+1)}-\frac{19}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-9*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*d) + (57*(a + a*Sec[c + d*
x])^(2/3)*Tan[c + d*x])/(80*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x])
^(5/3)*Tan[c + d*x])/(8*a*d) - (19*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 -
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 +
 Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]
*Tan[c + d*x])/(80*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-((
(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} + \frac{3\int \sec(c+dx)\left(\frac{5a}{3} - a\sec(c+dx\right.}{8a}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+d}{8ad}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+d}{8ad}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+d}{8ad}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+}{80d(1+\sec(c+dx))}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+}{80d(1+\sec(c+dx))}$$
$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+}{80d(1+\sec(c+dx))}$$

Mathematica [C] time = 0.34, size = 105, normalized size = 0.27

$$\frac{\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(38\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{\sec(c+dx)+1}\,\left(5\sec^2(c+dx)\right.\right.}{40d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(38*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
 (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(1/6)*(2 + 7*Sec[c + d*x] + 5
*Sec[c + d*x]^2))*Tan[c + d*x])/(40*d*(1 + Sec[c + d*x])^(7/6))
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(2/3),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**3, x)
```

3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=353

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right.}{5\sqrt[3]{2}}$$

```
[Out] 3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+
c)/d/(1+sec(d*x+c))-1/10*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2))
)^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*Elliptic
F((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*
(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+se
c(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*
x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3
)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3798, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right.}{5\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*(a + a*Sec[c + d*x])
^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)(a+a\sec(c+dx$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{\left(2(a+a\sec(c+dx))^{2/3}\right)\int \sec(c}{5(1+\sec(c+}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(2(a+a\sec(c+dx))^{2/3}\tan(c+d\right.}{5d\sqrt{1-\sec(c+d}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx}{5d(1+\sec(c+dx))}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx}{5d(1+\sec(c+dx))}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx}{5d(1+\sec(c+dx))}$$

Mathematica [C] time = 0.15, size = 85, normalized size = 0.24

$$\frac{\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(4\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3(\sec(c+dx)+1)^{7/6}\right)}{5d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(4*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
(1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(7/6))*Tan[c + d*x])/(5*d*(1 +
 Sec[c + d*x])^(7/6))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx+c)+a)^{\frac{2}{3}} \sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

```
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}} \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**2, x)
```

3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=326

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/\cdots}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}{2\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)}$$

```
[Out] 3/2*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-1/4*3^(3/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+
2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+
c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/\cdots}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}{2\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) - (3^(
3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*
Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^
2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}}\,dx, x, \sec(c+dx\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+\right.}{2d\sqrt{1-\sec(c}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{\left(3(a+a\sec(c+dx))^{2/3}\tan(c+\right.}{d\sqrt{1-\sec(}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(}}\right.\right.}{}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.20

$$\frac{2\sqrt[6]{2}\,\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int\sec(dx+c)\,(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

```
[Out] int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a(\sec(c+dx)+1))^{\frac{2}{3}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x), x)
```

3.146 $\int (a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] 3/7*AppellF1(7/6,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c
))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^{2/3}\,dx &= \frac{(a+a\sec(c+dx))^{2/3}\int (1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}\\
&= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}\,x}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}\\
&= \frac{3\sqrt{2}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan}{7d\sqrt{1-\sec(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 8.98, size = 694, normalized size = 9.01

$$\frac{}{ad\left(135F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\left(2\tan^2(c+dx)+3\right)\sec(c+dx)+3\right)+40\sin^2\left(\frac{1}{2}(c\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (45*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(a*
(1 + Sec[c + d*x]))^(5/3)*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)*Tan[c + d*x])/(a*d*(40*(3*AppellF1
[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/
2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Sec[c + d*x]^2*
Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - 6*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2*(10*Ap
pellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-7 + 16*
Cos[c + d*x] - 3*Cos[2*(c + d*x)]) + 15*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 - 16*Cos[c + d*x] + 3*Cos[2*(c + d*x)])
 + 24*(9*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] - 6*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 5*AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos
[c + d*x]*Tan[(c + d*x)/2]^2) + 135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + Sec[c + d*x]*(3 + 2*Tan[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int (a+a\sec (dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(2/3), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(c+dx\right)+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(2/3), x)
```

3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] -3/7*AppellF1(7/6,2,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+
c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3828, 3827, 136\}$

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[7/6, 1/2, 2, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \cos(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}x^2}\,dx, x, \sec(c+$$
$$\frac{}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$
$$= -\frac{3\sqrt{2}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c}{7d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 16.36, size = 2700, normalized size = 35.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(2/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*((Sec[(c +
d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/6 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1
 + Sec[c + d*x])^(2/3))/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Sec[c +
 d*x])^(2/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2
/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2
, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)
/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3
)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan
[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*
((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)
+ (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Ta
n[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3))
- (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (81*Cos[(c + d*x)/2]^2*(
-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/
(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2
*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
```

```
c + d*x)/2]^2) - (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-1/3
*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*T
an[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2
, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/
2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d*x)/2]*(A
ppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c +
 d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*A
ppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*A
ppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appe
llF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d
*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*
x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
1/3))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx+c)+a)^{\frac{2}{3}} \cos (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \cos (dx+c) (a+a \sec (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx+c)+a)^{\frac{2}{3}} \cos (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*cos(c + d*x), x)
```

3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=413

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{88d}+\frac{147a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{440d}+$$

```
[Out] 147/440*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+1029/880*a*(a+a*sec(d*x+c))^(
2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-9/88*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d+3
/11*(a+a*sec(d*x+c))^(8/3)*tan(d*x+c)/a/d-343/1760*3^(3/4)*a*((2^(1/3)-(1+s
ec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^
2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))
^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1
/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3
)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-
(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3800, 4001, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{88d}+\frac{147a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{440d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (147*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*d) + (1029*a*(a + a*Se
c[c + d*x])^(2/3)*Tan[c + d*x])/(880*d*(1 + Sec[c + d*x])) - (9*(a + a*Sec[
c + d*x])^(5/3)*Tan[c + d*x])/(88*d) + (3*(a + a*Sec[c + d*x])^(8/3)*Tan[c
+ d*x])/(11*a*d) - (343*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])
*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(880*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} + \frac{3\int \sec(c+dx)\left(\frac{8a}{3}-a\sec(c+dx\right.}{11a}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+}{11ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+}{11ad}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+}{11ad}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} - \frac{9(a+a\sec(c+dx))^{5/3}\tan(c+}{88d}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\text{ta}}{880d(1+\sec(c+dx)}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\text{ta}}{880d(1+\sec(c+dx)}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\text{ta}}{880d(1+\sec(c+dx)}$$

Mathematica [C] time = 0.34, size = 96, normalized size = 0.23

$$\frac{a\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(196\sqrt[6]{2}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3(8\sec(c+dx)+5)(\sec(c+dx)\right.}{88d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(196*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3
/2, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(13/6)*(5 + 8*Sec[c + d*x]
))*Tan[c + d*x])/(88*d*(1 + Sec[c + d*x])^(7/6))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(dx+c\right)^4+a\sec\left(dx+c\right)^3\right)\left(a\sec\left(dx+c\right)+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(a*sec(d*x + c) + a)^(2/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/3),x)
[Out] Timed out
```

3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=383

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}+\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{8d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{16d(\sec(c+dx)+1)}-$$

```
[Out] 3/8*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/16*a*(a+a*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d-7/32*3^(3
/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/
3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c
))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3798, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}+\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{8d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{16d(\sec(c+dx)+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(16*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 +
Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1
+ Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2
]*Tan[c + d*x])/(16*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(
((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{5}{8}\int \sec(c+dx)(a+a\sec(c+dx$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{\left(5a(a+a\sec(c+dx))^{2/3}\right)\int \sec(c}{8(1+\sec(c+}$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} - \frac{\left(5a(a+a\sec(c+dx))^{2/3}\tan(c+\right.}{8d\sqrt{1-\sec(c+}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+d}{8d}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+}{16d(1+\sec(c+dx))}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+}{16d(1+\sec(c+dx))}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+}{16d(1+\sec(c+dx))}$$

Mathematica [C] time = 0.45, size = 106, normalized size = 0.28

$$\frac{a\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(5\sqrt[6]{2}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt[6]{\text{se}}\right.}{2d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(5*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2
, (1 - Sec[c + d*x])/2] + 3*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(1 + Sec[c +
d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(7/6))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec(dx+c)^3 + a\sec(dx+c)^2\right)(a\sec(dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(a*sec(d*x + c) + a)^(2/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/3),x)
[Out] Timed out
```

3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{10d(\sec(c+dx)+1)}-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+}\right.}{}$$

```
[Out] 3/5*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/10*a*(a+a*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1+sec(d*x+c))-7/20*3^(3/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-
3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*
EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c)
)^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/
3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)
*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3828, 3827, 50, 63, 225}

$$\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{10d(\sec(c+dx)+1)}-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (7*3^(3/4)*a*Elliptic
F[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Se
c[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1 - Sec[c + d*x])*(1
+ Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \sec(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(7a(a+a\sec(c+dx))^{2/3}\tan(c\right.}{5d\sqrt{1-\sec(c}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c}{10d(1+\sec(c+dx))}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c}{10d(1+\sec(c+dx))}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c}{10d(1+\sec(c+dx))}$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.19

$$\frac{4\sqrt[6]{2}\tan(c+dx)(a(\sec(c+dx)+1))^{5/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
[Out] (4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(5/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(13/6))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec(dx+c)^2+a\sec(dx+c)\right)(a\sec(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*(a*sec(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sec (c + dx) + 1))^{\frac{5}{3}} \sec (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x), x)
```

3.151 $\int (a + a\sec(c + dx))^{5/3}\, dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

```
[Out] 3/13*a*AppellF1(13/6,1,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d*x
+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*a*AppellF1[13/6, 1/2, 1, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
 d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*Sq
rt[1 - Sec[c + d*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int (1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}\,x}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$
$$= \frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a}{13d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 16.09, size = 2694, normalized size = 31.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*Tan
[(c + d*x)/2])/(2*d*(1 + Sec[c + d*x])^(5/3)) + ((Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((3*Sec[(c + d*x)/2]^2*(1 + Sec[
c + d*x])^(2/3))/4 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2
/3))/2)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]
^2 + (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)*d*(1 + Sec[c + d*x])^(5/
3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2
, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
)))/(6*2^(1/3)) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]
*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2,
 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*AppellF1[3/2,
 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(
-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (135*AppellF1[1/
2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*S
in[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (135*Cos[(c + d*x)/2]^2*(-1/3*(AppellF1[3
/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2,
 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) -
```

```
(135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-1/3*(AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2
]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Tan[(c + d*x)/2]^2)^2))/(3*2^(1/3)) + (2^(2/3)*Tan[(c + d*x)/2]*(Appell
F1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]
*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*Appell
F1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*Appel
lF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1
[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2
]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3))
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int (a + a \sec (dx + c))^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/3),x)
[Out] int((a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos\left(c + d\,x\right)}\right)^{5/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(c + dx\right) + a\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(5/3), x)
```

3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

```
[Out] -3/13*a*AppellF1(13/6,2,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d*
x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3828, 3827, 136\}$

$$-\frac{3\sqrt{2}\,a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[2]*a*AppellF1[13/6, 1/2, 2, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c
+ d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*S
qrt[1 - Sec[c + d*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \cos(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}\,x^2}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$
$$= -\frac{3\sqrt{2}\,aF_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(}{13d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 16.33, size = 2700, normalized size = 31.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(5/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2*Sec[(c
+ d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/3 + (5*Cos[c + d*x]*Sec[(c + d*x)/2]^
2*(1 + Sec[c + d*x])^(2/3))/6)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Se
c[c + d*x])^(5/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*Appell
F1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)
/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
)*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)
/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/5) + (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(
1/3)) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (243*Cos[(c + d*x
)/2]^2*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/9))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
```

```
])*Tan[(c + d*x)/2]^2) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
- 9*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/9) + 2*Tan[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*App
ellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + Ap
pellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d
*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1
[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2
]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + C
os[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^(1/3))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx + c) + a)^{\frac{5}{3}} \cos(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \cos(dx + c)(a + a\sec(dx + c))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx + c) + a)^{\frac{5}{3}} \cos(dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=371

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40ad}+\frac{99\tan(c+dx)}{80d\sqrt[3]{a\sec(c+dx)+a}}+\frac{37\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}\right.}{}$$

```
[Out] 99/80*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/8*sec(d*x+c)^2*tan(d*x+c)/d/(a+
a*sec(d*x+c))^(1/3)-3/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/a/d+37/160*3^(3/
4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4
*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+s
ec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/
(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c
))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3824, 4010, 4001, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40ad}+\frac{99\tan(c+dx)}{80d\sqrt[3]{a\sec(c+dx)+a}}+\frac{37\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (99*Tan[c + d*x])/(80*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sec[c + d*x]^2*Tan
[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) - (3*(a + a*Sec[c + d*x])^(2/3)
*Tan[c + d*x])/(40*a*d) + (37*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3
) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(80*2^(1/3)*d*(1
- Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{1}{3}a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}dx}{8a}$$

$$= \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad} + \frac{9\int \frac{\sec(c+dx)\left(-\frac{2a^2}{9}+\right.}{\sqrt[3]{a+a\sec}}}{40a}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c}{40ad}$$

Mathematica [C] time = 0.41, size = 155, normalized size = 0.42

$$\frac{\tan(c+dx)\left(-4\sqrt[6]{2}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+16\sqrt[6]{2}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)-7\sqrt[6]{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\right.\right.}{8d\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((-4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] + 16*2
^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] - 7*2^(1/6)*
Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*Sec[c + d*x]^2*(
1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(8*d*(1 + Sec[c + d*x])^(1/6)*(a*(1
+ Sec[c + d*x]))^(1/3))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(dx+c\right)}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{4}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\sqrt[3]{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=336

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}-\frac{9\tan(c+dx)}{10d\sqrt[3]{a\sec(c+dx)+a}}-\frac{7\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c\cdots}{(\cdots}}}{10\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{\cdots}}$$

```
[Out] -9/10*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*
x+c)/a/d-7/20*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2
^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a
*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3800, 4001, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}-\frac{9\tan(c+dx)}{10d\sqrt[3]{a\sec(c+dx)+a}}-\frac{7\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c\cdots}{(\cdots}}}{10\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-9*Tan[c + d*x])/(10*d*(a + a*Sec[c + d*x])^(1/3)) + (3*(a + a*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*a*d) - (7*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)
*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2a}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}dx}{5a}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{7}{10}\int \frac{\sec(c+d}{\sqrt[3]{a+a\sec(c}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{\left(7\sqrt[3]{1+\sec(c+dx)}\right.}{10\sqrt[3]{a+a\,\mathrm{s}}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(7\tan(c+dx))\,\mathrm{Su}}{10d\sqrt{1-\sec(c+d}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(21\tan(c+dx))\,\mathrm{Su}}{5d\sqrt{1-\sec(c+dx)}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{7\,3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}}{\sqrt[3]{2}}\right.\right.}{}$$

Mathematica [C] time = 0.18, size = 95, normalized size = 0.28

$$\frac{\tan(c+dx)\left(7\sqrt[6]{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{\sec(c+dx)+1}\,(2\sec(c+dx)-1)\right)}{10d\sqrt[6]{\sec(c+dx)+1}\,\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((7*2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*(1 +
 Sec[c + d*x])^(1/6)*(-1 + 2*Sec[c + d*x]))*Tan[c + d*x])/(10*d*(1 + Sec[c
+ d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=306

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}+\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{2\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(\cdots}}$$

```
[Out] 3/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+1/4*3^(3/4)*((2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/
(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(
1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*
x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)
*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^
(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.30, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3798, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}+\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{2\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[ArcC
os[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx &= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{1}{2}\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx \\
&= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{2\sqrt[3]{a+a\sec(c+dx)}} \\
&= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} \\
&= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{(3\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} \\
&= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}\right.}{2\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt[3]{a+a\,\mathrm{s}}}
\end{aligned}$$

Mathematica [C]   time = 0.13, size = 85, normalized size = 0.28

$$\frac{\tan(c+dx)\left(3\sqrt[6]{\sec(c+dx)+1}-\sqrt[6]{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\right)}{2d\sqrt[6]{\sec(c+dx)+1}\,\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((-(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]) + 3*(1
+ Sec[c + d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/6)*(a*(1 +
Sec[c + d*x]))^(1/3))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(dx+c\right)}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{2}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\cos\left(c+d\,x\right)^{2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=276

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)\right)}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] -1/2*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2
)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^
(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+
c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3828, 3827, 63, 225}

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)\right)}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] -((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}\\
&= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}\\
&= -\frac{(6\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}\\
&= -\frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\cdots}}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+\cdots}}{\left(\sqrt[3]{2}-(1\cdots\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 65, normalized size = 0.24

$$\frac{\sqrt[6]{2}\,\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d\sqrt[6]{\sec(c+dx)+1}\,\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x
])/(d*(1 + Sec[c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] 3*AppellF1(1/6,1,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x
]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$
$$= \frac{3\sqrt{2}\,F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [B] time = 5.33, size = 718, normalized size = 9.57

$$\frac{}{d\sqrt[3]{a(\sec(c+dx)+1)}\left(40\sin^2\left(\frac{1}{2}(c+dx)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3F_1\left(\frac{3}{2};-\frac{1}{3},2;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (45*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Co
s[c + d*x]*(1 + Sec[c + d*x])^2*Tan[(c + d*x)/2]*(9*AppellF1[1/2, -1/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(3*AppellF1[3/2, -1/3, 2,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(d*(a*(1 +
Sec[c + d*x]))^(1/3)*(40*(3*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*
AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c
+ d*x]^2*Sin[(c + d*x)/2]^2*(-15*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 5*A
ppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*
Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 24*(9*AppellF1[5/2, -1/3, 3, 7/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2) + 135*Appell
F1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Sec
[c + d*x] - 3*Sin[c + d*x]*Tan[c + d*x] - Tan[c + d*x]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a\sec(dx + c))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx + c) + a)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\cos(c+d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(1/3),x)
```

```
[Out] int(1/(a + a/cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a\sec(c + dx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-1/3), x)
```

3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] -3*AppellF1(1/6,2,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[1/6, 1/2, 2, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\cos(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$
$$= -\frac{3\sqrt{2}F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [B] time = 2.04, size = 240, normalized size = 3.20

$$\frac{(a(\sec(c+dx)+1))^{2/3}\left(\frac{20\sin^3\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{3}{2};\frac{2}{3},1;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan}{6(\cos(c+dx)-1)\left(3F_1\left(\frac{5}{2};\frac{2}{3},2;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2F_1\left(\frac{5}{2};\frac{5}{3},1;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)+}\right.}{ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*((20*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^3)/(6*(3*Appe
llF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF
1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c +
 d*x]) + 45*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(1 + Cos[c + d*x])) + Sin[c + d*x] - Tan[(c + d*x)/2]))/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=766

$$\frac{375\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{28a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+\ldots}{28\,2^{2/3}\ldots}$$

```
[Out] -33/28*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+3/4*sec(d*x+c)^2*tan(d*x+c)/d/(a
+a*sec(d*x+c))^(5/3)+135/14*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+375/28*(a
+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d*x+c))^(2/3)/(2^(
1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-375/28*3^(1/4)*((2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(
1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a
+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+se
c(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)
/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-125/56*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1
/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(
d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^(1/3)*
(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*x+c))^
(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 766, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3824, 4008, 4000, 3828, 3827, 63, 308, 225, 1881}

$$\frac{375\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{28a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+\ldots}{28\,2^{2/3}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-33*Tan[c + d*x])/(28*d*(a + a*Sec[c + d*x])^(5/3)) + (3*Sec[c + d*x]^2*Ta
n[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + (135*Tan[c + d*x])/(14*a*d*(
a + a*Sec[c + d*x])^(2/3)) + (375*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3)*
Tan[c + d*x])/(28*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (375*3^(1/4)*EllipticE[ArcCos[(2^(1/3) - (1 - S
qrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + S
ec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*T
an[c + d*x])/(14*2^(2/3)*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
```

```
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (125*3^(3/4)*(1 - Sqr
t[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a
*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2
^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*a^2*d*(1 -
 Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
```

```
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{5}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{5/3}}dx}{4a}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} - \frac{9\int \frac{\sec(c+dx)\left(-\frac{55a^2}{9}+\frac{35}{9}a^2\sec\right.}{(a+a\sec(c+dx))^{2/3}}}{28a^3}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2}}$$

Mathematica [C] time = 0.56, size = 111, normalized size = 0.14

$$\frac{\tan(c+dx)\left(3\left(7\sec^2(c+dx)+90\sec(c+dx)+79\right)-250\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\ {}_2F\right.}{28d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-250*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Sec
[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6) + 3*(79 + 90*Sec[c + d*
x] + 7*Sec[c + d*x]^2))*Tan[c + d*x])/(28*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^4}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$-\frac{57\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{7a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}+\frac{19\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)}\right.}{7\,2^{2/3}a^2d(}$$

```
[Out] 3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)-36/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c)
)^(2/3)-57/7*(a+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d*x
+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))+57/7*3^(1/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+
2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(
d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)+19/14*3^(3/4)*((2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(
1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2
/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2
))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2
/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+
sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3799, 4000, 3828, 3827, 63, 308, 225, 1881}

$$-\frac{57\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{7a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}+\frac{19\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)}\right.}{7\,2^{2/3}a^2d(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (36*Tan[c + d*x])/(7*a*
d*(a + a*Sec[c + d*x])^(2/3)) - (57*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3
)*Tan[c + d*x])/(7*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))) + (57*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3)
+ (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))^2]*Tan[c + d*x])/(7*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*S
qrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (19*3^(3/4)*(1 - Sqrt[
```

```
3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a^2*d*(1 - Se
c[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))^2)])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{5a}{3}+\frac{7}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a^2}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{19\int \sec(c+dx)\sqrt[3]{a+a\sec(}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(19\sqrt[3]{a+a\sec(c+dx)}\right)\int \mathrm{se}}{7a^2\sqrt[3]{1+}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(19\sqrt[3]{a+a\sec(c+dx)}\,\tan(\right.}{7a^2d\sqrt{1-\mathrm{s}}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(114\sqrt[3]{a+a\sec(c+dx)}\,\tan\right.}{7a^2d\sqrt{1-}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(57\sqrt[3]{a+a\sec(c+dx)}\,\tan(\right.}{7a^2d\sqrt{}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{57\left(1+\sqrt{3}\right)\sqrt[3]{a+a}}{7a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}\right.}$$

Mathematica [C] time = 0.28, size = 98, normalized size = 0.13

$$\frac{\tan(c+dx)\left(38\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)-36\sec(c+d}{7d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-33 - 36*Sec[c + d*x] + 38*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1
/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*
Tan[c + d*x])/(7*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{3}}{a^{2}\sec(dx+c)^{2}+2\,a^{2}\sec(dx+c)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^{3}}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int\frac{\sec^{3}(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^{3}}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\cos(c+d\,x)^{3}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{3}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$\frac{15\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{15\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a}$$

```
[Out] -3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+15/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c
))^(2/3)+15/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-15/7*2^(1/3)*3^(1/4)*(
(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(
2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x
+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/14*3^(3/4)*((2^(1/3)-(1+sec(d*x
+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/
2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^
(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-sec(d*x+c))/(a+a*sec(d*
x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3797, 3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{15\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{15\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (15*Tan[c + d*x])/(7*a
*d*(a + a*Sec[c + d*x])^(2/3)) + (15*(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (15*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3) -
 (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (5*3^(3/4)*(1 - Sqrt[3])*
```

```
EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c +
 d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(
1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*
x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{\left(5(1+\sec(c+dx))^{2/3}\right)\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{2/3}}\,dx}{7a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{\left(5\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x}\right.}{7ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(5\sqrt[6]{1+\sec(c+dx)}\,\tan\right.}{7ad\sqrt{1-}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(30\sqrt[6]{1+\sec(c+dx)}\,\tan\right.}{7ad\sqrt{1}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(15\sqrt[6]{1+\sec(c+dx)}\,\tan\right.}{7a}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{15\left(1+\sqrt{3}\right)}{7ad(a+a\sec(c+dx))^{2}}$$

Mathematica [C] time = 0.36, size = 90, normalized size = 0.12

$$\frac{\tan(c+dx)\left(5\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\;{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)-3\right)}{7d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-3 + 5*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - S
ec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(7*d*(
a*(1 + Sec[c + d*x]))^(5/3))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^2}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=744

$$\frac{6\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{3\tan(c+dx)}{7ad(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}+\frac{6\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a}$$

```
[Out] 6/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+3/7*tan(d*x+c)/a/d/(1+sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)+6/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a/d
/(a+a*sec(d*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-6/7*2^(1
/3)*3^(1/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)
))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))
^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3
))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(3/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*
((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+se
c(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-sec(d*x+c))
/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3
))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 744, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{6\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{3\tan(c+dx)}{7ad(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}+\frac{6\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (6*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) + (3*Tan[c + d*x])/(7*a
*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)) + (6*(1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (6*2^(1/3)*3^(1/4)*EllipticE[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (2^(1/3)
```

```
*3^(3/4)*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + S
qrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sq
rt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*
(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2)])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{\left(2\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\mathrm{Subs}}{7ad\sqrt{1-\sec(c+dx)}\,(}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{\left(2\sqrt[6]{1+\mathrm{se}}\right.}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{\left(12\sqrt[6]{1+\right.}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{\left(6\sqrt[6]{1+\mathrm{se}}\right.}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{}{7ad(a+a}$$

Mathematica [C] time = 0.08, size = 68, normalized size = 0.09

$$\frac{\tan(c+dx)(\sec(c+dx)+1)^{7/6}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{2\sqrt[6]{2}\,d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Hypergeometric2F1[1/2, 13/6, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])
^(7/6)*Tan[c + d*x])/(2*2^(1/6)*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] -3/7*AppellF1(-7/6,1,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(
d*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[-7/6, 1/2, 1, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{1}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$
$$= -\frac{3\sqrt{2}\,F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 16.72, size = 3007, normalized size = 33.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((27*Sin[
c + d*x])/7 - (30*Tan[(c + d*x)/2])/7 + (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) + (2^(1/3)*(1 + Sec[c + d*x])^(5
/3)*((16*(1 + Sec[c + d*x])^(1/3))/7 - (27*Cos[c + d*x]*(1 + Sec[c + d*x])^
(1/3))/7)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2
, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3
/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9
)))))/(7*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(
5/3)*((Sec[(c + d*x)/2]^2*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
))))/(7*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (2^(1/3)*Tan[(c
+ d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (3*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (A
ppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) +
(2*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[
(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3) - Cos[
(c + d*x)/2]*Sin[(c + d*x)/2]*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
)) + Cos[(c + d*x)/2]^2*((5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[(c + d
*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
```

```
2])*Tan[(c + d*x)/2]^2)/9)) - (5*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/9))/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2, 1
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9))
+ (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-
1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*
(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + A
ppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2,
4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*AppellF1[5/
2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1 + Tan[(
c + d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])*Tan[(c + d*x)/2]^2)/9)^2))))/(7*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2
/3)) - (2*2^(1/3)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(Appe
llF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Ap
pellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF
1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)
/2]^2)/9))))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*(Cos[(c + d*x)/2]^2*Sec[c + d*x])
^(5/3))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/3),x)
[Out] int(1/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cos(c + d*x))^(5/3),x)
[Out] int(1/(a + a/cos(c + d*x))^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(c+dx)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/3),x)
[Out] Integral((a*sec(c + d*x) + a)**(-5/3), x)
```

3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] 3/7*AppellF1(-7/6,2,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d
*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[-7/6, 1/2, 2, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*
Sec[c + d*x])^(2/3))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\cos(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3\sqrt{2}F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 16.58, size = 3011, normalized size = 33.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((-48*Sin
[c + d*x])/7 + (51*Tan[(c + d*x)/2])/7 - (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) - (5*2^(1/3)*(1 + Sec[c + d*x])
^(5/3)*((-30*(1 + Sec[c + d*x])^(1/3))/7 + (55*Cos[c + d*x]*(1 + Sec[c + d*
x])^(1/3))/7)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[
1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF
1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2
, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
)/9)))))/(63*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]
))^(5/3)*((-5*Sec[(c + d*x)/2]^2*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF
1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]
^2)/9)))))/(63*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (5*2^(1/3
)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(2/3) - (11*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/5 + (AppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + (22*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(5/3)) - 9*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(-11 - AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*
(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(
-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2)/9))) + 9*Cos[(c + d*x)/2]^2*((AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-
1 + Tan[(c + d*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
```

```
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)) - (-1/3*(AppellF1[3/2, 1/3, 2, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)/((-1 + Tan[(c + d*x)/2]^2)*(Appel
lF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*App
ellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1
[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2)/9)) + (AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/9 + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*Appell
F1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*App
ellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1
 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)^2))))/(63*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(2/3)) + (10*2^(1/3)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*
x]*Sec[(c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/
3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]
^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
(2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2)/9))))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) +
Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(189*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^(5/3))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx + c)}{(a\sec(dx + c) + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)}{\left(a + \frac{a}{\cos(c+d\,x)}\right)^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3),x)
```

```
[Out] int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^{\frac{5}{2}}(c+dx)\,dx + a\int \sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{s$$

$$= \frac{6a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(}{}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a\sqrt{\sec(c+dx)}\,\sin(}{5d}$$

$$= -\frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [A] time = 0.25, size = 115, normalized size = 0.76

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)\left(9\sin(c+dx)+5\tan(c+dx)+5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-9\sqrt{\cos(}\right.}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(-9*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*Sin[c +
 d*x] + 5*Tan[c + d*x] + 3*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[Sec[c + d
*x]])
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sec\left(dx+c\right)^3+a\sec\left(dx+c\right)^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 5.94, size = 384, normalized size = 2.54

$$-\frac{a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{10\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{12\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^3-12/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+28/15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/5*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-1/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx+c) + a) \sec(dx+c)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(c + d\,x)}\right) \left(\frac{1}{\cos(c + d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\cdots}{\cdots}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^{\frac{3}{2}}(c+dx)\,dx + a\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a\int \sqrt{\sec$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big(a\sqrt{\cos(c$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\Big(\frac{1}{2}(}{}$$

Mathematica [A] time = 0.22, size = 83, normalized size = 0.67

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)+3\sin(2(c+dx))+2\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]))/(3*d)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(dx+c\right)^{2}+a\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 5.78, size = 369, normalized size = 3.00

$$\frac{2a\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(co
```

```
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sec^{\frac{3}{2}}(c+dx)\,dx+\int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

3.167 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))\,dx = a\int \sqrt{\sec(c+dx)}\,dx + a\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a\sqrt{\cos(c+d\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\,\sin}{d}$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [A] time = 0.17, size = 68, normalized size = 0.70

$$\frac{2a\sqrt{\sec(c+dx)}\left(\sin(c+dx)+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((a\sec(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [A] time = 3.89, size = 146, normalized size = 1.51

$$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2*a*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
```

```
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx+c)+a)\sqrt{\sec (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos (c+d\,x)}\right)\,\sqrt{\frac{1}{\cos (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{\sec (c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+a\int \sqrt{\sec(c+dx)}\,dx\\ &= \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.\\ &= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}\end{aligned}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 0.65

$$\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 2] + EllipticF[(c + d*x)/2,
 2])*Sqrt[Sec[c + d*x]])/d
```

fricas [F] time = 1.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 2.82, size = 150, normalized size = 2.00

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{\sec{(c+dx)}}}\,dx + \int \sqrt{\sec{(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+}\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.72

$$\frac{a\sqrt{\sec(c+dx)}\left(\sin(2(c+dx)) + 2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + 6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.33, size = 225, normalized size = 2.23

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec(dx + c) + a}{\sec(dx + c)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} \, dx + \int \frac{1}{\sqrt{\sec(c + dx)}} \, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ }{5d}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+ }{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a + a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

Mathematica [A] time = 0.22, size = 93, normalized size = 0.73

$$\frac{a\sqrt{\sec(c+dx)}\left(3\sin(c+dx) + 10\sin(2(c+dx)) + 3\sin(3(c+dx)) + 20\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + 36\sqrt{\cos}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(36*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*Sin[c + d*x] + 10*Sin[2*(
c + d*x)] + 3*Sin[3*(c + d*x)]))/(30*d)
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c) + a}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c) + a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 3.08, size = 219, normalized size = 1.72

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2}\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec(dx+c) + a}{\sec(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\, dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-5/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(}}{}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,d$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx}$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Mathematica [A] time = 0.29, size = 103, normalized size = 0.68

$$\frac{a\sqrt{\sec(c+dx)}\left(42\sin(c+dx)+130\sin(2(c+dx))+42\sin(3(c+dx))+15\sin(4(c+dx))+200\sqrt{\cos(c+dx}\right.}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(504*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
00*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 130*Sin
[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.51, size = 270, normalized size = 1.79

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-7/2), x) + Integral(sec(c + d*x)**(-5/2), x))
```

3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=187

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{12a^2\sin(c+dx)\sqrt{s}}{5d}$$

```
[Out] 8/7*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{12a^2\sin(c+dx)\sqrt{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(7*d) + (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(7*d) + (4*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6a^2\right)\ldots$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{4a^2\sec\ldots}{\ldots}$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{4a^2\sec\ldots}{\ldots}$$

$$= -\frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\ldots}{\ldots}$$

Mathematica [C] time = 2.65, size = 269, normalized size = 1.44

$$\frac{1}{35}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{42\csc(c)\cos(dx)+(14\cos(c+dx)+10\cos(2(c+dx))+15)\tan(c+dx)\ldots}{2d\sec^{\frac{3}{2}}(c+dx)}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Cos[c + d*x]^2*
(21*Sqrt[1 + E^((2*I)*(c + d*x))] + 21*(-1 + E^((2*I)*c))*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^((2
*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]) + (42*Cos[d*x]*Csc[c] + (15 + 14*Cos[c + d*x] + 10*Cos[2*(c + d
*x)])*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/35
```

fricas [F] time = 1.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec(dx+c)^4+2\,a^2\sec(dx+c)^3+a^2\sec(dx+c)^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

maple [B] time = 6.52, size = 439, normalized size = 2.35

$$-\frac{a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{28\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{d}{2}\right.\right.}}{7\left(-\frac{1}{2}+\cos^2\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^4-4/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+124/35*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/5*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(
1/2*d*x+1/2*c)^2)^3-24/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-12/5*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{s}}{}$$

```
[Out] 4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a\right.$$

$$= \frac{16a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2}{}$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2\sqrt{\sec(c+dx)}}{5d}$$

$$= -\frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 1.58, size = 269, normalized size = 1.67

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{24\csc(c)\cos(dx)+\tan(c+dx)(3\sec(c+dx)+10)}{\sec^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(12\left(-1+e^{2ic}\right)\sqrt{1-}\right.}{}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(12*(1 + E^((2*I)*(c + d*
x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I
)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (24*Cos[d*x]*Csc[c]
+ (10 + 3*Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(3/2)))/(30*d)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec(dx+c)^3 + 2a^2\sec(dx+c)^2 + a^2\sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

maple [B] time = 5.98, size = 386, normalized size = 2.40

$$-\frac{a^{2}\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{32\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\frac{68\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}\cdots\right.\right.}}{15\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}\cdots\right.\right.}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-32/5*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*
x+1/2*c)^2)^(1/2)+68/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-16/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{2}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.174 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=131

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-$$

```
[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right.$$

$$= -\frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [C] time = 1.25, size = 264, normalized size = 2.02

$$\frac{1}{3}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{\tan(c+dx)+6\csc(c)\cos(dx)}{2d\sec^{\frac{3}{2}}(c+dx)} - \frac{i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(3*(1 + E^((2*I)*(c + d*x))
) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1
/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
 + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (6*Cos[d*x]*Csc[c] + T
an[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

maple [B] time = 5.52, size = 371, normalized size = 2.83

$$4a^2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(4\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{d}{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{\sec(c+dx)}\,dx+\int 2\sec^{\frac{3}{2}}(c+dx)\,dx+\int\sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sqrt(sec(c + d*x)), x) + Integral(2*sec(c + d*x)**(3/2), x)
+ Integral(sec(c + d*x)**(5/2), x))
```

3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=64

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3788, 3771, 2641, 4043}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(2a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}}$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.75

$$\frac{2a^2\sqrt{\sec(c+dx)}\left(\sin(c+dx)+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[c + d*x]))/d
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

maple [A] time = 4.12, size = 104, normalized size = 1.62

$$-\frac{4a^2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -4*a^2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int 2\sqrt{\sec(c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
[Out] a**2*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(2*sqrt(sec(c + d*x)), x)
 + Integral(sec(c + d*x)**(3/2), x))
```

3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2639, 4045, 2641}

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(2a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\sqrt{\cos(c\right.$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [C] time = 0.85, size = 156, normalized size = 1.46

$$\frac{a^2\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)-i\cos\left(\frac{c}{2}\right)\right)\left(-\frac{24\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+8\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(Cos[c/2] - I*Sin[c/2])*((-I)*Cos[c/2] + Sin[c/2])*(12 - (24*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x)
)] + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*Sqrt[Sec[c + d*x]
])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.40, size = 228, normalized size = 2.13

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2}{\sqrt{\sec(c+dx)}}\,dx+\int\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-3/2), x) + Integral(2/sqrt(sec(c + d*x)), x)
 + Integral(sqrt(sec(c + d*x)), x))
```

3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{5d}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(2a^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}\left(8a^2\right)\int -$$
$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\text{cc}}}$$
$$= \frac{16a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] time = 1.14, size = 136, normalized size = 1.01

$$\frac{a^2\left(\frac{192i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 40i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 40\sin(c+dx) + 6\sin(2(c+\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin
[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2 + 2\,a^2\sec(dx+c) + a^2}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

maple [A] time = 3.47, size = 250, normalized size = 1.85

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{2}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-5/2), x) + Integral(2/sec(c + d*x)**(3/2), x
) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{c}}{}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(2a^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}\left(12a^2\right)\int \frac{}{\sec}$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,d$$
$$= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+d}{5d\sec^{\frac{3}{2}}(c+}$$
$$= \frac{12a^2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{7d}$$

Mathematica [C] time = 1.38, size = 149, normalized size = 0.93

$$\frac{a^2\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-80i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 85\sin(c+dx) + 28\sin(2(\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85
*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt[Sec
[c + d*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\sec\left(dx+c\right)^2 + 2\,a^2\sec\left(dx+c\right) + a^2}{\sec\left(dx+c\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.60, size = 272, normalized size = 1.69

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{2}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-7/2), x) + Integral(2/sec(c + d*x)**(5/2), x
) + Integral(sec(c + d*x)**(-3/2), x))
```

3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=187

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)}{5d}$$

```
[Out] 52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3791, 3768, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+a^3\sec^{\frac{9}{2}}(c+$$

$$= a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx + a^3\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{6a^3\sec^{\frac{5}{2}}}{}$$

$$= \frac{28a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6a^3\mathrm{se}}{}$$

$$= -\frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

$$= -\frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 2.26, size = 287, normalized size = 1.53

$$\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(\frac{294\csc(c)\cos(dx)+(63\cos(c+dx)+65\cos(2(c+dx))+80)\tan(c+dx)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{}}{}\right.}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(147*(1 + E^((2*I)*(c + d
*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (294*Cos[d*x]*Csc
[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d
*x])/Sec[c + d*x]^(5/2)))/(420*d)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec(dx+c)^4+3\,a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+a^3\sec(dx+c)\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2
+ a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

maple [B] time = 6.37, size = 439, normalized size = 2.35

$$-\frac{a^3\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{10\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots\right)}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-3/10*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^3-56/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*co
s(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+848/105*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-28/5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-26/21*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^3\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.180 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\,\sqrt{s}}{}$$

```
[Out] 2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+3
6/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2641, 3768, 2639}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\,\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + 3a^3\sec^{\frac{5}{2}}(c+dx) + a^3\sec^{\frac{7}{2}}\right.$$
$$= a^3\int \sqrt{\sec(c+dx)}\,dx + a^3\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sec^{\frac{3}{2}}(c+\ldots$$
$$= \frac{6a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{2a^3\ldots}{\ldots}$$
$$= \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{36a^3\sqrt{\sec(c+dx\ldots}}{5d}$$
$$= -\frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4a^3\sqrt{\cos(c+dx\ldots}}{\ldots}$$
$$= -\frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+\ldots}}{\ldots}$$

Mathematica [C] time = 1.91, size = 267, normalized size = 1.70

$$\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(\frac{18\csc(c)\cos(dx)+\tan(c+dx)(\sec(c+dx)+5)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(\ldots}}\right.}{\ldots}\right.}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(9*(1 + E^((2*I)*(c + d*x
))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*
(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (18*Cos[d*x]*Csc[c] +
(5 + Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(20*d)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec(dx+c)^3 + 3\,a^3\sec(dx+c)^2 + 3\,a^3\sec(dx+c) + a^3\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

maple [B] time = 6.51, size = 386, normalized size = 2.46

$$-\frac{a^3\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{\cos\left(\frac{dx}{2}+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(56/5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^3-72/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/
(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-36/5*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2639, 2641, 3768}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \int \left(\frac{a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sec^{\frac{5}{2}}(c+dx)\right)dx\\ &= a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+a^3\int \sec^{\frac{5}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sqrt{\sec(c+dx)}\,dx+\left(3a^3\right)\\ &= \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}a^3\int \sqrt{\sec(c+dx)}\\ &= \frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\\ &= -\frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}\end{aligned}$$

Mathematica [C] time = 1.41, size = 187, normalized size = 1.43

$$\frac{a^3e^{-2i(c+dx)}\sec^{\frac{3}{2}}(c+dx)(\sin(2(c+dx))-i\cos(2(c+dx)))\left(6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d*
x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeometric
2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*Sin[2
*(c + d*x)])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d*E^((2*I)*(c +
 d*x)))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

maple [B] time = 5.60, size = 371, normalized size = 2.83

$$\frac{4a^3\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(10\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int 3\sqrt{\sec(c+dx)}\,dx+\int 3\sec^{\frac{3}{2}}(c+dx)\,dx+\int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(3*sqrt(sec(c + d*x)), x)
 + Integral(3*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+}{}$$

```
[Out] 2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4
*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int\left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$= a^3\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+a^3\int\sec^{\frac{3}{2}}(c+dx)\,dx+\left(3a^3\right)\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx+\left(3a^3\right.$$

$$= \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{3}a^3\int\sqrt{\sec(c+dx)}\,dx-a^3$$

$$= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{d}$$

$$= \frac{4a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] time = 1.09, size = 169, normalized size = 1.29

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-10i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+d\right.\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((24*I)*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] +
 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x])))
/(3*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.68, size = 172, normalized size = 1.31

$$-\frac{4a^3\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{3}{\sqrt{\sec(c+dx)}}\,dx+\int 3\sqrt{\sec(c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-3/2), x) + Integral(3/sqrt(sec(c + d*x)), x)
 + Integral(3*sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

```
[Out] 2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+3
6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sqrt{\sec(c+dx)}} + a^3\sqrt{\sec(c+dx)}\right) dx$$
$$= a^3 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^3 \int \sqrt{\sec(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int$$
$$= \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a^3\right) \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3 \int \sqrt{\sec(c+}$$
$$= \frac{6a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$
$$= \frac{36a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

Mathematica [C] time = 1.07, size = 171, normalized size = 1.31

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{144i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-20i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right.\right.}{10d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((144*I)*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))]
+ 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/
2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)]))))/(10*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

maple [A] time = 3.76, size = 250, normalized size = 1.91

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)

[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)

[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{3}{\sqrt{\sec(c+dx)}}\,dx+\int \sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)

[Out] a**3*(Integral(sec(c + d*x)**(-5/2), x) + Integral(3/sec(c + d*x)**(3/2), x) + Integral(3/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))

3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{c}}{}$$

```
[Out] 2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}dx = \int\left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx + a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx + \left(3a^3\right)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx + \left(3a\right.$$

$$= \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^3\right)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

Mathematica [C] time = 1.52, size = 146, normalized size = 0.91

$$\frac{a^3\left(\frac{4704i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 1040i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 1070\sin(c+dx) + 252\,\mathrm{s}\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1
070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt
[Sec[c + d*x]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\sec(dx+c)^3 + 3a^3\sec(dx+c)^2 + 3a^3\sec(dx+c) + a^3}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)

maple [A] time = 3.56, size = 272, normalized size = 1.69

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)

[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos (1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2 *c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2) ^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)

[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{3}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)

[Out] a**3*(Integral(sec(c + d*x)**(-7/2), x) + Integral(3/sec(c + d*x)**(5/2), x) + Integral(3/sec(c + d*x)**(3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))

3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\right.}{21d}$$

```
[Out] 2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+44/21*a^3*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right) dx$$

$$= a^3 \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right) \int$$

$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a^3 \int$$

$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{15}(7$$

$$= \frac{18a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

$$= \frac{68a^3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Mathematica [C] time = 1.98, size = 156, normalized size = 0.83

$$\frac{a^3\left(\frac{22848i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 5280i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 5820\sin(c+dx) + 2044\,\mathrm{si}\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[
4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\sec(dx+c)^3 + 3\,a^3\sec(dx+c)^2 + 3\,a^3\sec(dx+c) + a^3}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

maple [A] time = 4.05, size = 260, normalized size = 1.39

$$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^{3}\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^{7}\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{3}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.186 $\int \sec^{\frac{3}{2}}(c + dx)(a + a\sec(c + dx))^4\, dx$

Optimal. Leaf size=213

$$\frac{2a^4\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4\sin(c+dx)\mathrm{sec}}{7d}$$

```
[Out] 32/7*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+122/45*a^4*sec(d*x+c)^(5/2)*sin(d*x+
c)/d+8/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*a^4*sec(d*x+c)^(9/2)*sin(d*x
+c)/d+152/15*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-152/15*a^4*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3791, 3768, 3771, 2639, 2641}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4\sin(c+dx)\mathrm{sec}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^4*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4*Sec[
c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^{\frac{3}{2}}(c+dx)+4a^4\sec^{\frac{5}{2}}(c+dx)+6a^4\sec^{\frac{7}{2}}(c+dx)+4a^4\sec^{\frac{9}{2}}\right.$$
$$= a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + a^4\int \sec^{\frac{11}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{5}{2}}(c+dx$$
$$= \frac{2a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{12a^4}{}$$
$$= \frac{46a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{32a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{12}{}$$
$$= -\frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}}{}$$
$$= -\frac{46a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+}}{}$$
$$= -\frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 5.09, size = 271, normalized size = 1.27

$$\frac{1}{210}a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{1596\csc(c)\cos(dx)+\tan(c+dx)\left(35\sec^3(c+dx)+180\sec^2(c+dx\right.}{12d\sec^{\frac{7}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-I)*Sqrt[2]*Cos[c + d*x]^4*
(133*Sqrt[1 + E^((2*I)*(c + d*x))] + 133*(-1 + E^((2*I)*c))*Hypergeometric2
F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 60*E^(I*(c + d*x))*(-1 + E^((2*I
)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^(
(2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]) + (1596*Cos[d*x]*Csc[c] + (720 + 427*Sec[c + d*x] + 180*Sec[c
 + d*x]^2 + 35*Sec[c + d*x]^3)*Tan[c + d*x])/(12*d*Sec[c + d*x]^(7/2))))/21
0
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4\sec(dx+c)^5+4a^4\sec(dx+c)^4+6a^4\sec(dx+c)^3+4a^4\sec(dx+c)^2+a^4\sec(dx+c)\right)\sqrt{\sec(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^5 + 4*a^4*sec(d*x + c)^4 + 6*a^4*sec(d*x + c)^3
+ 4*a^4*sec(d*x + c)^2 + a^4*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

maple [B] time = 7.02, size = 492, normalized size = 2.31

$$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{72\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{61\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}{90\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^5-61/90*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-304/15*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)
^2)^(1/2)+1544/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-152/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1
/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^4-16/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^4\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.187 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=187

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{}}{5d}$$

```
[Out] 94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2641, 3768, 2639}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + 6a^4\sec^{\frac{5}{2}}(c+dx) + 4a^4\sec^{\frac{7}{2}}(\right.$$
$$= a^4\int \sqrt{\sec(c+dx)}\,dx + a^4\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{3}{2}}(c+dx)$$
$$= \frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{4a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{8a^4\sec}{}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{64a^4\sqrt{\sec(c+dx)}\,\mathrm{si}}{5d}$$
$$= -\frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^4\sqrt{\cos(c+dx)}\,F}{}$$
$$= -\frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+d}}{}$$

Mathematica [C] time = 2.72, size = 279, normalized size = 1.49

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{672\csc(c)\cos(dx)+\tan(c+dx)\left(15\sec^2(c+dx)+84\sec(c+dx)+235\right)}{\sec^{\frac{7}{2}}(c+dx)} - \frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(\right.}{}\right.}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(168*(1 + E^((2*I)*(c + d
*x))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (672*Cos[d*x]*Csc
[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])/Sec[c + d*x
]^(7/2)))/(840*d)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4\sec(dx+c)^4 + 4a^4\sec(dx+c)^3 + 6a^4\sec(dx+c)^2 + 4a^4\sec(dx+c) + a^4\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)

maple [B] time = 6.31, size = 439, normalized size = 2.35

$$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2024\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x)

[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2024/105*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2/5*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-128/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-64/5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-47/21*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^4\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4,x)

[Out] Timed out

3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\,\sqrt{}$$

```
[Out] 8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A] time = 0.18, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3791, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\,\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int\left(\frac{a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + 6a^4\sec^{\frac{3}{2}}(c+dx) + 4a^4\sec^{\frac{5}{2}}(c+dx) + a^4$$
$$= a^4\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^4\int\sec^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int\sqrt{\sec(c+dx)}\,dx + \left(4a\right.$$
$$= \frac{12a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{d}$$
$$= -\frac{10a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$
$$= -\frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] time = 3.27, size = 286, normalized size = 1.78

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{30\cos(c)\sin(dx)-3(5\cos(2c)-61)\csc(c)\cos(dx)+2\tan(c+dx)(3\sec(c+dx)+20)}{\sec^{\frac{7}{2}}(c+dx)} - \frac{8i\sqrt{2}e^{-i(c+d}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(21*(1 + E^((2*I)*(c + d*
x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (-3*(-61 + 5*Cos[2*
c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[c +
d*x])/Sec[c + d*x]^(7/2)))/(240*d)
```

fricas [F] time = 1.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4 + 4\,a^4\sec(dx+c)^3 + 6\,a^4\sec(dx+c)^2 + 4\,a^4\sec(dx+c) + a^4}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

maple [B] time = 6.32, size = 386, normalized size = 2.40

$$-\frac{a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\text{EllipticE}\right.}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-56/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+328/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-132/5*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1
/2)-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3d}$$

```
[Out] 2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+8*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}dx = \int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + 6a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + a^4\sec^{\frac{5}{2}}($$

$$= a^4\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx + a^4\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx + \left(4a^4\right)\int$$

$$= \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + 2$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{12a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}$$

$$= \frac{40a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+}}{}$$

Mathematica [A] time = 0.32, size = 70, normalized size = 0.59

$$\frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+dx))+80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4+4\,a^4\sec(dx+c)^3+6\,a^4\sec(dx+c)^2+4\,a^4\sec(dx+c)+a^4}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)

maple [B] time = 4.75, size = 292, normalized size = 2.47

$$\frac{8a^4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)

[Out] 8/3*a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{4}{\sqrt{\sec(c+dx)}}\,dx + \int 6\sqrt{\sec(c+dx)}\,dx + \int 4\sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)

[Out] a**4*(Integral(sec(c + d*x)**(-3/2), x) + Integral(4/sqrt(sec(c + d*x)), x) + Integral(6*sqrt(sec(c + d*x)), x) + Integral(4*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))

3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

```
[Out] 2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3791, 3769, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + a^4\sec^{\frac{3}{2}}(\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right.$$

$$= \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{5}\left(3a^4\right)\int$$

$$= \frac{12a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{d}$$

$$= \frac{56a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [C] time = 1.24, size = 184, normalized size = 1.16

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 320i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-336*I + ((672*I)*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c
+ d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2
, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c + d*x
]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4 + 4\,a^4\sec(dx+c)^3 + 6\,a^4\sec(dx+c)^2 + 4\,a^4\sec(dx+c) + a^4}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

maple [A] time = 3.59, size = 194, normalized size = 1.22

$$-\frac{8a^4\left(-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+26\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{15\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)
```

```
[Out] -8/15*a^4*(-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+26*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{4}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{6}{\sqrt{\sec(c+dx)}}\,dx+\int 4\sqrt{\sec(c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**(-5/2), x) + Integral(4/sec(c + d*x)**(3/2), x
) + Integral(6/sqrt(sec(c + d*x)), x) + Integral(4*sqrt(sec(c + d*x)), x) +
 Integral(sec(c + d*x)**(3/2), x))
```

3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{}}{}$$

```
[Out] 2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int\left(\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + a^4\sqrt{\sec(c+\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^4\int\sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(4a^4\right.$$

$$= \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^4\right)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{6a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 1.32, size = 180, normalized size = 1.12

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{10752i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2720i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+\right.\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-5376*I + ((10752*I)*
Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*
(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336*Sin
[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4 + 4\,a^4\sec(dx+c)^3 + 6\,a^4\sec(dx+c)^2 + 4\,a^4\sec(dx+c) + a^4}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.29, size = 272, normalized size = 1.69

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int\frac{4}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{6}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{4}{\sqrt{\sec(c+dx)}}\,dx+\int\sqrt{\sec(c+dx)}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**(-7/2), x) + Integral(4/sec(c + d*x)**(5/2), x
) + Integral(6/sec(c + d*x)**(3/2), x) + Integral(4/sqrt(sec(c + d*x)), x)
+ Integral(sqrt(sec(c + d*x)), x))
```

3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}\right.}{7d}$$

```
[Out] 2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c)^
(1/2)+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}\right.}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\sec(c+dx)}}\right)$$

$$= a^4\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right)$$

$$= \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{9}\left(7a^4\right)$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx}{7d\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{46a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

$$= \frac{152a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{7d}$$

Mathematica [C] time = 1.85, size = 156, normalized size = 0.83

$$\frac{a^4\left(\frac{51072i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 11520i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 12240\sin(c+dx) + 3556\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Si
n[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{a^4\sec(dx+c)^4 + 4a^4\sec(dx+c)^3 + 6a^4\sec(dx+c)^2 + 4a^4\sec(dx+c) + a^4}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

maple [A] time = 3.21, size = 260, normalized size = 1.39

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sec\left(dx+c\right)+a\right)^4}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{904a^4\sqrt{\cos(c+}}{}$$

```
[Out] 2/11*a^4*sin(d*x+c)/d/sec(d*x+c)^(9/2)+8/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2
)+150/77*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+128/45*a^4*sin(d*x+c)/d/sec(d*x+
c)^(3/2)+904/231*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+128/15*a^4*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{904a^4\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec[c +
 d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904*a^4*
Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{11}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$= a^4 \int \frac{1}{\sec^{\frac{11}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(4a\right.$$

$$= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^4}{3d\sqrt{}}$$

$$= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{24a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3d}$$

$$= \frac{128a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{74a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{21d}$$

$$= \frac{128a^4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{904a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{231d}$$

Mathematica [C] time = 3.38, size = 296, normalized size = 1.39

$$a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{137055\sin(2(c+dx))+48664\sin(3(c+dx))+14760\sin(4(c+dx))+3080\sin(5(c+dx))+315\sin(6(c\right.}{384d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*((I*Sqrt[2]*Cos[c + d*x]^4*(12
32*Sqrt[1 + E^((2*I)*(c + d*x))] + 1232*(-1 + E^((2*I)*c))*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 565*E^(I*(c + d*x))*(-1 + E^((2*I
)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*(-1 + E^(
(2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]) + (-213752*Cos[d*x]*Csc[c] - 259336*Cos[2*c + d*x]*Csc[c] + 1
37055*Sin[2*(c + d*x)] + 48664*Sin[3*(c + d*x)] + 14760*Sin[4*(c + d*x)] +
3080*Sin[5*(c + d*x)] + 315*Sin[6*(c + d*x)])/(384*d*Sec[c + d*x]^(7/2))))/
2310
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4 + 4\,a^4\sec(dx+c)^3 + 6\,a^4\sec(dx+c)^2 + 4\,a^4\sec(dx+c) + a^4}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

maple [A] time = 3.74, size = 273, normalized size = 1.28

$$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^4\left(5040\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5320\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^9\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*c
os(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+3
26*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+
1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*
d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^4}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{3ad}$$

```
[Out] 5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(
d*x+c))-3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3818, 3787, 3768, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{5\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{d(a+a\sec(c+dx))}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx}{d(a+a\sec(c+dx))}$$

$$= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}}{3ad}$$

Mathematica [C] time = 3.21, size = 291, normalized size = 1.77

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\sqrt{\sec(c+dx)}\left(18\csc(c)\cos(dx)+\sec(c+dx)\left(\tan\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d
*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))/(3*a*d
*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

maple [B] time = 6.28, size = 413, normalized size = 2.52

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticF}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*x
+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-3\sqrt{\cos(c+$$

[Out] -sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-3\sqrt{\cos(c+$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]

[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{3}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos\right.}}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(}{d(a}$$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{ad}$$

Mathematica [C] time = 1.78, size = 262, normalized size = 1.93

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{\sqrt{\sec(c+dx)}\left(6\csc(c)\cos(dx)-2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F\right.}{}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(
c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c
 + d*x)/2]))/d))/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

maple [A] time = 4.25, size = 253, normalized size = 1.86

$$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(sec(c + d*x) + 1), x)/a
```

3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\ldots\right.}{ad}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3818, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\ldots\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\left(\sqrt{\cos(c\right.}{}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+\right.}{ad}$$

Mathematica [C] time = 0.59, size = 201, normalized size = 1.83

$$-\frac{2ie^{-i(c+dx)}\left(-\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2}\right.}{ad\left(1+e^{i(c+dx)}\right)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x)))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]
^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.46, size = 200, normalized size = 1.82

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\mathrm{EllipticF}\left(\mathrm{co}\right.\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{ad}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3820, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{2a^2}$$
$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}$$
$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos(\right.}{}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{ad}$$

Mathematica [C] time = 0.56, size = 202, normalized size = 1.84

$$-\frac{2ie^{-i(c+dx)}\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\right.\right.}{ad\left(1+e^{i(c+dx)}\right)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x
]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.30, size = 198, normalized size = 1.80

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\mathrm{EllipticF}\left(\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{ad}$$

```
[Out] -sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3819, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{2a}$$

$$= \frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{ad}$$

Mathematica [C] time = 1.57, size = 317, normalized size = 2.83

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+2\cos\left(\frac{1}{2}(c+3dx)\right)+\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+d}}{2d}\right.}{a(\sec(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2
] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d
*x]])/(2*d))*Sec[c + d*x])/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)^2 + a\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^2 + a*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 3.72, size = 199, normalized size = 1.78

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\text{EllipticF}\left(\cos\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x)/a
```

3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=140

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+}}{}$$

[Out] 5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]

[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+d}}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{5\int \sqrt{\sec(c}}{6a}$$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} -$$

$$= -\frac{3\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 4.44, size = 318, normalized size = 2.27

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\sqrt{\sec(c+dx)}\,\left(\sin(2c)\cos(2dx)-6\cos(c)\sin(dx)+\cos(2c)\sin(2dx)+3(\cos(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(
c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*
x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*C
sc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*
Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Sec[c + d
*x]))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)^3+a\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 3.61, size = 215, normalized size = 1.54

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(5\,\mathrm{EllipticF}\left(c\right.\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
[Out] -1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(1/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x)/a
```

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=168

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3ad}$$

```
[Out] 7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(
d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{7\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= \frac{21\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3a}$$

Mathematica [C] time = 2.58, size = 347, normalized size = 2.07

$$\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\sqrt{\sec(c+dx)}\left(18(11\cos(2c)+17)\csc(c)\cos(dx)+4\left(10\sin(2c)\cos(2dx)-3\sin\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c +
d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d
*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2
]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*
x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)^4 + a\sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec (dx + c) + a) \sec (dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 3.78, size = 229, normalized size = 1.36

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(25\,\mathrm{Ellipt}\right.\right.}{15 a \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
[Out] -1/15/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d
*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)
^4+23*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec (dx + c) + a) \sec (dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x)/a
```

3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(}}{3\alpha}$$

```
[Out] 10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/
d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*sin
(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(}}{3\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{21a^2}{2}-15a\right.}{3a^4}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{7\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a^2} + \frac{5\int}{}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx}$$

$$= \frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3a^2d}$$

Mathematica [C] time = 3.84, size = 287, normalized size = 1.42

$$-\frac{\left(-1+e^{ic}\right)\csc\left(\frac{c}{2}\right)e^{-\frac{1}{2}i(4c+3dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(7e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(1+e^{i(c+dx)}\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) -
 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) -
```

```
53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*
x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3
/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5
/2))/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Sec[c +
d*x])^2)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(9/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 6.52, size = 413, normalized size = 2.04

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\text{co}}}{}$$

[Out] -5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d

Rubi [A] time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{5a^2}{2}-6a^2\right.}{3a^4}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\sec(c+dx)}\,dx}{6a^2} + \frac{2}{}$$

$$= \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{5\sec}{3a}$$

$$= -\frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3a^2d}$$

Mathematica [C] time = 1.41, size = 252, normalized size = 1.43

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-4ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-4*I)*(1 + E^(I*(c + d*x)))^3*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*C
```

```
os[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c +
d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)*(1 +
Sec[c + d*x])^2)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{7}{2}}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 4.00, size = 405, normalized size = 2.30

$$-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{5}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} + \frac{\int \frac{}{\sqrt{\mathrm{s}}}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}\right.}{3a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3a^2d}$$

Mathematica [C] time = 1.27, size = 242, normalized size = 1.62

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d
*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin
[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 3.55, size = 257, normalized size = 1.72

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{6a^2\cos\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] 1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3815, 21, 3771, 2641}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{6a^2}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}$$
$$= \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [A] time = 0.43, size = 98, normalized size = 1.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+4\sqrt{\cos(c+dx)}\cos^3\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\right.\right.}{3a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3
*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [B] time = 3.40, size = 188, normalized size = 2.44

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{a^2d}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c)
^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} - \frac{\int \frac{1}{\ldots}}{\ldots}$$

$$= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\ldots}{3a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\ldots}{3a^2d}$$

Mathematica [C] time = 1.50, size = 239, normalized size = 1.60

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(
-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c
+ 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 3.98, size = 257, normalized size = 1.72

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ell}}{6a^2\sqrt{-2\left(\mathrm{sin}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/
2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x
+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] -5/3*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-6a^2+\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\mathrm{se}\cdots}}{\cdots}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(5\sqrt{\cos\cdots}\right.}{\cdots}$$

$$= \frac{4\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.95, size = 260, normalized size = 1.71

$$-\frac{i\sqrt{2}\,e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(4e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-16e^{i(c+dx)}-23e^{2i(c+dx)}\cdots\right.}{3a^2d\left(1+e^{i(c+d\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-1/3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16*
E^(I*(c + d*x)) - 23*E^((2*I)*(c + d*x)) - 25*E^((3*I)*(c + d*x)) - 20*E^((
4*I)*(c + d*x)) - 9*E^((5*I)*(c + d*x)) - (5*I)*E^(I*(c + d*x))*(1 + E^(I*(
c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 4*E^((2*I)*(c +
 d*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(a^2*d*E^(I*(c + d*x))*(1 + E^(I
*(c + d*x)))^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^3+2\,a^2\sec\left(dx+c\right)^2+a^2\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^
2*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

maple [A] time = 4.02, size = 257, normalized size = 1.69

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ell}\right.}{6a^2\sqrt{-2\left(\mathrm{si}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2
*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d
*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x)
)), x)/a**2
```

3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=178

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(}}{}$$

```
[Out] 10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))/
sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.22, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}dx = -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c$$

$$= \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{}{3d\sqrt{\sec(}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [C] time = 1.99, size = 257, normalized size = 1.44

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(7ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+dx)}\right)^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[(
c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 8
0*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*
```

```
(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2]
 + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]
))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^4+2\,a^2\sec\left(dx+c\right)^3+a^2\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^
2*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 3.99, size = 270, normalized size = 1.52

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/
2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2
*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*
d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3/
2)), x)/a**2
```

3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{a^2d}$$

```
[Out] 56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)
/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*sin(
d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.24, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec($$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec($$

$$= \frac{56\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [C] time = 1.98, size = 271, normalized size = 1.36

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-112ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos[
(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2]
 - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (
(112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Sin[(c
+ d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7
*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Sec[c +
d*x])^2)
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{5}+2\,a^{2}\sec\left(dx+c\right)^{4}+a^{2}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^5 + 2*a^2*sec(d*x + c)^4 + a^
2*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 4.29, size = 283, normalized size = 1.42

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1
/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+2\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(7/2) + sec(c + d*x)**(5/
2)), x)/a**2
```

3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$-\frac{119\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{2}$$

```
[Out] 11/2*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a
+a*sec(d*x+c))^3-2/3*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-119
/30*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)*se
c(d*x+c)^(1/2)/a^3/d+119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.36, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{119\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) - (119*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + (11*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5
*d*(a + a*Sec[c + d*x])^3) - (2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x])^2) - (119*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a
^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7a}{2}-\frac{13}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(25a^2-\frac{69}{2}a^2\sec\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c}{30d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c}{30d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(}{5d(a+a\sec(c+}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(}{5d(a+a\sec(c+}$$

$$= \frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{2a^3d}$$

Mathematica [C] time = 5.34, size = 378, normalized size = 1.53

$$\csc\left(\frac{c}{2}\right)e^{-idx}\left(\frac{(-1+e^{ic})e^{-\frac{3}{2}i(2c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(944e^{i(c+dx)}+2476e^{2i(c+dx)}+4148e^{3i(c+dx)}+5134e^{4i(c+dx)}+4664e^{5i(c+dx)}+3340e^{6i(c+dx)}+\right.}{16\left(1+e^{2i(c+dx)}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*(-119*Sqrt[2]*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2
]^6*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c +
 d*x]^3 + ((-1 + E^(I*c))*Cos[(c + d*x)/2]*(165 + 944*E^(I*(c + d*x)) + 247
6*E^((2*I)*(c + d*x)) + 4148*E^((3*I)*(c + d*x)) + 5134*E^((4*I)*(c + d*x))
 + 4664*E^((5*I)*(c + d*x)) + 3340*E^((6*I)*(c + d*x)) + 1620*E^((7*I)*(c +
 d*x)) + 357*E^((8*I)*(c + d*x)) - (165*I)*(1 + E^(I*(c + d*x)))^5*(1 + E^(
(2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sec[c + d*x
]^(7/2))/(16*E^(((3*I)/2)*(2*c + d*x))*(1 + E^((2*I)*(c + d*x))))))/(15*a^3
*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{11}{2}}}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(11/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3
*a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^{\frac{11}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 6.16, size = 453, normalized size = 1.83

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{32\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{118\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(32/15*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+11
8/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
```

```
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*
c)^2)^(1/2)+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)^5-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\sqrt{}}{}$$

```
[Out] -1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+
a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.34, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(12a^2-\frac{41}{2}a^2\sec\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+}{6d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+}{6d\left(a^3+a^3\sec(c+d\right.}$$

$$= \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+}{15ad(a+a\sec(c+dx}$$

$$= -\frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} -$$

$$= -\frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{6a^3d}$$

Mathematica [C] time = 2.12, size = 371, normalized size = 1.68

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c
 + d*x])^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{9}{2}}}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(9/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [B] time = 4.24, size = 555, normalized size = 2.51

$$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(65\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(3a^2-\frac{21}{2}a^2\sec\right.}{a+a\sec(c+dx)}}{15a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+}{10d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+}{10d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+}{10d\left(a^3+a^3\sec(c+d\right.}$$

$$= \frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{2a^3d}$$

Mathematica [C] time = 4.84, size = 274, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-3ie^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(34 +
69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c +
d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{7}{2}}}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 3.89, size = 268, normalized size = 1.37

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{Elli}}{20a^3\ \text{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-2a^2-\frac{9}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx}}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+dx\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+dx\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{6d\left(a^3+a^3\sec(c+dx\right.}$$

$$= \frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{6a^3d}$$

Mathematica [C] time = 1.82, size = 371, normalized size = 1.90

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+7\cos\left(\frac{1}{2}(c+3a\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^
((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c + 3*d
*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(7*c + 5
*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqr
t[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 4.12, size = 270, normalized size = 1.38

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{Elli}}{60a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{10a^3d}$$

```
[Out] 1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
```

```
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}+3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,d}{15a^4}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx}{6d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx}{6d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx}{6d\left(a^3+a^3\sec(c+dx)\right)}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{6a^3d}$$

Mathematica [C] time = 1.97, size = 371, normalized size = 1.90

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+17\cos\left(\frac{1}{2}(c+3d\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[(c + 3
*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos[(7*c +
 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*S
qrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 4.04, size = 270, normalized size = 1.38

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)*sec(d*
x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^
3*sec(d*x+c))-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.33, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

```
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{9a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{3a^2-\frac{9}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx}}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{2d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{2d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\,\sin(c+}{2d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{9\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

Mathematica [C] time = 5.62, size = 272, normalized size = 1.39

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(3e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\left(1+e^{i(c+}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*
(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1
 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] +
 (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*
Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 4.67, size = 270, normalized size = 1.38

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{E}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1
/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2
*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{10a^3d}$$

```
[Out] -1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13
/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

```
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{41}{2}}{\sqrt{\sec(}}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec}}{6d\left(a^3\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec}}{6d\left(a^3\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec}}{6d\left(a^3\right.}$$

$$= \frac{49\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\,F}{}$$

Mathematica [C] time = 2.07, size = 386, normalized size = 1.98

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1134\cos\left(\frac{1}{2}(c-dx)\right)+1071\cos\left(\frac{1}{2}(3c+dx)\right)+923\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*
Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] +
265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/
```

```
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*
x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^4 + 3a^3\sec(dx+c)^3 + 3a^3\sec(dx+c)^2 + a^3\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*
a^3*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

maple [A] time = 4.28, size = 270, normalized size = 1.38

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1
/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos
(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(
3/2) + sqrt(sec(c + d*x))), x)/a**3
```

3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119}{}$$

```
[Out] 11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3/
sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-119
/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.35, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec($$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec($$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec($$

$$= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{}{3ad\sqrt{\mathrm{se}}}$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+d}}$$

$$= -\frac{119\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}}{}$$

Mathematica [C] time = 2.47, size = 285, normalized size = 1.29

$$e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(119ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos
[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/
2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2]
+ 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x)
)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^5+3\,a^3\sec\left(dx+c\right)^4+3\,a^3\sec\left(dx+c\right)^3+a^3\sec\left(dx+c\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + 3*
a^3*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 4.53, size = 283, normalized size = 1.28

$$-\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(
1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*
cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(9/2) + 3*sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(
5/2) + sec(c + d*x)**(3/2)), x)/a**3
```

3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$-\frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2a^3d}$$

```
[Out] 77/10*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-1/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(
a+a*sec(d*x+c))^3-4/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-63
/10*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)/a^3/
d/sec(d*x+c)^(1/2)+231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/
d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.37, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (231*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (21*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) + (77*Sin[c + d*x])/(10*a^3*d*Sec[c + d*x]^(3/2)) - (21*Sin[c +
 d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*Sec[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^3) - (4*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^2) - (63*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*S
ec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\se$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\se$$

$$= \frac{231\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{21\sqrt{\cos(c+dx)}\,F$$

Mathematica [C] time = 2.86, size = 297, normalized size = 1.20

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(77ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -1/40*(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-3465*
I)*Cos[(c + d*x)/2] - (2541*I)*Cos[(3*(c + d*x))/2] - (1155*I)*Cos[(5*(c +
d*x))/2] - (231*I)*Cos[(7*(c + d*x))/2] + 3360*Cos[(c + d*x)/2]^5*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((77*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(((3*I)/2)*(c + d*x)) + 125*Sin[(c + d*x)/2] + 359*Sin[(3*(c + d*x)
)/2] + 350*Sin[(5*(c + d*x))/2] + 138*Sin[(7*(c + d*x))/2] + 5*Sin[(9*(c +
d*x))/2] - Sin[(11*(c + d*x))/2]))/(a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^3\sec(dx+c)^6+3\,a^3\sec(dx+c)^5+3\,a^3\sec(dx+c)^4+a^3\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^6 + 3*a^3*sec(d*x + c)^5 + 3*
a^3*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 4.24, size = 296, normalized size = 1.20

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1
/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos
(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^{\frac{11}{2}}(c+dx)+3\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(11/2) + 3*sec(c + d*x)**(9/2) + 3*sec(c + d*x)**
(7/2) + sec(c + d*x)**(5/2)), x)/a**3
```

3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{3a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

```
[Out] 3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sec(
d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*sec(d*x+c)^(5/2)*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3803, 3801, 215}

$$\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{3a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (3*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{8}\int \sqrt{\sec$$

$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{3\,\mathrm{Subst}\Big(}{}$$

$$= \frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\sec^{\frac{5}{2}}(c}{2d\sqrt{a}}$$

Mathematica [A] time = 0.56, size = 100, normalized size = 0.86

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\left(\frac{1}{8}\cos(c+dx)(3\cos(c+dx)+2)+\frac{3\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.82, size = 356, normalized size = 3.07

$$\left[\frac{3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*c
os(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos
(d*x + c)), 1/8*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 1.77, size = 219, normalized size = 1.89

$$\frac{(-1+\cos(dx+c))\left(3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\mathrm{co}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-3*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-6*(-2/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2
))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(
d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
```

maxima [B] time = 1.06, size = 1264, normalized size = 10.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
```

```
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sec(d*x+c)^(
3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3803, 3801, 215}

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx&=\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx\\&=\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\&=\frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.25, size = 75, normalized size = 1.04

$$\frac{a\tan(c+dx)\left(\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x
])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [B] time = 0.64, size = 302, normalized size = 4.19

$$\left[\frac{\sqrt{a}\left(\cos(dx+c)+1\right)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,s}{\sqrt{\cos(dx+c}}}{4\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(sqrt(-a)*(cos(d*x + c) + 1)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 1.63, size = 188, normalized size = 2.61

$$-\frac{\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1-\sin(dx+c)\right)\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1+\sin(dx+c)\right)\sqrt{2}}{4}\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1
/2))*2^(1/2)*cos(d*x+c)-arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
```

```
/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-
2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 0.70, size = 662, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2), x)
```

3.220 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3801, 215}

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 1.46

$$-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])
/(d*Sqrt[1 - Sec[c + d*x]])
```

fricas [B] time = 0.64, size = 189, normalized size = 5.11

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(\ldots}}{a\cos(dx+c)^2-a\cos(d\ldots}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.42, size = 150, normalized size = 4.05

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{\frac{1}{\cos(dx+c)}}\cos(dx+c)\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(d\ldots}{\ldots}\right.\right.}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*(
arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2)))*(-2/(
1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)
```

maxima [B] time = 0.65, size = 241, normalized size = 6.51

$$\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
```

```
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a + \frac{a}{\cos(c + d\,x)}}\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec(c + dx) + 1\right)}\,\sqrt{\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3804}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.09, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.54, size = 49, normalized size = 1.36

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 1.47, size = 52, normalized size = 1.44

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{d\sin\left(dx+c\right)\sqrt{\frac{1}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
[Out] -2/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(
d*x+c))^(1/2)
```

maxima [A] time = 0.62, size = 20, normalized size = 0.56

$$\frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

mupad [B] time = 0.54, size = 53, normalized size = 1.47

$$\frac{\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
[Out] (sin(2*c + 2*d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2))/(d*(cos(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/3*a*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3805, 3804}

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2}{3}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 49, normalized size = 0.64

$$\frac{2(\cos(c+dx)+2)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*Sqr
t[Sec[c + d*x]])
```

fricas [A] time = 0.55, size = 66, normalized size = 0.86

$$\frac{2\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(d*x + c)^2 + 2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.60, size = 68, normalized size = 0.88

$$-\frac{2\left(\cos^{2}\left(dx+c\right)+\cos\left(dx+c\right)-2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{2}\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/c
os(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)
```

maxima [A] time = 0.58, size = 113, normalized size = 1.47

$$\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

mupad [B] time = 1.30, size = 69, normalized size = 0.90

$$\frac{\cos\left(c+d\,x\right)\,\left(4\,\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{3\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)*(4*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)*((
a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+c
)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)^(
1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3805, 3804}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{4}{5}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{8}{15}\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a}{1}\end{aligned}$$

Mathematica [A] time = 0.19, size = 61, normalized size = 0.53

$$\frac{(8\cos(c+dx)+3\cos(2(c+dx))+19)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.59, size = 78, normalized size = 0.68

$$\frac{2 \left(3 \cos \left(dx+c\right)^{3}+4 \cos \left(dx+c\right)^{2}+8 \cos \left(dx+c\right)\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin \left(dx+c\right)}{15 \left(d \cos \left(dx+c\right)+d\right) \sqrt{\cos \left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 8*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x +
c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec \left(dx+c\right)+a}}{\sec \left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 1.54, size = 80, normalized size = 0.70

$$-\frac{2 \left(3 \left(\cos^{3}\left(dx+c\right)\right)+\cos^{2}\left(dx+c\right)+4 \cos \left(dx+c\right)-8\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\, \left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}} \left(\cos^{3}\left(dx+c\right)\right)}{15 d \sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c)
```

maxima [B] time = 0.67, size = 203, normalized size = 1.77

$$\sqrt{2} \left(30 \cos \left(\frac{4}{5} \arctan \left(\sin \left(\frac{5}{2} dx+\frac{5}{2} c\right), \cos \left(\frac{5}{2} dx+\frac{5}{2} c\right)\right)\right) \sin \left(\frac{5}{2} dx+\frac{5}{2} c\right)+5 \cos \left(\frac{2}{5} \arctan \left(\sin \left(\frac{5}{2} dx+\frac{5}{2} c\right),\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

mupad [B] time = 1.67, size = 82, normalized size = 0.71

$$\frac{\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(c+d\,x)+1\right)}{\cos(c+d\,x)}}\,\left(35\,\sin\left(c+d\,x\right)+8\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)\right)}{30\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(35*sin(c + d*x) + 8*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(30*d*(c
os(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(5/2), x)
```

3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{}{35d\sqrt{\mathrm{s}}}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+12/35*a*sin(d*x+
c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/sec(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*s
ec(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3805, 3804}

$$\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{}{35d\sqrt{\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (32*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{6}{7}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{24}{35}\int\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{35d\sqrt{\mathrm{s}}}\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{35d\sqrt{\mathrm{s}}}\end{aligned}$$

Mathematica [A] time = 0.27, size = 71, normalized size = 0.46

$$\frac{(47\cos(c+dx)+12\cos(2(c+dx))+5\cos(3(c+dx))+76)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{70d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((76 + 47*Cos[c + d*x] + 12*Cos[2*(c + d*x)] + 5*Cos[3*(c + d*x)])*Sqrt[a*(
1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(70*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.80, size = 88, normalized size = 0.58

$$\frac{2\left(5\cos(dx+c)^4+6\cos(dx+c)^3+8\cos(dx+c)^2+16\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{35\left(d\cos(dx+c)+d\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 16*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c)
+ d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 1.55, size = 90, normalized size = 0.59

$$-\frac{2\left(5\left(\cos^4(dx+c)\right)+\cos^3(dx+c)+2\left(\cos^2(dx+c)\right)+8\cos(dx+c)-16\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\cos^4(dx+c)\right)}{35d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

maxima [B] time = 0.76, size = 293, normalized size = 1.92

$$\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
```

```
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)
*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*sqrt(a)/d
```

mupad [B] time = 2.20, size = 93, normalized size = 0.61

$$\frac{\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(140\,\sin\left(c+d\,x\right)+42\,\sin\left(2\,c+2\,d\,x\right)+12\,\sin\left(3\,c+3\,d\,x\right)+5\,\sin\left(4\,c\right.\right.}{140\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(140*sin(c + d*x) + 42*sin(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin
(4*c + 4*d*x)))/(140*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=160

$$\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(7/2)*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3814, 21, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (11*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (11*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}a\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(c+d\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(11a)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

$$= \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(11a)$$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec}{3d}$$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec}{3d}$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2 s}{12}$$

Mathematica [A] time = 0.59, size = 112, normalized size = 0.70

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[
2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] +
 11*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

fricas [A] time = 1.67, size = 400, normalized size = 2.50

$$\left[\frac{33\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{96\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(33*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
```

```
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x +
 c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(33*(a*cos(d*x + c)^3
+ a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) + 2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3 + d*cos(d*x + c)^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.52, size = 244, normalized size = 1.52

$$(-1+\cos(dx+c))\left(33\sqrt{2}\,\left(\cos^3(dx+c)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)-33\sqrt{2}\,\left(\cos^3(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/48/d*(-1+cos(d*x+c))*(33*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))-33*2^(1/2)*cos(d*x+c)^3*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-66*cos(d*
x+c)^2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-44*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*sin(d*x+c)-16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+
c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*
x+c)^2*a
```

maxima [B] time = 0.91, size = 2361, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
```

```
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos \left(c+d x\right)}\right)^{3/2} \left(\frac{1}{\cos \left(c+d x\right)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*se
c(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(5/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3814, 21, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (7*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
```

```
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c+\right.}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}(7a)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(7a)$$

$$= \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(7a)\,\mathrm{S}}{}$$

$$= \frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\,\mathrm{se}}{2d}$$

Mathematica [A] time = 0.45, size = 99, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2
]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7
*Sin[(3*(c + d*x))/2]))/(8*d)
```

fricas [A] time = 0.66, size = 370, normalized size = 3.08

$$\left[\frac{7\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos
(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
```

```
 + d*cos(d*x + c)), 1/8*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(7*a*cos(d*x + c
) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 1.53, size = 214, normalized size = 1.78

$$-\frac{(-1+\cos(dx+c))\left(7\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-7\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+14*(-2/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2*a
```

maxima [B] time = 0.88, size = 2244, normalized size = 18.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqr
t(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
```

```
*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*
log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((2*(2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arctan2(
```

```
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.227 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3814, 21, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + a\int \frac{\sqrt{\sec(c+dx)}\left(\frac{3a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(3a)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+a}$$

$$= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}dx, x, -\frac{a\tan(c+dx}{\sqrt{a+a\sec(c+}}\right.}{d}$$

$$= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.29, size = 75, normalized size = 1.00

$$\frac{a^2\tan(c+dx)\left(\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}-3\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-3*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*
x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [B] time = 0.65, size = 310, normalized size = 4.13

$$\left[\frac{3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{\sqrt{\cos(dx+}}}{4\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(3*(a*cos(d*x + c) + a)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*a*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.49, size = 182, normalized size = 2.43

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c}{2d\sin(dx+c)^{2}\sqrt{-\frac{}{1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+
c))*(3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/
2))*2^(1/2)*cos(d*x+c)-3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1
+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)*a
```

maxima [B] time = 0.68, size = 1143, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
```

```
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos \left(c+d\,x\right)}\right)^{3/2}\,\sqrt{\frac{1}{\cos \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec \left(c + dx\right) + 1\right)\right)^{\frac{3}{2}} \sqrt{\sec \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x)), x)
```

3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*sin(d*
x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3813, 21, 3801, 215}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (2a)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + a\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.38, size = 86, normalized size = 1.13

$$\frac{2a^2\left(\sin(c+dx)\sqrt{-((\sec(c+dx)-1)\sec(c+dx))} + \tan(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*(Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + ArcSin[Sqr
t[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + S
ec[c + d*x])])
```

fricas [B] time = 0.62, size = 307, normalized size = 4.04

$$\left[\frac{4a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\sin(dx+c) + (a\cos(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 - \frac{4\left(\cos(dx+c)^2 - 2\cos(d\right.}{}}{\cos(dx+c)^3+\cos(dx}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) +
d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.51, size = 174, normalized size = 2.29

$$-\frac{\left(\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin(dx+c)-\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(}{}\right.\right.}{2d\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+4*cos(d*x+c)-4)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*
x+c)/(1/cos(d*x+c))^(1/2)*a
```

maxima [B] time = 0.87, size = 274, normalized size = 3.61

$$\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 8/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+
c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3809, 3804}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}(4a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.22, size = 50, normalized size = 0.63

$$\frac{2a(\cos(c+dx)+5)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*S
qrt[Sec[c + d*x]])
```

fricas [A] time = 0.63, size = 69, normalized size = 0.87

$$\frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.56, size = 71, normalized size = 0.90

$$-\frac{2\left(\cos^{2}\left(dx+c\right)+4\cos\left(dx+c\right)-5\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+4*cos(d*x+c)-5)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*co
s(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a
```

maxima [A] time = 0.57, size = 38, normalized size = 0.48

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

mupad [B] time = 1.41, size = 70, normalized size = 0.89

$$\frac{a\,\cos\left(c+d\,x\right)\,\left(10\,\sin\left(c+d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{3\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (a*cos(c + d*x)*(10*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)
*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/5*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+a*sec(d*x+c)
)^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3812, 3809, 3804}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*(a
+ a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{3}{5}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(4a)$$
$$= \frac{8a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec}{5}$$

Mathematica [A] time = 0.29, size = 60, normalized size = 0.52

$$\frac{a(6\cos(c+dx)+\cos(2(c+dx))+13)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(5*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.56, size = 80, normalized size = 0.69

$$\frac{2\left(a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 + 6*a*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
 + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 1.67, size = 81, normalized size = 0.70

$$-\frac{2\left(\cos^{3}\left(dx+c\right)+2\left(\cos^{2}\left(dx+c\right)\right)+3\cos\left(dx+c\right)-6\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^{3}\left(dx+c\right)\right)a}{5d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5/d*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*(a*(1+cos(d*x+c))/cos(d
*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c)*a
```

maxima [B] time = 0.87, size = 210, normalized size = 1.81

$$\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

mupad [B] time = 1.77, size = 81, normalized size = 0.70

$$\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(25\,\sin\left(c+d\,x\right)+6\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{10\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*(25*sin(c + d*x) + 6*sin(2*c + 2*d*x
) + sin(3*c + 3*d*x))*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(10*d*(c
os(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(5/2), x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{}{1}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+26/35*a^2*sin(
d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d/s
ec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3813, 21, 3805, 3804}

$$\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(2a)\int \frac{\frac{13a}{2}+\frac{13}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}(13a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}(5$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{105d}$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{105d}$$

Mathematica [A] time = 0.37, size = 72, normalized size = 0.45

$$\frac{a(253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx))+494)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqr
t[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 1.48, size = 92, normalized size = 0.57

$$\frac{2\left(15\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3+52\,a\cos\left(dx+c\right)^2+104\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 10
4*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*
cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 1.70, size = 93, normalized size = 0.58

$$-\frac{2\left(15\left(\cos^4\left(dx+c\right)\right)+24\left(\cos^3\left(dx+c\right)\right)+13\left(\cos^2\left(dx+c\right)\right)+52\cos\left(dx+c\right)-104\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^4\left(d\right.\right.}{105d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-104
)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin
(d*x+c)*a
```

maxima [B] time = 0.61, size = 303, normalized size = 1.88

$$\frac{\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+175\,a\cos\left(\frac{4}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

mupad [B] time = 2.33, size = 94, normalized size = 0.58

$$\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(910\,\sin\left(c+d\,x\right)+238\,\sin\left(2\,c+2\,d\,x\right)+78\,\sin\left(3\,c+3\,d\,x\right)+15\right.}{420\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(910*sin(c + d*x) + 238*sin(2*c + 2*d*x) + 78*sin(3*c + 3*d*x) + 15
*sin(4*c + 4*d*x)))/(420*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)} \, dx$

Optimal. Leaf size=201

$$\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+34/63*a^2*sin(
d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+68/105*a^2*sin(d*x+c)/d/se
c(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+272/315*a^2*sin(d*x+c)/d/sec(d*x+c)^(
1/2)/(a+a*sec(d*x+c))^(1/2)+544/315*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*
sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3813, 21, 3805, 3804}

$$\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (3
4*a^2*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6
8*a^2*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
272*a^2*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
 (544*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
```

```
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{9}(2a)\int\frac{\frac{17a}{2}+\frac{17}{2}a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{9}(17a)\int\frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx\\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{21}\\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{10}\\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{10}\\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{}{10}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 80, normalized size = 0.40

$$\frac{2a^2\sin(c+dx)\left(272\sec^4(c+dx)+136\sec^3(c+dx)+102\sec^2(c+dx)+85\sec(c+dx)+35\right)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(35 + 85*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 136*Sec[c + d*x]^3 + 27
2*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c
 + d*x])])
```

fricas [A] time = 0.46, size = 103, normalized size = 0.51

$$\frac{2\left(35\,a\cos\left(dx+c\right)^5+85\,a\cos\left(dx+c\right)^4+102\,a\cos\left(dx+c\right)^3+136\,a\cos\left(dx+c\right)^2+272\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos}{c}}}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 + 85*a*cos(d*x + c)^4 + 102*a*cos(d*x + c)^3 + 1
36*a*cos(d*x + c)^2 + 272*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2), x)

maple [A] time = 1.69, size = 103, normalized size = 0.51

$$-\frac{2\left(35\left(\cos^{5}(dx+c)\right)+50\left(\cos^{4}(dx+c)\right)+17\left(\cos^{3}(dx+c)\right)+34\left(\cos^{2}(dx+c)\right)+136\cos(dx+c)-272\right)\sqrt{}}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x)

[Out] -2/315/d*(35*cos(d*x+c)^5+50*cos(d*x+c)^4+17*cos(d*x+c)^3+34*cos(d*x+c)^2+136*cos(d*x+c)-272)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a

maxima [B] time = 1.10, size = 396, normalized size = 1.97

$$\frac{\sqrt{2}\left(3780\,a\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+1050\,a\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}dx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/5040*sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(a)/d

mupad [B] time = 3.02, size = 105, normalized size = 0.52

$$\frac{a\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(4830\sin(c+d\,x)+1428\sin(2\,c+2\,d\,x)+513\sin(3\,c+3\,d\,x)+1\right.}{2520\,d\,\left(\cos(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)

[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(4830*sin(c + d*x) + 1428*sin(2*c + 2*d*x) + 513*sin(3*c + 3*d*x) + 170*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)

[Out] Timed out
```

3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=200

$$\frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx}{64d\sqrt{a\sec(c}$$

```
[Out] 163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*
a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*sec(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^(7/2)*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*
sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3814, 4016, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{17a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*
d) + (163*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (163*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (17*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d}+\frac{1}{4}a\int \sec^{\frac{5}{2}}(c+dx$$

$$= \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}{4d}$$

$$= \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+\frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+$$

Mathematica [C] time = 8.50, size = 582, normalized size = 2.91

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(7824i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(\sqrt{2}-1\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)+7824i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx\right.}{\left(\sqrt{2}-1\right)\cos}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/6144*(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((7824*I)*ArcTan[(
Cos[(c + d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c +
 d*x)/4] - Sin[(c + d*x)/4])] + (7824*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sq
rt[2])*Sin[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4
])] + Sec[c + d*x]^4*(-2934*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] + 1467*Log[2
- Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - 1956*Cos[2*(c + d*
x)]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2]
 - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*S
in[(c + d*x)/2]]) - 489*Cos[4*(c + d*x)]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2
]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 +
 Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 1467*Log[2 + Sqrt[
2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] + 2060*Sqrt[2]*Sin[(c + d*x
)/2] - 6204*Sqrt[2]*Sin[(3*(c + d*x))/2] - 652*Sqrt[2]*Sin[(5*(c + d*x))/2]
 - 1956*Sqrt[2]*Sin[(7*(c + d*x))/2])))/(Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.65, size = 446, normalized size = 2.23

$$\left[\frac{489\left(a^{2}\cos\left(dx+c\right)^{4}+a^{2}\cos\left(dx+c\right)^{3}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{768\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 326*a
^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3), 1/384*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(489*a^2*cos(
d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.76, size = 286, normalized size = 1.43

$$\frac{\left(489\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^{4}\left(dx+c\right)\right)-489\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1-\sin\ldots}{4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/768/d*(489*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c))*2^(1/2))*cos(d*x+c)^4-489*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^4+978*(-2/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^3*sin(d*x+c)+652*cos(d*x+c)^2*sin(d*x+c)*(-2/(1+cos(d*x+
c)))^(1/2)+368*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x
+c))^(5/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-
1)*a^2
```

maxima [B] time = 1.58, size = 3860, normalized size = 19.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x
 + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 4
8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c
) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x +
 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos
(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a
^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^
2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x +
2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*
x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*
d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2
+ 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x
 + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x
+ 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*
x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*si
n(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
```

```
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*
cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2
+ a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4
*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^
2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*
d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(
4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*c
os(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2
*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*si
n(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*
(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*
a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d
*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c)
 + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2
)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c
) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2
)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c)
 + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(
6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sq
rt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos (c+d x)}\right)^{5/2} \left(\frac{1}{\cos (c+d x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{13a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}}{}$$

```
[Out] 25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)^(
5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3814, 4016, 3803, 3801, 215}

$$\frac{13a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (25*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (13*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}a\int \sec^{\frac{3}{2}}(c+dx$$

$$= \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}{3d}$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2}{}$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2}{}$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13}{}$$

Mathematica [C] time = 8.06, size = 458, normalized size = 2.86

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-600i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(\sqrt{2}-1\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)-600i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-}{\left(\sqrt{2}-1\right)\cos\left(\frac{1}{4}\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((-600*I)*ArcTan[(Cos[(c +
 d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])] - (600*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Si
n[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])] + Sec
[c + d*x]^3*(225*Cos[c + d*x]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2
- Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Co
s[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 75*Cos[3*(c + d*x)]*(2*Log[Sq
rt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Si
n[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/
2]]) + 4*Sqrt[2]*(114*Sin[(c + d*x)/2] - 7*Sin[(3*(c + d*x))/2] + 75*Sin[(5
*(c + d*x))/2]))))/(384*Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 1.60, size = 420, normalized size = 2.62

$$\left[\frac{75\left(a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{96\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(75*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a^2*c
os(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(75*(a^2*cos
(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) + 2*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
+ 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.65, size = 254, normalized size = 1.59

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)\left(75\sqrt{2}\,\left(\cos^{3}\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-75\sqrt{2}\,\left(\cos^{3}\left(dx+c\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(75*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-75*2^(1/2)*cos(d*x+c)^3*arcta
n(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+150*cos(
d*x+c)^2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+68*(-2/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*sin(d*x+c)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c)/sin(d*x+c)^2*a^2
```

maxima [B] time = 0.99, size = 3469, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)
*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4
56*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*
d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^
2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*
x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*
x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7
5*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
```

^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((cos(6*d*x + 6*c)^2 + 6*(cos(6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(6*d*x + 6*c) + 1)*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.235 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=120

$$\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] 19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*s
ec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(3/2
)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3814, 4016, 3801, 215}

$$\frac{9a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
 + (9*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}a\int\sqrt{\sec(c+a}$$

$$= \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}{2d}$$

$$= \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}{2d}$$

$$= \frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

Mathematica [A] time = 0.51, size = 106, normalized size = 0.88

$$\frac{a^3\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+11\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}-19\sin^{-1}\left(\sqrt{\sec(c+d}\right.\right.}{4d\sqrt{1-\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(-19*ArcSin[Sqrt[Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x
]^(3/2) + 11*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*d*
Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.69, size = 386, normalized size = 3.22

$$\left[\frac{19\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c)), 1/8*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c
))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(
11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.44, size = 224, normalized size = 1.87

$$\frac{(-1+\cos(dx+c))\left(19\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-19\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}}{}\right.\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(19*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-19*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-22*(-2/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(-2/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c)/sin(d*x+c)^2*a^2
```

maxima [B] time = 5.13, size = 2826, normalized size = 23.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^
2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 7
6*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
```

```
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)
^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)
- 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*
(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) +
 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) -
 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2
*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d
*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=112

$$\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{d}$$

```
[Out] 5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^3*sin(d*x+
c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+a^2*sin(d*x+c)*sec(d*x+c)^(1/2
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3814, 4015, 3801, 215}

$$\frac{a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqr
t[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + a\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{a}{2}+\frac{5}{2}a\right.}{\sqrt{\sec(c+dx)}}$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} - $$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\,\sqrt{a+}}{}$$

Mathematica [A] time = 0.75, size = 91, normalized size = 0.81

$$\frac{a^3\tan(c+dx)\left((2\cos(c+dx)+1)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+5\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]] + (1 + 2*Cos[c + d*x])*Sqrt[(-1 + Co
s[c + d*x])*Sec[c + d*x]^2])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.39, size = 346, normalized size = 3.09

$$\left[\frac{5\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(2a^2\cos(dx+\right.}{}}{4\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(
5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)) + 2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.59, size = 199, normalized size = 1.78

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(5\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)\sqrt{2}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)*sin(d*x+c)*2^(1/2)-5*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1
/2)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*sin
(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a*s
ec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3813, 4015, 3801, 215}

$$\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
4*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{7a}{2}+\frac{3}{2}a\sec(\right.}{\sqrt{\sec(c+dx)}}$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + a^2\int \sqrt{\mathrm{se}}$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{(2a^2)\,\mathrm{Su}}{}$$

$$= \frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(}}{3d\sqrt{\mathrm{se}}}$$

Mathematica [A] time = 0.45, size = 103, normalized size = 0.87

$$\frac{2a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(8\sec(c+dx)+1)+3\sec^{\frac{3}{2}}(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^3*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[
c + d*x]]*(1 + 8*Sec[c + d*x]))*Sin[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x
])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.25, size = 364, normalized size = 3.08

$$\left[\frac{3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(a^2\cos(d\right.}{}}{6\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c) + d), 1/3*(3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c) + d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.66, size = 195, normalized size = 1.65

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin(dx+c)-3\sqrt{2}\ \arctan\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*2^(1/2)*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2+28*
cos(d*x+c)-32)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a^2
```

maxima [B] time = 0.66, size = 593, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+64/15*a^3*sin(d*
x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*se
c(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3809, 3804}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}(8a)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{15}(3\\ &= \frac{64a^3\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2a(a+a\,\mathrm{se}}{5}\end{aligned}$$

Mathematica [A] time = 0.33, size = 64, normalized size = 0.54

$$\frac{a^2(28\cos(c+dx)+3\cos(2(c+dx))+89)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]
*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.84, size = 87, normalized size = 0.73

$$\frac{2\left(3\,a^{2}\cos\left(dx+c\right)^{3}+14\,a^{2}\cos\left(dx+c\right)^{2}+43\,a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 43*a^2*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*s
qrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 1.68, size = 85, normalized size = 0.71

$$-\frac{2\left(3\left(\cos^{3}\left(dx+c\right)\right)+11\left(\cos^{2}\left(dx+c\right)\right)+29\cos\left(dx+c\right)-43\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^{3}\left(dx+c\right)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{15d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)*a^2
```

maxima [A] time = 0.59, size = 60, normalized size = 0.50

$$\frac{\left(3\,\sqrt{2}\,a^{2}\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\,\sqrt{2}\,a^{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\,\sqrt{2}\,a^{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [B] time = 1.83, size = 85, normalized size = 0.71

$$\frac{a^{2}\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(175\,\sin\left(c+d\,x\right)+28\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)\right)}{30\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(175*sin(c + d*x) + 28*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(3
0*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\,\mathrm{s}}{}$$

```
[Out] 2/7*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*(a+a*sec(d*x
+c))^(5/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+64/21*a^3*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/
sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3812, 3809, 3804}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3/2)) + (2
*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{5}{7}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(8a)$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + 2(a+$$
$$= \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + 2a(a+a\,\mathrm{se}$$

Mathematica [A] time = 0.37, size = 74, normalized size = 0.47

$$\frac{a^2(101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx))+208)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{42d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)])*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d*Sqrt[Sec[c + d*x]])
```

fricas [A] time = 0.69, size = 100, normalized size = 0.64

$$\frac{2\left(3a^2\cos(dx+c)^4+12a^2\cos(dx+c)^3+23a^2\cos(dx+c)^2+46a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{21(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2
+ 46*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 1.74, size = 95, normalized size = 0.61

$$-\frac{2\left(3\left(\cos^4(dx+c)\right)+9\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+23\cos(dx+c)-46\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\ \left(\cos^4(dx+\right.}{21d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/21/d*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46)*(a
*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x
+c)*a^2
```

maxima [B] time = 0.75, size = 323, normalized size = 2.07

$$\frac{\sqrt{2}\left(315\,a^{2}\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^{2}\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,a\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

mupad [B] time = 2.32, size = 96, normalized size = 0.62

$$\frac{a^{2}\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(392\,\sin\left(c+d\,x\right)+98\,\sin\left(2\,c+2\,d\,x\right)+24\,\sin\left(3\,c+3\,d\,x\right)+3\,\right.}{84\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(392*sin(c + d*x) + 98*sin(2*c + 2*d*x) + 24*sin(3*c + 3*d*x) + 3
*sin(4*c + 4*d*x)))/(84*d*(cos(c + d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{1168a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] 38/63*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+146/105*a^3*
sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)
/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)*sec(d*x+
c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)
/d/sec(d*x+c)^(7/2)
```

Rubi [A] time = 0.33, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3813, 4015, 3805, 3804}

$$\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
))
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}(2a)\int\frac{\sqrt{a+a\sec(c+dx)}\left(\frac{19a}{2}+\frac{15}{2}a\,\mathrm{se}\right.}{\sec^{\frac{7}{2}}(c+dx)}\\ &= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{21}\\ &= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\\ &= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\\ &= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\end{aligned}$$

Mathematica [A] time = 0.60, size = 80, normalized size = 0.40

$$\frac{2a^3\sin(c+dx)\left(584\sec^4(c+dx)+292\sec^3(c+dx)+219\sec^2(c+dx)+130\sec(c+dx)+35\right)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(35 + 130*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 292*Sec[c + d*x]^3 + 5
84*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[
c + d*x])])
```

fricas [A] time = 0.55, size = 113, normalized size = 0.56

$$\frac{2\left(35\,a^2\cos(dx+c)^5+130\,a^2\cos(dx+c)^4+219\,a^2\cos(dx+c)^3+292\,a^2\cos(dx+c)^2+584\,a^2\cos(dx+c)\right.}{315\,(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 + 130*a^2*cos(d*x + c)^4 + 219*a^2*cos(d*x + c
)^3 + 292*a^2*cos(d*x + c)^2 + 584*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)

maple [A] time = 1.75, size = 105, normalized size = 0.52

$$-\frac{2\left(35\left(\cos^{5}(dx+c)\right)+95\left(\cos^{4}(dx+c)\right)+89\left(\cos^{3}(dx+c)\right)+73\left(\cos^{2}(dx+c)\right)+292\cos(dx+c)-584\right)\sqrt{}}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)

[Out] -2/315/d*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)^2+2
92*cos(d*x+c)-584)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)
*cos(d*x+c)^5/sin(d*x+c)*a^2

maxima [B] time = 0.72, size = 422, normalized size = 2.10

$$\frac{\sqrt{2}\left(8190\,a^{2}\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+2100\,a^{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d

mupad [B] time = 2.96, size = 107, normalized size = 0.53

$$\frac{a^{2}\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,\left(10290\,\sin(c+d\,x)+2856\,\sin(2\,c+2\,d\,x)+981\,\sin(3\,c+3\,d\,x)+\right.}{2520\,d\,\left(\cos(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)

[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(10290*sin(c + d*x) + 2856*sin(2*c + 2*d*x) + 981*sin(3*c + 3*d*x
) + 260*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1)
)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}$$

```
[Out] 46/99*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+710/693*a^3*
sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+284/231*a^3*sin(d*x+c)
/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1136/693*a^3*sin(d*x+c)/d/sec(d*
x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2272/693*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/
d/(a+a*sec(d*x+c))^(1/2)+2/11*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d
*x+c)^(9/2)
```

Rubi [A] time = 0.40, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3813, 4015, 3805, 3804}

$$\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (46*a^3*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) +
(710*a^3*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
+ (284*a^3*Sin[c + d*x])/(231*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (1136*a^3*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (2272*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2))
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{1}{11}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{23a}{2}+\frac{19}{2}a\,\mathrm{s}\right.}{\sec^{\frac{9}{2}}(c+dx)}\\
&= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{1}{99}\\
&= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\\
&= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\\
&= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\\
&= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\end{aligned}$$

Mathematica [A] time = 0.40, size = 90, normalized size = 0.37

$$\frac{2a^3\sin(c+dx)\left(1136\sec^5(c+dx)+568\sec^4(c+dx)+426\sec^3(c+dx)+355\sec^2(c+dx)+224\sec(c+dx\right.}{693d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*a^3*(63 + 224*Sec[c + d*x] + 355*Sec[c + d*x]^2 + 426*Sec[c + d*x]^3 + 5
68*Sec[c + d*x]^4 + 1136*Sec[c + d*x]^5)*Sin[c + d*x])/(693*d*Sec[c + d*x]^
(9/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.51, size = 126, normalized size = 0.52

$$\frac{2\left(63\,a^2\cos(dx+c)^6+224\,a^2\cos(dx+c)^5+355\,a^2\cos(dx+c)^4+426\,a^2\cos(dx+c)^3+568\,a^2\cos(dx+c\right.}{693\,(d\cos(dx+c)+d)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 + 355*a^2*cos(d*x + c
)^4 + 426*a^2*cos(d*x + c)^3 + 568*a^2*cos(d*x + c)^2 + 1136*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) +
 d)*sqrt(cos(d*x + c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)
```

maple [A] time = 1.74, size = 115, normalized size = 0.48

$$-\frac{2\left(63\left(\cos^{6}(dx+c)\right)+161\left(\cos^{5}(dx+c)\right)+131\left(\cos^{4}(dx+c)\right)+71\left(\cos^{3}(dx+c)\right)+142\left(\cos^{2}(dx+c)\right)+56}{693d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/693/d*(63*cos(d*x+c)^6+161*cos(d*x+c)^5+131*cos(d*x+c)^4+71*cos(d*x+c)^3
+142*cos(d*x+c)^2+568*cos(d*x+c)-1136)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*
(1/cos(d*x+c))^(11/2)*cos(d*x+c)^6/sin(d*x+c)*a^2
```

maxima [B] time = 0.76, size = 521, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/22176*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a
^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11
/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a
^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/
2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c
)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*
a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/
2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c)
+ 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))
 + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)
)) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
11/2*c))))*sqrt(a)/d
```

mupad [B] time = 6.93, size = 356, normalized size = 1.48

$$\frac{\sqrt{a-\frac{a}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1}}\left(2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1i-1\right)\left(\frac{23a^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-2\sin\left(\frac{11c}{4}+\frac{11dx}{4}\right)^2+\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{4d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] ((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i
+ 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((23*a^2*sin(c/2 + (d*x)/2)*(sin((11*
c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (19*a^
2*sin((3*c)/2 + (3*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4
+ (11*d*x)/4)^2 + 1))/(12*d) + (5*a^2*sin((5*c)/2 + (5*d*x)/2)*(sin((11*c)/
2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(8*d) + (13*a^2*s
in((7*c)/2 + (7*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (
11*d*x)/4)^2 + 1))/(56*d) + (5*a^2*sin((9*c)/2 + (9*d*x)/2)*(sin((11*c)/2 +
 (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a^2*sin((1
1*c)/2 + (11*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*
d*x)/4)^2 + 1))/(88*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(
c/4 + (d*x)/4)^2 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 4*a^2*sec(d*x+c)^(3/4)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3814, 8}

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

```
[Out] (4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx &= \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(4a)\int 0\,dx\\ &= \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 1.18

$$\frac{4\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}{d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

```
[Out] (4*Sec[c + d*x]^(3/4)*(a*(1 + Sec[c + d*x]))^(3/2)*Sin[c + d*x])/(d*(1 + Se
c[c + d*x])^2)
```

fricas [A] time = 0.70, size = 50, normalized size = 1.32

$$\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\cos\left(dx+c\right)^{\frac{1}{4}}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)^(1/4)*sin(d*x + c)
/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

```
[Out] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

maxima [B] time = 0.59, size = 121, normalized size = 3.18

$$\frac{4\left(\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\,a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

mupad [B] time = 0.75, size = 55, normalized size = 1.45

$$\frac{2\,a\,\sin\left(2\,c+2\,d\,x\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/4}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/4),x)
```

```
[Out] (2*a*sin(2*c + 2*d*x)*(1/cos(c + d*x))^(3/4)*((a*(cos(c + d*x) + 1))/cos(c
+ d*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3801, 215}

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [A] time = 0.15, size = 54, normalized size = 1.46

$$-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sin^{-1}\left(\sqrt{\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (-2*ArcSin[Sqrt[Sec[e + f*x]]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])
/(f*Sqrt[1 - Sec[e + f*x]])
```

fricas [B] time = 0.77, size = 189, normalized size = 5.11

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}-\frac{4\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{\sqrt{\cos\left(fx+e\right)}}+8\,a}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}}\right)}{2\,f}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\cos\left(fx+e\right)}\ldots}{a\cos\left(fx+e\right)^{2}-a\cos\ldots}\right)}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e)/sqrt(cos(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x +
 e))*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\sqrt{\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(sec(f*x + e)), x)
```

maple [B] time = 1.61, size = 150, normalized size = 4.05

$$\frac{\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{1}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1+\sin\left(fx+e\right)\right)\sqrt{2}}{4}\right)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos\left(fx+e\right)}}\left(\mathrm{c}\ldots\right.}{\ldots}\right.\right.}{2f\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e))^(1/2)*cos(f*x+e)*(
arctan(1/4*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e))*2^(1/2))-arc
tan(1/4*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*x+e))*2^(1/2)))*(-2/(
1+cos(f*x+e)))^(1/2)/sin(f*x+e)^2*(-1+cos(f*x+e)^2)*2^(1/2)
```

maxima [B] time = 0.62, size = 241, normalized size = 6.51

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)-\log\left(\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sq
rt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) - log(2*co
s(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sqrt(2)*cos(1/2*f*x + 1
/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) + log(2*cos(1/2*f*x + 1/2*e)^2
+ 2*sin(1/2*f*x + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin
(1/2*f*x + 1/2*e) + 2) - log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2
*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2
))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(sec(e + f*x)), x)
```

3.244 $\int \sqrt{-\sec(e+fx)}\,\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(f*x+e)/(a-a*sec(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{3801, 215\}$

$$\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\sec(e+fx)}\,\sqrt{a-a\sec(e+fx)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \frac{a\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [C] time = 2.06, size = 299, normalized size = 7.87

$$\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}\left(\log\left(-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)+2\right)-\log\left(-\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*((-2*I)*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[2])*Sin[(e
+ f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + (2*I)*Arc
```

```
Tan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/((-1 + Sqrt[2])*Cos
[(e + f*x)/4] - Sin[(e + f*x)/4])] - 4*ArcTanh[Sqrt[2]*Cos[(2*e + f*x)/4]*S
ec[(f*x)/4] + Tan[(f*x)/4]] + Log[2 - Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Si
n[(e + f*x)/2]] - Log[2 + Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Sin[(e + f*x)/
2]])*Sqrt[a - a*Sec[e + f*x]])/(2*Sqrt[2]*f*Sqrt[-Sec[e + f*x]])
```

fricas [B] time = 0.70, size = 215, normalized size = 5.66

$$\left[\frac{\sqrt{a}\log\left(\frac{4\left(\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{\cos\left(fx+e\right)}}+\left(a\cos\left(fx+e\right)^{2}+8\,a\cos\left(fx+e\right)+8\,a\right)\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}\right)}{2\,f}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 2*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e)) + (a*co
s(f*x + e)^2 + 8*a*cos(f*x + e) + 8*a)*sin(f*x + e))/(cos(f*x + e)^2*sin(f*
x + e)))/f, -sqrt(-a)*arctan(2*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e))/((a*cos(f*x + e)
 + 2*a)*sin(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)2*sqrt(2)*(-a^2*atan(sqrt(a)/sqrt(-a))/sqrt(2)/sqrt(-a)+a^2*atan(sq
rt(a*tan(1/2*(f*x+exp(1)))^2+a)/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a))*abs(a)*
sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/a^2/f
```

maple [B] time = 1.70, size = 126, normalized size = 3.32

$$\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\,\left(\cos\left(fx+e\right)+1+\sin\left(fx+e\right)\right)}{2}\right)-\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}\,\left(-\cos\left(fx+e\right)-1+\sin\left(fx+e\right)\right)}{2}\right)\right)\cos\left(fx+e\right)\sqrt{-}}{f\sin\left(fx+e\right)\sqrt{\frac{1}{1+\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))-arctan
h(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e))))*cos(f*x+e)*(-1/
cos(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(1/(1+cos
(f*x+e)))^(1/2)
```

maxima [B] time = 0.99, size = 353, normalized size = 9.29

$$-\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)^{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) + log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{-\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sec\left(e+fx\right)}\,\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**(1/2)*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-sec(e + f*x))*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=128

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/
d/a^(1/2)+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3822, 4023, 3808, 206, 3801, 215}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
```

```
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,(a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,dx}{2a} + \int \frac{\sqrt{\sec(c+}}{\sqrt{a+a\sec(c}}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+}{d\sqrt{a+a\sec(c+d}}$$

Mathematica [A] time = 0.26, size = 125, normalized size = 0.98

$$\frac{\tan(c+dx)\left(\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\,\tan^{-1}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.16, size = 481, normalized size = 3.76

$$\left[\frac{\sqrt{a}\,(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\,\sqrt{2}\,(a\cos(dx+c)+a)}{}}{4\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)

maple [B] time = 1.58, size = 224, normalized size = 1.75

$$\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^2(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)-\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/4/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+4*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a

maxima [B] time = 0.69, size = 876, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos

```
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3821, 3801, 215, 3808, 206}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
```

```
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.10, size = 89, normalized size = 0.94

$$\frac{\tan(c+dx)\left(\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) - 2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

fricas [A] time = 0.64, size = 351, normalized size = 3.69

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2 + \frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right) + \sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 - \frac{4\left(\cos(dx+c)^2 - 2\right.}{}}{\cos(dx+c)^3 + c}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/s
in(d*x + c)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

maple [B] time = 1.55, size = 184, normalized size = 1.94

$$\frac{\left(\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)-2\arctan \cdots}{2d\sin(dx+ \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
))*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2*(-2/(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a
```

maxima [B] time = 0.82, size = 476, normalized size = 5.01

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^{2}+2\sin\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3808, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 1.34

$$-\frac{\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))

fricas [A] time = 0.68, size = 160, normalized size = 2.86

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))/d]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)

maple [B] time = 1.38, size = 99, normalized size = 1.77

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\cos(dx+c)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)-1\right)}{d\sin(dx+c)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/d*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a

maxima [A] time = 0.67, size = 90, normalized size = 1.61

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx\right.\right.}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{a(\sec{(c+dx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3812, 3808, 206}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(}{\sqrt{a+a\sec(c+d}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.23, size = 102, normalized size = 1.10

$$\frac{\tan(c+dx)\left(\frac{2\sqrt{1-\sec(c+dx)}}{\sqrt{\sec(c+dx)}} + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + (2*
Sqrt[1 - Sec[c + d*x]])/Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c
 + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 1.47, size = 281, normalized size = 3.02

$$\left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}} + 4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx}{2(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 1.52, size = 100, normalized size = 1.08

$$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-2\cos(dx+c)+2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{d\sqrt{\frac{1}{\cos(dx+c)}}\,\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)-2*cos(d*x+c)+2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(
d*x+c))^(1/2)/sin(d*x+c)/a
```

maxima [A] time = 0.59, size = 104, normalized size = 1.12

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx\right.\right.}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec(c+dx)+1\right)}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(
1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3823, 4013, 3808, 206}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \int$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\mathrm{S}}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.26, size = 120, normalized size = 0.92

$$\frac{\tan(c+dx)\left(2(\cos(c+dx)-1)\sqrt{1-\sec(c+dx)}-3\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])
])
```

fricas [A] time = 0.51, size = 318, normalized size = 2.43

$$\left[\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*(c
os(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*co
s(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(cos(d*x + c)^2 - co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 1.63, size = 120, normalized size = 0.92

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)-4\cos(dx+}{3d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*c
os(d*x+c)+2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/a
```

maxima [B] time = 0.66, size = 282, normalized size = 2.15

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\mathrm{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec(c+dx)+1\right)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\ t}{}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^
(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+26/15*sin(d
*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3823, 4022, 4013, 3808, 206}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\ t}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
```

```
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

Mathematica [A] time = 1.21, size = 117, normalized size = 0.69

$$\frac{\frac{15\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c+dx)(-2\cos(c+dx)+3\cos(2(c+dx))+29)\sqrt{\sec(c+dx)}}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
 + (15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*
Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.63, size = 342, normalized size = 2.02

$$\left[\frac{\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}} + \frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos}{\cos}}}{\sqrt{\cos(dx+c)}}}{30\left(ad\cos(dx+c)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*
(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
), 1/15*(15*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))
 + 2*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\ \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 1.75, size = 130, normalized size = 0.77

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(6\left(\cos^3(dx+c)\right)-15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin(dx+c)-8\left(\cos^2(dx\right.\right.}{15d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8
*cos(d*x+c)^2+28*cos(d*x+c)-26)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c
)/a
```

maxima [B] time = 0.84, size = 357, normalized size = 2.11

$$\frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
```

```
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(5/2)), x)
```

3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{2ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -3*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/
2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3816, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{2ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-3a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{3a^2}{2}+3a^2\sec(c\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^3}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int \sqrt{\sec(c+dx)}\sqrt{a+a}}{2a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x,\right.}{a^2d}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d(a+a\sec(c+d}$$

Mathematica [A] time = 0.68, size = 252, normalized size = 1.45

$$\frac{4\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+6\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-9\sqrt{2}\tan(c+dx)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec[
c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[
```

```
Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqr
t[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
+ 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 18*Arc
Sin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.61, size = 579, normalized size = 3.33

$$\left[\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\cdots}{8\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 6*(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x
 + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.53, size = 282, normalized size = 1.62

$$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^3(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\cos(dx+c)\sin(dx\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^3*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
*(3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))
*cos(d*x+c)*sin(d*x+c)*2^(1/2)-3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-3*(-2/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2+9*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
)*cos(d*x+c)*sin(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+2*(-2/(1+cos(d
*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

maxima [B] time = 1.03, size = 4934, normalized size = 28.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
```

```
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c
)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2
*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
```

```
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(
2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))/((sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*cos(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(
2*d*x + 2*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sqrt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out
```

3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-5/4*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3816, 4023, 3808, 206, 3801, 215}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$
$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\sqrt{\sec}}{\sqrt{a+a\,s}}}{4}$$
$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d} + \frac{5\,\mathrm{Subs}}{}$$
$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c}{2d(a+a\sec(c+}$$

Mathematica [A] time = 0.62, size = 220, normalized size = 1.64

$$\frac{-2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+5\sqrt{2}\tan(c+dx)\sec(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+5\sqrt{2}\tan(}{4d\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 5*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 5*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]*Tan[c + d*x] - 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[
c + d*x] - 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(
4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [B] time = 0.81, size = 559, normalized size = 4.17

$$\left[\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) -
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.49, size = 240, normalized size = 1.79

$$\left(2\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)-2\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1-\sin(dx+c))\sqrt{}}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c))*2^(1/2))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-(-2/(1+c
os(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(
d*x+c))^(5/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

maxima [B] time = 1.49, size = 2122, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
```

```
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d
*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
in(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*si
n(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3810, 3808, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [B] time = 0.64, size = 220, normalized size = 2.27

$$\frac{2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) - \sqrt{2}\tan(c+dx)\sec(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) - \sqrt{2}\tan(c+dx)}{4d\sqrt{1-\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - Sqrt[2]
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*T
an[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d
*x] + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sq
rt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.55, size = 338, normalized size = 3.48

$$\left[\frac{\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2a\cos(dx+c) - 3a}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)}{8\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.47, size = 146, normalized size = 1.51

$$\frac{\left(\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)-\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin\left(dx+c\right)-\sqrt{-\frac{2}{1+\cos(dx+c)}}\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\ (-1}{2d\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin\left(dx+c\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*((-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/c
os(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(-2/(1+c
os(d*x+c)))^(1/2)/sin(d*x+c)^3/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3811, 3808, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.19, size = 120, normalized size = 1.24

$$\frac{-2\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx) - 3\sqrt{2}\,\tan(c+dx)(\sec(c+dx)+1)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{4d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*
Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.72, size = 340, normalized size = 3.51

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^2 + 2\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2a\cos(dx+c)}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{8\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.38, size = 146, normalized size = 1.51

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos(dx+c)+3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c}{4d\sin(dx+c)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(
(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

maxima [B] time = 1.50, size = 1031, normalized size = 10.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c + dx\right)}}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -7/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+
c))^(3/2)+5/2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3817, 4013, 3808, 206}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{5a}{2}+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{7\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{7\,\mathrm{Sub}}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5}{}$$

Mathematica [A] time = 0.56, size = 145, normalized size = 1.06

$$\frac{2\sin(c+dx)\left(5\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+4\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\right)+7\sqrt{2}\,\tan(c+dx)(\sec(}{4d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(5*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x] + 7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d
*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1
- Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.55, size = 378, normalized size = 2.76

$$\left[\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)^2 + 5*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 1.66, size = 175, normalized size = 1.28

$$-\frac{\left(7\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)-7\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\cdots}\right.}{4d\sin(dx+c)^3\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(7*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-7*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)
))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*cos(d*x+c)^3+6*cos(d*x+c)^
2+12*cos(d*x+c)-10)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos
(d*x+c))^(1/2)/a^2
```

maxima [B] time = 1.00, size = 7176, normalized size = 52.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)
^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) -
 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
 + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*
x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2
*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(
1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/
2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
```

/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(
1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
+ 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*s
in(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2
*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^
2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*
c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8*
(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co

s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*cos(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*(3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x + 1/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x + 1/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1

/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*cos(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/((4*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x + 5/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2*d

```
*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2*d
*x + 3/2*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\,(\sec{(c+dx)}+1))^{\frac{3}{2}}\,\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{\cdots}{2d\sqrt{\sec\cdots}}$$

```
[Out] 11/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/sec(d*x
+c)^(1/2)+7/6*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-19/6*s
in(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3817, 4022, 4013, 3808, 206}

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{\cdots}{2d\sqrt{\sec\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{7a}{2}+2a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec}}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.98, size = 150, normalized size = 0.85

$$\frac{\sqrt{1-\sec(c+dx)}\,(4\sin(c+dx)-\tan(c+dx)(19\sec(c+dx)+12))-33\sqrt{2}\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+}{6d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\,(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-33*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
4*Sin[c + d*x] - (12 + 19*Sec[c + d*x])*Tan[c + d*x]))/(6*d*Sqrt[-((-1 + Se
c[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.73, size = 398, normalized size = 2.25

$$\left[\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
 2*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 1.76, size = 193, normalized size = 1.09

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(33\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)+8\left(\cos^4(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(33*arctan(1/2*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+8*co
s(d*x+c)^4-33*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-40*cos(d*x+c)^3+18*cos(d*x+c)^2+52*cos(d*x+c)-38)*
(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)^3/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**(3/2)), x)
```

3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] -1/2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)+9/10*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)-13/1
0*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+c)*s
ec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3817, 4022, 4013, 3808, 206}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sec[c + d*x]^(3/
2)*(a + a*Sec[c + d*x])^(3/2)) + (9*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) - (13*Sin[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{9a}{2}+3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}$$

Mathematica [A] time = 1.38, size = 163, normalized size = 0.75

$$\frac{(39\cos(c+dx)-2\cos(2(c+dx))+\cos(3(c+dx))+47)\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)+75\sqrt{2}\sin(c}{10d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}(a(\sec(c+dx)+1))^{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (47 + 39*Cos[c + d*x] - 2
*Cos[2*(c + d*x)] + Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(10*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

fricas [A] time = 0.68, size = 418, normalized size = 1.93

$$\left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{40\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x +
c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, 1/20*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x + c)
^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 1.82, size = 203, normalized size = 0.94

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(8\left(\cos^{5}\left(dx+c\right)\right)-75\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(8*cos(d*x+c)^5-75*arctan(1/2*si
n(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*
sin(d*x+c)-24*cos(d*x+c)^4+96*cos(d*x+c)^3+75*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-54*cos(d*x+c)^2-12
4*cos(d*x+c)+98)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c}{4d(a\sec(c+dx)+}$$

```
[Out] -5*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*x
+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sec(d*x+c)^(5/2)*sin(d*
x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+115/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d
*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sec(d*x
+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3816, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{35\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c}{4d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-5a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{4}-\frac{35}{2}a^2 \ldots\right)}{\sqrt{a+a\sec(c+d \ldots}}}{8a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(\ldots}{16a^2d\sqrt{a+a\sec(c \ldots}}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(\ldots}{16a^2d\sqrt{a+a\sec(c \ldots}}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(\ldots}{16a^2d\sqrt{a+a\sec(c \ldots}}$$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sec^{\frac{7}{2}}(c+dx)\,\mathrm{s}\ldots}{4d(a+a\sec(\ldots}$$

Mathematica [A] time = 1.35, size = 340, normalized size = 1.59

$$\frac{32\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{7}{2}}(c+dx) + 110\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx) + 70\sin(c+dx) \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (70*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Se
c[c + d*x]^(7/2)*Sin[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*
x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 115*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]^2*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2
*Tan[c + d*x] + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c +
 d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 1.42, size = 667, normalized size = 3.12

$$\left[\frac{115\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c \ldots}}{\cos(dx+c)^2 + 2\cos(dx+ \ldots}\right) \ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(115*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
+ c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.62, size = 454, normalized size = 2.12

$$-\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}\left(\cos^4(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(-40\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\sqrt{-\frac{}{1+\cos}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(1/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/
2)*(-1+cos(d*x+c))^2*(-40*cos(d*x+c)^2*sin(d*x+c)*arctan(1/4*(-2/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+40*cos(d*x+c)^2*sin(
d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/
2))*2^(1/2)+35*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)-115*cos(d*x+c)^2*sin(
d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-40*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+
c)*2^(1/2)+40*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
)*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+20*(-2/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)^2-115*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin
(d*x+c)-39*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-16*(-2/(1+cos(d*x+c)))^(1/2
))/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

maxima [B] time = 6.10, size = 9048, normalized size = 42.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
6*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*si
n(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(
sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*
sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x +
4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c
)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x +
 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c)
 + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*
cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c)
+ 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c)
+ 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
```

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c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4
*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*s
qrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2
)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*
x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(
2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4
*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*
x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(
2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7
*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x
 + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(s
qrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x
+ 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x
+ 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) +
 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c
) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c
) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4
*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*si
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n(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt
(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d
*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c
) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*s
in(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) +
7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x +
 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(c
os(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(
6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*s
in(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x +

6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(75*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 32*(24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 560*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 560*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a^2

```
*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*a^2*co
s(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 64*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*sin(4*d*x + 4*c)^
2 + 98*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*a^2*sin(2
*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 64*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 14*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(7*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6
*c) + 14*(7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) +
8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2
)*a^2*cos(2*d*x + 2*c) + 8*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2)*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 16*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqr
t(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*
c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 8*sq
rt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*
a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*
sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a
^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}} dx$

Optimal. Leaf size=174

$$-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*sec(d*x+c)^(3/2)*sin(d*x
+c)/a/d/(a+a*sec(d*x+c))^(3/2)-43/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x
+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3816, 4019, 4023, 3808, 206, 3801, 215}

$$-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{11a^2}{4}-8a^2 s\right.}{\sqrt{a+a\sec(c+dx}}}{8a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}\sqrt{a-}}{a^3}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx\right.}{a^3}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin}{4d(a+a\sec(c+}$$

Mathematica [A] time = 0.73, size = 308, normalized size = 1.77

$$-30\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx) - 22\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) + 43\sqrt{2}\tan(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-22*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*S
```

```
qrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c +
d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]
]]*Sec[c + d*x]^2*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x] - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x]
)^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

fricas [B] time = 0.99, size = 665, normalized size = 3.82

$$\left[\frac{43\,\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) - 4*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(c
os(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*
sin(d*x + c))) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*(1
1*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.58, size = 406, normalized size = 2.33

$$\frac{\left(-1+\cos\left(dx+c\right)\right)^2\left(16\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\sqrt{2}-16\arctan\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-16*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+
c)*2^(1/2)+16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c))*2^(1/2))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+11*(-2/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^2-43*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)*sin(d*x+c)+4*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-43*arctan(1/2*sin(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-15*(-2/(1+cos(d*x+c)))^(1/2))*(a*
(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+
c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

maxima [B] time = 2.56, size = 4988, normalized size = 28.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2
)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
```

```
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2
*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)
*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6
*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x
 + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 +
36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt
(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(
4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*si
n(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin
(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4
*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) +
 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) +
6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2
```

```
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*l
og(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) + 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 1
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x
+ 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*
sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*(19*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 19*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
1*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*
c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2
*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*s
in(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 176*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 +
16*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*
sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8
*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)
*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d
*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*s
qrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*sec(d*x+c)^(3
/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/
2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3810, 3808, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$

$$= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a}{\cdots}\right.}{16a^2d}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c\cdots}{16ad(a+a\sec(c+\cdots}$$

Mathematica [B] time = 0.80, size = 308, normalized size = 2.25

$$\frac{14\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx) + 6\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx) - 3\sqrt{2}\,\tan(c+dx)\,\text{se}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Sec
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[(Sq
rt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
 - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^2*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d
*x])^2*Tan[c + d*x] + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan
[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 1.15, size = 426, normalized size = 3.11

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d\cdots}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{64\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)^2 + 7*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3
*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 7*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(a\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.51, size = 210, normalized size = 1.53

$$-\frac{(-1+\cos (dx+c))^2\left(3\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2 (dx+c)\right)-3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\cos (dx+c)\sin (dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(3*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-3*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)+4*(-2/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*sin(d*x+c)-7*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1
/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3811, 3810, 3808, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$
$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{5\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$
$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{16}$$
$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c}$$

Mathematica [A] time = 1.98, size = 266, normalized size = 1.94

$$-\frac{10\sin(c+dx)\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)+8\sin(c+dx)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/32*(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 10*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])*Sin[c + d*x] - 10*Sq
rt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x] -
 10*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] - 10*A
rcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 5*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])
^2*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.77, size = 422, normalized size = 3.08

$$\left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
), -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.38, size = 210, normalized size = 1.53

$$-\frac{(-1+\cos(dx+c))^2\left(5\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)-5\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\cos(dx+c)\sin(dx+c)\cdots\right.}{16d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(5*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-5*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*sin(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(
1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/
2)/a^3
```

maxima [B] time = 1.47, size = 2875, normalized size = 20.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin
(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +
 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x +
 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c
os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d
*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1
2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*
(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
```

)^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))/((16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2

```
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) +
sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*co
s(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^2*s
in(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] -1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-9/16*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3817, 4012, 3808, 206}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (9*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*
a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
```

-1]

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{7a}{2}+a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{19\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{16a^2}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(}{16ad(a+a\sec(c+}$$

Mathematica [A] time = 1.00, size = 146, normalized size = 1.07

$$\frac{-\sin(c+dx)(13\cos(c+dx)+9)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx) - 76\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-76*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] - (9 + 13*Cos[c + d*x])*Sq
rt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 1.05, size = 426, normalized size = 3.11

$$\left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(13*cos(d*x + c)^2 + 9*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d), -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c
) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(13*cos(d*x + c)^2 + 9*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx + c)}}{(a\sec(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.34, size = 208, normalized size = 1.52

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(-1+\cos(dx+c))^2\left(13\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)+19\arctan\left(\frac{\sin}{\ldots}\right.\right.}{16\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*
(-1+cos(d*x+c))^2*(13*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*arctan(1/2*
sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)+19*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))
*sin(d*x+c)-9*(-2/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(
1/2)/a^3
```

maxima [B] time = 1.06, size = 3049, normalized size = 22.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
```

```
3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2*
d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
+ 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
+ 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1
/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*sin
(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a^2*
cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x + 4
*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d*x
+ c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*
```

```
d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*cos
(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*
c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x
 + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*
sin(3*d*x + 3*c))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{\left(c + dx \right)}}}{\left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\sec(}}{4d(a\sec(c+dx)}$$

```
[Out] -75/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*
x+c))^(5/2)-13/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+49
/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3817, 4020, 4013, 3808, 206}

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\sec(c}}{4d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a*d*(a + a*Sec[c + d*x])^(3/2)) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
```

```
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{9a}{2}+2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cdot}{\sqrt{se}}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{s}}{16a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\,\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{s}}{16a}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} -$$

Mathematica [A] time = 1.39, size = 186, normalized size = 1.05

$$\frac{\sin(c+dx)\left(49\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)+85\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+32\sqrt{-((\sec(c+dx)-1)\,se}\right.}{16d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (300*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + (85*Sqrt[1 - Sec[c + d*x
]]*Sec[c + d*x]^(3/2) + 49*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 32*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 1.54, size = 446, normalized size = 2.52

$$\left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{\cos(dx+c)^2+2\cos(dx+c)}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 + 85*cos(d*x +
 c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d
*x + c)^3 + 85*cos(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 1.66, size = 236, normalized size = 1.33

$$-\frac{(-1+\cos(dx+c))^2\left(-75\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)-150\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))^2*(-75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-150*cos(d*x+c)*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)+64*cos(d*x+c)^3-75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(
-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+106*cos(d*x+c)^2-72*cos(d*x+c)-98)*(a*(
1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sec(c+dx)+1))^{\frac{5}{2}}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(5/2)*sqrt(sec(c + d*x))), x)
```

3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{16ad}$$

```
[Out] 163/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/sec(d
*x+c)^(1/2)-17/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+95
/48*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-299/48*sin(d*x
+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3817, 4020, 4022, 4013, 3808, 206}

$$-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{11a}{2}+3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+d}{16ad\sqrt{\sec(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+d}{16ad\sqrt{\sec(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+d}{16ad\sqrt{\sec(c+dx)}\,(a+a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+d}{16ad\sqrt{\sec(c+dx)}\,(a+a}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+}$$

Mathematica [A] time = 2.49, size = 165, normalized size = 0.76

$$-\frac{\sec(c+dx)\Big(\tan(c+dx)\sqrt{1-\sec(c+dx)}\,(379\sec(c+dx)+16\cos(2(c+dx))(5\sec(c+dx)-1)+487)+97}{48d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\,(a(\sec(c+a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/48*(Sec[c + d*x]*(978*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt
[1 - Sec[c + d*x]]*(487 + 379*Sec[c + d*x] + 16*Cos[2*(c + d*x)]*(-1 + 5*Se
c[c + d*x]))*Tan[c + d*x]))/(d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a
*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.97, size = 466, normalized size = 2.15

$$\left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^4 - 160*cos(d*
x + c)^3 - 503*cos(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) - 2*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 503*cos(d*x + c
)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^
3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 1.70, size = 254, normalized size = 1.17

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(489\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*arctan(1
/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)^2*sin(d*x+c)+64*cos(d*x+c)^4+978*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)-384*cos(d*x+c)^3+
489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1
```

```
/2)*sin(d*x+c)-686*cos(d*x+c)^2+408*cos(d*x+c)+598)*cos(d*x+c)^2*(1/cos(d*x
+c))^(3/2)/sin(d*x+c)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{4d}$$

```
[Out] 7/4*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d*
x+c)))*2^(1/2)/d-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)+1/2
*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3822, 4021, 4023, 3807, 215, 3801}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (7*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/(4*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
4*d*Sqrt[1 + Sec[c + d*x]]) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1
 + Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{(3-\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}+\right.}{\sqrt{1+\sec(c-}}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{7}{8}\int \sqrt{\sec(c+dx)}\sqrt{1+\mathrm{se}}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\right.}{4d}$$

$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\mathrm{s}}{}$$

Mathematica [A] time = 0.46, size = 140, normalized size = 1.11

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+\sin^{-1}\left(\right.\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 8*ArcSin[Sqrt[Sec[c + d*x]]
] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] -
 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Sqrt[-((-1 + Sec[c + d*x])*S
ec[c + d*x])])*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

fricas [B] time = 0.87, size = 337, normalized size = 2.67

$$\frac{8\left(\sqrt{2}\cos(dx+c)^2+\sqrt{2}\cos(dx+c)\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*log(-(2*sqrt(2)*sqr
t((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*
x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*(
cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos
(d*x + c) + 1)) + 7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 -
2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - c
os(d*x + c) - 2)/(cos(d*x + c) + 1)) - 4*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*(cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(sec(d*x + c) + 1), x)
```

maple [B] time = 1.46, size = 253, normalized size = 2.01

$$-\frac{(-1+\cos(dx+c))\left(7\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-7\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-16*cos(d*x+
c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-2*(-2/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*((1+co
s(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2
```

maxima [B] time = 1.08, size = 1643, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
```

```
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(1+sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

```
[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d+arcsinh(tan(d*x+c)/(1+sec(d*x+c
)))*2^(1/2)/d+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3822, 4023, 3807, 215, 3801}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - ArcSinh[Tan[c + d*x]
/Sqrt[1 + Sec[c + d*x]]]/d + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 +
Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}} + \frac{1}{2}\int \frac{(1-\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}} - \frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\,dx + \int \frac{\sqrt{\sec(c+}}{\sqrt{1+\sec(c}}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} - \frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

Mathematica [A] time = 0.31, size = 111, normalized size = 1.31

$$\frac{\tan(c+dx)\left(\sqrt{-((\sec(c+dx)-1)\sec(c+dx))} + \sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right) + 2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) - \sqrt{2}\tan\right.}{d\sqrt{-\tan^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[-Tan[c + d*x]^2])
```

fricas [B] time = 0.97, size = 297, normalized size = 3.49

$$2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+(\cos(dx+c)+1)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (cos(d*x + c) + 1)*l
og(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - (cos(d*x + c)
+ 1)*log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) + 4*sqrt((
cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(sec(d*x + c) + 1), x)
```

maple [B] time = 1.42, size = 220, normalized size = 2.59

$$\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^2(dx+c)\right)\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)-\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(
arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(
1/2)*cos(d*x+c)-arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c))*2^(1/2))*2^(1/2)*cos(d*x+c)+4*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+c
os(d*x+c)))^(1/2))+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 1.25, size = 873, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] 2*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d*x+
c)))*2^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3821, 3801, 215, 3807}

$$\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$
$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}$$

Mathematica [A] time = 0.09, size = 76, normalized size = 1.41

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] ((2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]]
)/Sqrt[1 - Sec[c + d*x]]])*Cot[c + d*x]*Sqrt[-Tan[c + d*x]^2])/d
```

fricas [B] time = 0.92, size = 223, normalized size = 4.13

$$\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\log\left(-\frac{\cos(dx+c)^2+2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d}{\cos(dx+c)+1}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d
*x + c) + 1)) + log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1))
)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(sec(d*x + c) + 1), x)
```

maple [B] time = 1.54, size = 180, normalized size = 3.33

$$\frac{\left(\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)-2\arcta\right.}{2d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
))*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*((1+co
s(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)*(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 0.93, size = 473, normalized size = 8.76

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+2\sin\left(\frac{1}{2}\text{a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(sec(c + d*x) + 1), x)
```

3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3807, 215}

$$\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx &= -\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\cos(c+dx)+1}}\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]*Sqrt[(1 + Cos[c + d*x])^(-1)]
)/d
```

fricas [B] time = 0.62, size = 88, normalized size = 3.26

$$\frac{\sqrt{2}\,\log\left(\frac{2\,\sqrt{2}\,\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-\cos(dx+c)^{2}+2\,\cos(dx+c)+3}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(d*x + c) + 3)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{\sec\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(sec(d*x + c) + 1), x)
```

maple [B] time = 1.36, size = 95, normalized size = 3.52

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\cos\left(dx+c\right)\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(dx+c\right)-1\right)}{d\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d
*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 1.29, size = 87, normalized size = 3.22

$$\frac{\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(sec(c + d*x) + 1), x)
```

3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2*sin(d*x+c)*sec(d*x+c)^(1/2)
/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3812, 3807, 215}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx\\
&= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\
&= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 90, normalized size = 1.45

$$\frac{2\sin(c+dx)\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{-\tan^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqrt[
-Tan[c + d*x]^2])
```

fricas [B] time = 0.70, size = 144, normalized size = 2.32

$$\frac{\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos}}{2\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d
*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(dx+c)+1}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 1.50, size = 98, normalized size = 1.58

$$-\frac{\left(-\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\cos(dx+c)-2\right)\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}}{d\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+2*cos(d*x+c)-2)*((1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x
+c)/(1/cos(d*x+c))^(1/2)
```

maxima [A] time = 1.09, size = 101, normalized size = 1.63

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec{\left(c + dx \right)} + 1}\,\sqrt{\sec{\left(c + dx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\sec(c+dx)+1}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/sec(d*x+c)^(1
/2)/(1+sec(d*x+c))^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3823, 4013, 3807, 215}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\sec(c+dx)+1}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (2*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[1 + Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{1}{3}\int \frac{1-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$
$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}} + \int \frac{\sqrt{}}{\sqrt{1}}$$
$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}} - \frac{\sqrt{2}\,\text{Su}}{}$$
$$= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c-}}{3d\sqrt{1}}$$

Mathematica [A] time = 0.26, size = 118, normalized size = 1.20

$$\frac{\tan(c+dx)\left(2(\cos(c+dx)-1)\sqrt{1-\sec(c+dx)}-3\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}\sqrt{\sec(c+dx)+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[1 + Sec[c + d*x]])
```

fricas [A] time = 1.22, size = 163, normalized size = 1.66

$$\frac{3\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{\text{co}}{\text{c}}}}{\sqrt{\cos(dx+c)}}}{6\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 -
cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

maple [A] time = 1.64, size = 116, normalized size = 1.18

$$-\frac{\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\sin(dx+c)+2\left(\cos^2(dx+c)\right)-4\cos(dx+c)+\right.}{3d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos
(d*x+c)+2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)
```

maxima [B] time = 0.92, size = 279, normalized size = 2.85

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\text{arct}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(c+dx)+1}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=134

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\sec(c+dx)+1}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}}{}$$

```
[Out] -arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/sec(d*x+c)^(
3/2)/(1+sec(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c))
^(1/2)+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3823, 4022, 4013, 3807, 215}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\sec(c+dx)+1}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(15*d*Sqrt[1 + Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{1}{5}\int \frac{1-4\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx}}$$

$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{}{15d\sqrt{}}$$

Mathematica [A] time = 0.31, size = 122, normalized size = 0.91

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(13\sec^2(c+dx)-\sec(c+dx)+3\right)+15\sqrt{2}\sec^{\frac{5}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right.}{15d\sqrt{-\tan^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c + d*x] + 13*Sec[c +
d*x]^2))*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)*Sqrt[-Tan[c + d*x]^2])
```

fricas [A] time = 0.47, size = 174, normalized size = 1.30

$$\frac{15\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(3\cos(dx+c)^3-\cos(dx\right.}{}}{30\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c
) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)
^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(dx+c)+1}\,\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(5/2)), x)
```

maple [A] time = 1.75, size = 126, normalized size = 0.94

$$-\frac{\sqrt{\frac{1+\cos(dx+c)}{\cos(dx+c)}}\left(6\left(\cos^{3}(dx+c)\right)-15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-8\left(\cos^{2}(dx+c\right.\right.}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*((1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2*sin
(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*c
os(d*x+c)^2+28*cos(d*x+c)-26)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

maxima [B] time = 0.76, size = 354, normalized size = 2.64

$$\frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(c+dx)+1}\,\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(5/2)), x)
```

3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2e\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{5d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 6/5*a*e*(e*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/5*3^(3/4
)*a^2*e*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+
c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3)
)*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d
*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+
c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c
))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3806, 50, 63, 218}

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2e\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{5d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
+ (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*e*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3
) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3)
)], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1
/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*d*(a - a*Sec[c + d*x])*Sqrt[
a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt[3]{ex}}{\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2a^2e^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{}{(ex}\right.}{5d\sqrt{a-a\sec(c+dx)}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(6a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{5d\sqrt{a-a\sec(c+dx)}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2eF\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{}$$

Mathematica [C] time = 0.28, size = 71, normalized size = 0.22

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)}{d\sec^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(4/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(4/3))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}e\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)*e*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(dx+c) + a}\, (e \sec(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int (e \sec(dx+c))^{\frac{4}{3}} \sqrt{a + a \sec(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(dx+c) + a}\, (e \sec(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\cos(c + d\,x)}} \left(\frac{e}{\cos(c + d\,x)}\right)^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(4/3)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.273 $\int \sqrt[3]{e \sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\cdots}}\right.\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 2*3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*se
c(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))
^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e
*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*ta
n(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec
(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3806, 63, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\cdots}}\right.\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))],
-7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*
(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a + a*
Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
```

```
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2 e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\right.}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+a}}$$

Mathematica [C] time = 0.16, size = 71, normalized size = 0.25

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\sqrt[3]{e\sec(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};1-\sec(c+dx)\right)}{d\sqrt[3]{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(1/3))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

maple [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{\frac{1}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec (dx+c)+a}\,(e \sec (dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos (c+d\,x)}}\,{\left(\frac{e}{\cos (c+d\,x)}\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sec (c+dx)+1)}\,\sqrt[3]{e \sec (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3), x)
```

3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=326

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}\right.\right.}{2de(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 3/2*a*tan(d*x+c)/d/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2)+1/2*3^(3/4)*
a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^
(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1
/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c
))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d
/e/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))
^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3806, 51, 63, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}\right.\right.}{2de(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(2*d*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]) +
(3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e
*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7
 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e
*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = -\frac{\left(a^2 e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{5/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,dx \ldots\right.}{4d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec \ldots}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x \ldots\right.}{4de\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec \ldots}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\,\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\,\ldots}}\ldots\right.\right.}{2de(a-\ldots}$$

Mathematica [C] time = 0.17, size = 71, normalized size = 0.22

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{2}{3}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\;{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{3}{2};1-\sec(c+dx)\right)}{d(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(2/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(2/3))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}}{e\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)/(e*sec(d*x + c)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\sec(dx+c)}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="maxima"
)
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3),x)
[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(c+dx)+1)}}{(e\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(2/3),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(2/3), x)
```

3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=716

$$-\frac{80\sqrt{2}\,3^{3/4}a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}\right.\right.}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 60/91*a*e^2*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+6/13*a
*e*(e*sec(d*x+c))^(5/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-240/91*a*e^3*ta
n(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/
2)-80/91*3^(3/4)*a^2*e^(7/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^
(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)
-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*se
c(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d
*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*
x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+120/91*3^
(1/4)*a^2*e^(7/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(
e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x
+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3
)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e
*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 50, 63, 303, 218, 1877}

$$-\frac{80\sqrt{2}\,3^{3/4}a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}\right.\right.}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (60*a*e^2*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x
]]) + (6*a*e*(e*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(13*d*Sqrt[a + a*Sec[c +
d*x]]) - (240*a*e^3*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))) + (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^
2*e^(7/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3
) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3)
+ (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2]*Tan[c + d*x])/(91*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]) - (80*Sqrt[2]*3^(3/4)*a^2*e^(7/3)*EllipticF[ArcSin[((
1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*S
qrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(91*d*(a - a
*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\mathrm{Subst}\left(\int\frac{(ex)^{5/3}}{\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{\left(10a^2e^2\tan(c+dx)\right)\mathrm{Subst}\left(\int\right.}{13d\sqrt{a-a\sec(c+dx)}\,\cdot}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+}{13d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+}{13d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+}{13d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+}{13d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.26, size = 71, normalized size = 0.10

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{8/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)}{d\sec^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-5/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(8/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(8/3))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}e^2\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e^2*sec(d*x + c)^2
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int (e \sec (dx+c))^{\frac{8}{3}} \sqrt{a+a \sec (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec (dx+c)+a}\, (e \sec (dx+c))^{\frac{8}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos (c+d\,x)}}\, {\left(\frac{e}{\cos (c+d\,x)}\right)}^{8/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(8/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=673

$$-\frac{8\sqrt{2}\,3^{3/4}a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\Bigg(\sin^{-1}\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 6/7*a*e*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-24/7*a*e^2
*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^
(1/2)-8/7*3^(3/4)*a^2*e^(4/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3
^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*s
ec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(
d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d
*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+12/7*3^(
1/4)*a^2*e^(4/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e
*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+
c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)
+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*
sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3806, 50, 63, 303, 218, 1877}

$$-\frac{8\sqrt{2}\,3^{3/4}a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\Bigg(\sin^{-1}\Bigg(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
- (24*a*e^2*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1
/3) - (e*Sec[c + d*x])^(1/3))) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(4/3)*
EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c
 + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c +
 d*x])/(7*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) - (8*Sqrt[2]*3^(3/4)*a^2*e^(4/3)*EllipticF[ArcSin[((1 - Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x]
)^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*
e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{(ex)^{2/3}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{\left(4a^2e^2\tan(c+dx)\right)\text{Subst}\left(\int \cdots\right.}{7d\sqrt{a-a\sec(c+dx)}\cdots}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{\left(12a^2e\tan(c+dx)\right)\text{Subst}\left(\int \cdots\right.}{7d\sqrt{a-a\sec(c+dx)}\cdots}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(12a^2e\tan(c+dx)\right)\text{Subst}\left(\int \cdots\right.}{7d\sqrt{a-a\sec(c+\cdots}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{24ae^2\tan(\cdots}{7d\sqrt{a+a\sec(c+dx)}\,\left(\left(1+\sqrt{\cdots}\right.\right.}$$

Mathematica [C] time = 0.25, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)}{d\sec^{\frac{5}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*Hypergeometric2F1[-2/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(5/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(5/3))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,\left(e\sec(dx+c)\right)^{\frac{2}{3}}e\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\left(e\sec(dx+c)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{\frac{5}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=624

$$-\frac{2\sqrt{2}\,3^{3/4}a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}\right.\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] -6*a*e*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*
x+c))^(1/2)-2*3^(3/4)*a^2*e^(1/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*
(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^
(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+
(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*
tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*s
ec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+3*3^
(1/4)*a^2*e^(1/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(
e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x
+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3
)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e
*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3806, 63, 303, 218, 1877}

$$-\frac{2\sqrt{2}\,3^{3/4}a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}}\right.\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-6*a*e*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(1/3)*Ellipti
cE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x]
)^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/
(d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (
e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]
) - (2*Sqrt[2]*3^(3/4)*a^2*e^(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3)
- (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3
)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{ex}\,\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int\frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{\left(3\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\,\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)} + \frac{3\sqrt[4]{3}\,\sqrt{2-\cdots}}{\cdots}$$

Mathematica [C] time = 0.16, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};1-\sec(c+dx)\right)}{d\sec^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (2*Hypergeometric2F1[1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(2/3))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{a\sec\left(dx+c\right)+a}\,\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(dx+c\right)+a}\,\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

maple [F] time = 1.25, size = 0, normalized size = 0.00

$$\int\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sqrt{a+a\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(dx+c\right)+a}\,\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3), x)
```

3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=662

$$\frac{\sqrt{2}\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\,\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 3*a*tan(d*x+c)/d/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+3*a*tan(d*x+c)
/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+3^(3/
4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c
))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))
*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*
sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(2/3)/(a-a*s
ec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(
-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*a^2*Ellipti
cE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)-
1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(
-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(2/3)/(a
-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3
))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{\sqrt{2}\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\,\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]) + (3
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE[ArcSin[((1
 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sq
rt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
+ Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e^(2/3)*
(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*S
ec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]) +
 (Sqrt[2]*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c +
d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt
[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c +
d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(d*e^(2/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec
[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = -\frac{(a^2e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{(ex)^{4/3}\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{(a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\sqrt{a-ax}}\,dx,x,\right.}{2d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{(3a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\right.}{2de\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{(3a^2\tan(c+dx))\,\text{Subst}\left(\int \frac{(1-\sqrt{3})\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,\right.}{2de\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\,\mathrm{s}}\right.}$$

Mathematica [C] time = 0.16, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt[3]{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\sec(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(1/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(1/3))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}}{e\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e*sec(d*x + c)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="giac")
```

[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a\sec\left(dx + c\right)}}{\left(e\sec\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)

[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec\left(dx + c\right) + a}}{\left(e\sec\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="maxima"
)

[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\cos(c+d\,x)}}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3),x)

[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c + dx\right) + 1\right)}}{\sqrt[3]{e\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/3),x)

[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(1/3), x)

3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=715

$$\frac{5\, 3^{3/4} a^2 \tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|\right.}{4\sqrt{2}\,de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] 3/4*a*tan(d*x+c)/d/(e*sec(d*x+c))^(4/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d
*x+c)/d/e/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d*x+c)/d/e
/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+5/8*3^(
3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x
+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3
))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*
x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(5/3)/(a-a*sec(d*x
+c))*2^(1/2)/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))
/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-15/16*3^(1/4)*a^2*Ell
ipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/2*6^(1
/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3
))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(5/3
)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^
(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{5\, 3^{3/4} a^2 \tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|\right.}{4\sqrt{2}\,de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(4*d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]]) +
(15*a*Tan[c + d*x])/(8*d*e*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
 + (15*a*Tan[c + d*x])/(8*d*e*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/
3) - (e*Sec[c + d*x])^(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x]
)^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^
(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(1
6*d*e^(5/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) + (5*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Se
c[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*Sqrt[2]*d*e^(5/3)*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{7/3}\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(5a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{4/3}\sqrt{a-ax}}\,c\right.}{8d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\,\text{se}}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} -$$

Mathematica [C] time = 0.20, size = 71, normalized size = 0.10

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{4}{3}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\;{}_2F_1\left(\frac{1}{2},\frac{7}{3};\frac{3}{2};1-\sec(c+dx)\right)}{d(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 7/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(4/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(4/3))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}}{e^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e^2*sec(d*x + c)^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

maple [F] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a\sec\left(dx + c\right)}}{\left(e\sec\left(dx + c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec\left(dx + c\right) + a}}{\left(e\sec\left(dx + c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \left(\sec{\left(c + dx \right)} + 1\right)}}{\left(e \sec{\left(c + dx \right)}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(4/3), x)
```

3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\tan(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -3/2*AppellF1(2/3,1,1/2,5/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(2/3)*ta
n(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3828, 3827, 130, 510}

$$-\frac{3\tan(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[2/3, 1/2, 1, 5/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt[3]{ex}\,(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(3\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{3F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 9.25, size = 750, normalized size = 9.62

$$\overline{ad\left(270\cos^2\left(\frac{1}{2}(c+dx)\right)(2\cos(c+dx)+1)F_1\left(\frac{1}{2};\frac{1}{6},\frac{1}{3};\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2+10\cos(c+dx\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (90*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[c + d*x]^2*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*
x)/2]*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2))/(a*d*(270*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^2*(1 + 2*Cos[c + d*x]) + 3*
AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*(-10*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) +
5*AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos
[(c + d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) - 6*(16*AppellF1[5/
2, 1/6, 7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*AppellF1[5/2
, 7/6, 4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 7*AppellF1[5/2,
 13/6, 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sin
[(c + d*x)/2]^2)*Tan[(c + d*x)/2]^2 + 10*(-2*AppellF1[3/2, 1/6, 4/3, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Tan[(c + d*x)/2]^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sec \left(dx+c\right)\right)^{\frac{2}{3}}}{\sqrt{a+a \sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sec \left(dx+c\right)\right)^{\frac{2}{3}}}{\sqrt{a \sec \left(dx+c\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{2/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sec \left(c+dx\right)\right)^{\frac{2}{3}}}{\sqrt{a \left(\sec \left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.281 $\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+a \sec(c+dx)}} \, dx$

Optimal. Leaf size=76

$$-\frac{3 \tan(c+dx) F_1\left(\frac{1}{3}; \frac{1}{2}, 1; \frac{4}{3}; \sec(c+dx), -\sec(c+dx)\right) \sqrt[3]{e \sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -3*AppellF1(1/3,1,1/2,4/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(1/3)*tan(
d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 429}

$$-\frac{3 \tan(c+dx) F_1\left(\frac{1}{3}; \frac{1}{2}, 1; \frac{4}{3}; \sec(c+dx), -\sec(c+dx)\right) \sqrt[3]{e \sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[1/3, 1/2, 1, 4/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{2/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(3\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 8.12, size = 749, normalized size = 9.86

$$\frac{}{d\sqrt{a(\sec(c+dx)+1)}\,(e\sec(c+dx))^{2/3}\left(4320(4\cos(c+dx)-1)\cos^6\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{1}{2};-\frac{1}{6},\frac{2}{3};\frac{3}{2};\tan^2\left(\frac{1}{2}(c+d\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (720*e*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x])^2*Sin[(c + d*x)/2]*(9*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - (4*AppellF1[3/2,
 -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5
/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
)/(d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])]*(4320*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]
^6*(-1 + 4*Cos[c + d*x]) + 160*(4*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^4 + 12*AppellF1
[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x
)/2]^2*(20*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(7 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) + 5*
AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 +
14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) - 24*(40*AppellF
1[5/2, -1/6, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 8*AppellF
1[5/2, 5/6, 5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1
[5/2, 11/6, 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x
]*Sin[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(e \sec (dx+c))^{\frac{1}{3}}}{\sqrt{a+a \sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sec (dx+c))^{\frac{1}{3}}}{\sqrt{a \sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{e \sec (c+dx)}}{\sqrt{a (\sec (c+dx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{3\tan(c+dx)F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}\,\sqrt[3]{e\sec(c+dx)}}$$

```
[Out] 3*AppellF1(-1/3,1,1/2,2/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*x+c
))^(1/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 510}

$$\frac{3\tan(c+dx)F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}\,\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-1/3, 1/2, 1, 2/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{4/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(3\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 20.59, size = 3346, normalized size = 44.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -(((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c +
d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*
(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/
3)))/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Sec[(c + d*x)/
2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]^2*((2*AppellF
1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x
)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2,
1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*
x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3))) - (Sec[(c + d*x)/
2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*(-1 + Tan[(c + d*x)/2]^2)*((2*
AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(
c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF
1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[
(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))/2 - (Cos[(
c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2
)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/
6) + (2*Tan[(c + d*x)/2]^2*(-1/5*(AppellF1[5/2, 1/6, 4/3, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (Appell
F1[5/2, 7/6, 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/10))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) - (5
*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[
(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
```

```
d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(11/6)) -
 (Tan[(c + d*x)/2]*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/
3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[
(c + d*x)/2]^2)^(1/3) + (3*((-3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[
(c + d*x)/2]^2)^2*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) + (3*(-1/9*(AppellF1[3/2, 1/6, 4/3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]) + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18))/((-1 + Tan[(c + d*x)/2]^2)*(9*
AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2
*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
 + d*x)/2]^2)) - (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*((-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-1/9*(AppellF1[3/2, 1
/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]) + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18) + Tan[(c + d*x)/2]^
2*(-1/5*(AppellF1[5/2, 7/6, 4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (7*AppellF1[5/2, 13/6, 1/3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/10 - 2*((-4*AppellF1[5/2, 1/6, 7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 7/6, 4/3
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/10))))/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2)))/(Sec[(c +
 d*x)/2]^2)^(1/3)) - (Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*((2*Appell
F1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*
x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2,
 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d
*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3))*(-(Cos[(c + d*x)/
2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c +
 d*x]))/(6*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")

[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{a+a\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\,\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{3\tan(c+dx)F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\,(e\sec(c+dx))^{2/3}}$$

```
[Out] 3/2*AppellF1(-2/3,1,1/2,1/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*x
+c))^(2/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3828, 3827, 130, 510}

$$\frac{3\tan(c+dx)F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\,(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-2/3, 1/2, 1, 1/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
2*d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(ex)^{5/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$
$$= -\frac{(3\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{3F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\,(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 7.47, size = 585, normalized size = 7.50

$$\sec^{\frac{7}{6}}(c+dx)\left(\frac{5\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\cos(c+dx)+1}}\,(3\cos(c+dx)-1)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{5/}}{5\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)\left(3-4\sqrt{2}\left(\frac{1}{\cos(c+dx)+1}\right)^{2/3}\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)^{5/6}\tan^4\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{5}{2};\frac{11}{6},\frac{2}{3};\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)-120\sin}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sec[c + d*x]^(7/6)*((-3*Cos[(c + d*x)/2]*Sec[c + d*x]^(5/6)*(Sin[(c + d*x)
/2] - Sin[(3*(c + d*x))/2]))/2 + (5*Sqrt[(1 + Cos[c + d*x])^(-1)]*(-1 + 3*C
os[c + d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)*Sin[(c + d*x)/2]*(-3*C
os[c + d*x]^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, 2*Sin[(c + d*x)/2]^2]*(S
ec[(c + d*x)/2]^2)^(1/3) + 2*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6)*Tan[(c + d*
x)/2]^2))/(-120*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x])
)^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2] + 32*AppellF1[5/2, 5/6, 5/3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(
Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2]^3
+ 5*Sqrt[2]*Cos[(c + d*x)/2]*(3 - 4*Sqrt[2]*AppellF1[5/2, 11/6, 2/3, 7/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Co
s[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Tan[(c + d*x)/2]^4))))/(d*(e*Sec[c + d
*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\sqrt{a+a\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\,\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)),x)
```

```
[Out] int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3)), x)
```

3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

```
[Out] 2^(5/6)*AppellF1(1/2,-1/3,1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*sec
(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3825, 133}

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])
/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \frac{\sqrt[3]{a+a\sec(c+dx)}\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{1+\sec(c+dx)}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$

$$= \frac{\left(\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{\sqrt[6]{2-x}\sqrt{x}}\,dx, x, 1-\sec(c+\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{5/6}}$$

$$= \frac{2^{5/6}F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+}}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [B] time = 15.01, size = 1982, normalized size = 25.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c
 + d*x]))^(1/3)*Sin[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/3)) + (3*(a*(1 + S
ec[c + d*x]))^(1/3)*(-((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(2/3)) + (Sec[
c + d*x]^(1/3)*(1 + Sec[c + d*x])^(1/3))/2)*Tan[(c + d*x)/2]*(-1 + (3*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*Ap
pellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hype
rgeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))
/(2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1
/3)*(1 + Sec[c + d*x])^(1/3)*((3*(Sec[(c + d*x)/2]^2)^(1/3)*(-1 + (3*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*App
ellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hyper
geometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))/
(2*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)) - (2^(1/3)*Tan[(c + d*x
)/2]^2*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*
Tan[(c + d*x)/2]^2)))/((Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^(1/3)) + (3*Tan[(c + d*x)/2]*((3*((-2*AppellF1[3/2, -1/3, 5/3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2) - (3*AppellF1[1/2, -1/3, 2/3, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-2*(2*AppellF1[3/2, -1/3, 5/3, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4},
 {7/4}, Tan[(c + d*x)/2]^4])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*((-2*A
ppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) - 2*Tan[(c + d*x
)/2]^2*(2*(-(AppellF1[5/2, -1/3, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) - (AppellF1[5/2, 2/3, 5/3, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/5) + (3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-HypergeometricPFQ[{2/3, 3
/4}, {7/4}, Tan[(c + d*x)/2]^4] + (1 - Tan[(c + d*x)/2]^4)^(-2/3)))/2)))/(9
*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
```

```
*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2
]^2)^2))/(2^(2/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(1/3)) - (Tan[(c + d*x)/2]*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4
}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*
x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(2^(2
/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(4/3))))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

maple [F] time = 1.41, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)(a+a\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{1/3}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

```
[Out] 2*2^(1/6)*AppellF1(1/2,-1/3,-1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3828, 3825, 133}

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{4}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}\,\sqrt[6]{2-x}}{\sqrt{x}}\,dx, x, 1-\text{se}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$
$$= \frac{2\sqrt[6]{2}\,F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec}{d(1+\sec(c+dx))^{7/6}}$$

Mathematica [C] time = 20.39, size = 2618, normalized size = 33.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c +
 d*x]))^(2/3)*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^(2/3)) + (15*AppellF1
[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(a*(1 + Sec[c
 + d*x]))^(2/3)*(9*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
+ d*x)/4]^2])*Tan[(c + d*x)/4]^2)*Tan[(c + d*x)/2])/(d*(Sec[c + d*x]/(1 + S
ec[c + d*x]))^(2/3)*(1 + Sec[c + d*x])^(2/3)*((135*AppellF1[1/2, 2/3, 1/3,
3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]^2*(5 + Cos[c + d*x]))/2 + 20*
(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*
AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])^2*Co
s[(c + d*x)/2]*Tan[(c + d*x)/4]^4 - 3*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c +
 d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Tan[(c + d*x)/4]^2*(5*AppellF1[3/2, 2/3, 4
/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2]
+ Cos[c + d*x]) - 10*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[
(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2] + Cos[c + d*x]) + 24*(2*AppellF1[5
/2, 2/3, 7/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*AppellF1[5/
2, 5/3, 4/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 5*AppellF1[5/2
, 8/3, 1/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Cos[(c + d*x)/2]
*Tan[(c + d*x)/4]^2))) - ((Sec[(c + d*x)/2]^2)^(4/3)*Sec[c + d*x]^(1/3)*(a*
(1 + Sec[c + d*x]))^(2/3)*(AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + T
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2
])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*
x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)
/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3))))
/(2^(1/3)*d*(-(((Sec[(c + d*x)/2]^2)^(1/3)*Tan[(c + d*x)/2]*(AppellF1[-2/3,
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + T
an[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/
3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((
-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2]
)/(1 + Tan[(c + d*x)/2]))^(1/3))))/2^(1/3)) - (3*(Sec[(c + d*x)/2]^2)^(1/3)
*((((1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (-1 - I)/(-1 + Tan[(c + d*x)/
2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + Tan[(c + d
*x)/2])^2 + ((1/3 + I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (-1 - I)/(-1 + Tan[(
c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + T
an[(c + d*x)/2])^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3
)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3,
```

```
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)
/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1
 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*(Sec[(c + d*x)/2]^2
/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/
(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)) - (
((-1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
 (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])
^2 - ((1/3 - I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/
2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2])^2)/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*(-1/2*(
Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[
(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])/(1
+ Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^
(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (
1 + I)/(1 + Tan[(c + d*x)/2])]*(-1/2*(Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)
/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2
]))))/(3*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(4/3))))/2^(1/3)))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx+c)+a)^{\frac{2}{3}}\sec (dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)(a+a\sec (dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx+c)+a)^{\frac{2}{3}}\sec (dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{2/3}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(2/3),x)
[Out] Timed out
```

3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=327

$$\frac{\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}}{8d\sqrt[3]{\frac{1}{\cos(c+dx)+1}}\,(\sec(c+dx)+1)^{4/3}}-\frac{5\tan^3(c+dx)(\cdots}{\cdots}$$

```
[Out] -3/2*a*sec(d*x+c)^(5/3)*sin(d*x+c)/d/(a*(1+sec(d*x+c)))^(1/3)+9/4*sec(d*x+c
)^(2/3)*(a*(1+sec(d*x+c)))^(2/3)*sin(d*x+c)/d-9/4*(a*(1+sec(d*x+c)))^(2/3)*
tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(7/3)+1/8*hypergeom([1
/4, 1/3],[5/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)*sec(1/2*d*x+1/2*c)^4)^(1/3
)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x
+c))^(4/3)-5/8*hypergeom([1/3, 3/4],[7/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)
*sec(1/2*d*x+1/2*c)^4)^(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)^3/d/(1/(1+
cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(10/3)
```

Rubi [C] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 0.24, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3825, 133}

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -2/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{5}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{2/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx, x, 1-\text{s}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7/6}}$$

$$= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec}{d(1+\sec(c+dx))^{7/6}}$$

Mathematica [A] time = 8.33, size = 274, normalized size = 0.84

$$\frac{(a(\sec(c+dx)+1))^{2/3}\left(\sqrt[3]{2}\tan\left(\frac{1}{2}(c+dx)\right){}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt[3]{\text{co}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(-3*Sec[(c + d*x)/2]^3*Sec[c + d*x]*(1 + Sec[
c + d*x])^(1/3)*(Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]) + 2^(1/3)*Hyper
geometric2F1[1/4, 1/3, 5/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d*x)/2] - 5*2
^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*S
ec[(c + d*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d
*x)/2]^3))/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(2/3))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}(dx+c)\right)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+a*sec(d*x+c))**(2/3),x)
[Out] Timed out
```

3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2\,2^{5/6}a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

```
[Out] 2*2^(5/6)*a*AppellF1(1/2,4/3,-5/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a
*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3825, 133}

$$\frac{2\,2^{5/6}a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6
))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\right)\int \frac{(1+\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$
$$= \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^{4/3}\sqrt{x}}\,dx, x, 1-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{5/6}}$$
$$= \frac{2\,2^{5/6}aF_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\,\tan(c}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [C] time = 15.18, size = 2325, normalized size = 29.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (-3*(a*(1 + Sec[c + d*x]))^(4/3)*((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(1/
3) + Sec[c + d*x]^(2/3)*(1 + Sec[c + d*x])^(1/3))*(-8*Tan[(c + d*x)/2] + (A
ppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/
(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))
^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
(1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)
/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[
(c + d*x)/2])^2))/(4*2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(1/3)*(1 + Sec[c + d*x]
)^(4/3)*((Tan[(c + d*x)/2]*(-8*Tan[(c + d*x)/2] + (AppellF1[-4/3, -2/3, -2/
3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])
]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2
/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3
, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d
*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2))/(4*2^
(2/3)*(Sec[(c + d*x)/2]^2)^(1/3)) - (3*(-4*Sec[(c + d*x)/2]^2 + (Sec[(c + d
*x)/2]^2*(((-4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (-1 - I)/(-1 + T
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1
 + Tan[(c + d*x)/2])^2 - ((4/3 + (4*I)/3)*AppellF1[-1/3, 1/3, -2/3, 2/3, (-
1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c +
d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2))/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[
(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)
) + (AppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1
 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(((-I +
 Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(
-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(
1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*(
(-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2
])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]) - (2*AppellF1[-4/3,
 -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c
+ d*x)/2])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2
])) - (Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2
])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)*((I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - (2*AppellF1[-4/3, -2/3, -
2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2
])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (S
ec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(
3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*
```

```
x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)) - ((-I + Tan[(c + d*x)/2])/(1 + Tan[
(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*
(1 + Tan[(c + d*x)/2])^2*(((4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (
1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x
)/2]^2)/(1 + Tan[(c + d*x)/2])^2 + ((4/3 - (4*I)/3)*AppellF1[-1/3, 1/3, -2/
3, 2/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])^2) - (AppellF1[-4/3, -2/3, -2/3, -1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((I + T
an[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-1
/2*(Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 +
Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])
/(1 + Tan[(c + d*x)/2]))^(2/3)) - (AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)
/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*
x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-1/2*(Sec[(c
 + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/(1 + Tan[(c + d*x)/2])^2 + Sec[(c + d*
x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(2/3))))/(4*2^(2/3)*(Sec[(c + d*x)/2]^2)^(1/3))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sec(dx + c))^{\frac{4}{3}}}{\sec(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
[Out] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx + c) + a)^{\frac{4}{3}}}{\sec(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{4/3}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)/sec(d*x+c)**(1/3),x)
[Out] Timed out
```

3.288 $\int \sec^n(e + fx)(a + a\sec(e + fx))^4\, dx$

Optimal. Leaf size=304

$$-\frac{a^4\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)(n+3)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(2n+3)\sin(e+fx)\sec^n(e}{fn(n+2}$$

```
[Out] a^4*(4*n^2+21*n+30)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(n^3+6*n^2+11*n+6)+sec(f*
x+e)^(1+n)*(a^2+a^2*sec(f*x+e))^2*sin(f*x+e)/f/(3+n)+2*(4+n)*sec(f*x+e)^(1+
n)*(a^4+a^4*sec(f*x+e))*sin(f*x+e)/f/(2+n)/(3+n)-a^4*(8*n^2+24*n+3)*hyperge
om([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/
f/(3+n)/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+4*a^4*(3+2*n)*hypergeom([1/2, -1/2*n]
,[1-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(
1/2)
```

Rubi [A] time = 0.48, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3814, 4018, 3997, 3787, 3772, 2643}

$$-\frac{a^4\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)(n+3)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(2n+3)\sin(e+fx)\sec^n(e}{fn(n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

```
[Out] (a^4*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 +
 n)*(3 + n)) + (Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x
])/(f*(3 + n)) + (2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^4 + a^4*Sec[e + f*x])*S
in[e + f*x])/(f*(2 + n)*(3 + n)) - (a^4*(3 + 24*n + 8*n^2)*Hypergeometric2F
1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e +
f*x])/(f*(1 - n)*(1 + n)*(3 + n)*Sqrt[Sin[e + f*x]^2]) + (4*a^4*(3 + 2*n)*H
ypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e
 + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{a\int \sec^n(e+fx)(a\cdots}{\cdots}$$

$$= \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{2(4+n)\sec^{1+n}(e+\cdots}{\cdots}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+\cdots\right.}{f\cdots}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+\cdots\right.}{f\cdots}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+\cdots\right.}{f\cdots}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+\cdots\right.}{f\cdots}$$

Mathematica [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
[Out] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4 \sec\left(fx+e\right)^4 + 4\,a^4 \sec\left(fx+e\right)^3 + 6\,a^4 \sec\left(fx+e\right)^2 + 4\,a^4 \sec\left(fx+e\right) + a^4\right) \sec\left(fx+e\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(f*x + e)^4 + 4*a^4*sec(f*x + e)^3 + 6*a^4*sec(f*x + e)^2
+ 4*a^4*sec(f*x + e) + a^4)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sec\left(fx+e\right) + a\right)^4 \sec\left(fx+e\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

maple [F] time = 2.92, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right) \left(a + a \sec\left(fx+e\right)\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sec\left(fx+e\right) + a\right)^4 \sec\left(fx+e\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^4\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4 \left(\int 4 \sec\left(e + fx\right) \sec^n\left(e + fx\right) dx + \int 6 \sec^2\left(e + fx\right) \sec^n\left(e + fx\right) dx + \int 4 \sec^3\left(e + fx\right) \sec^n\left(e + fx\right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**4,x)
```

```
[Out] a**4*(Integral(4*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(6*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(4*sec(e + f*x)**3*sec(e + f*x)**n, x) +
Integral(sec(e + f*x)**4*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=230

$$-\frac{a^3(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4n+7)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{fn(n+2)\sqrt{\sin^2(e+\right.}}$$

```
[Out] a^3*(5+2*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)/(2+n)+sec(f*x+e)^(1+n)*(a^3
+a^3*sec(f*x+e))*sin(f*x+e)/f/(2+n)-a^3*(1+4*n)*hypergeom([1/2, 1/2-1/2*n],
[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+
e)^2)^(1/2)+a^3*(7+4*n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec
(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3814, 3997, 3787, 3772, 2643}

$$-\frac{a^3(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4n+7)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{fn(n+2)\sqrt{\sin^2(e+\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 + n)) + (Se
c[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) - (a^
3*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Se
c[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (a^3
*(7 + 4*n)*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^n*Sin[e + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)} + \frac{a\int \sec^n(e+fx)(\ldots}{\ldots}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(\ldots\right.}{f(2+n)\ldots}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(\ldots\right.}{f(2+n)\ldots}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(\ldots\right.}{f(2+n)\ldots}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(\ldots\right.}{f(2+n)\ldots}$$

Mathematica [C] time = 1.54, size = 286, normalized size = 1.24

$$-\frac{ia^3 2^{n-3}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\left(\frac{8e^{3i(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{n+5}{2};-e^{2i(e+fx)}\right)}{(n+3)\left(1+e^{2i(e+fx)}\right)^2}+\frac{6e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{n+3}{2}\ldots\right)}{n+1}\ldots\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((-I)*2^(-3 + n)*a^3*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + Cos
[e + f*x])^3*((8*E^((3*I)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (5 +
n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))^2*(3 + n)) + (6*E^(
I*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (3 + n)/2, -E^((2*I)*(e + f*x)
)])/(1 + n) + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, 1 - n/2, (2 +
 n)/2, -E^((2*I)*(e + f*x))])/n + (12*E^((2*I)*(e + f*x))*Hypergeometric2F1
[1, -1/2*n, (4 + n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))*(2
 + n)))*Sec[(e + f*x)/2]^6)/f
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3\right)\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

maple [F] time = 7.98, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int 3\sec^{2}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int \sec^{3}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(3*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**3*sec(e + f*x)**n, x) + In
tegral(sec(e + f*x)**n, x))
```

3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=172

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\right.}{fn\sqrt{\sin^2(e+fx)}}$$

```
[Out] a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)-a^2*(1+2*n)*hypergeom([1/2, 1/2-1/2
*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(
f*x+e)^2)^(1/2)+2*a^2*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f
*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3788, 3772, 2643, 4046}

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\right.}{fn\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyperg
eometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n
)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hypergeometric2
F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*n*
Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \left(2a^2\right)\int \sec^{1+n}(e+fx)\,dx + \int \sec^n(e+fx)\left(a^2+a^2\sec^2(e+fx)\right)\,dx$$
$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} + \frac{\left(a^2(1+2n)\right)\int \sec^n(e+fx)\,dx}{1+n} + \left(2a^2 c\right.$$
$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} + \frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)\sec^n(}{fn\sqrt{\sin^2(e+fx)}}$$
$$= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} - \frac{a^2(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx\right.}{f\left(1-n^2\right)\sqrt{\sin^2(}}$$

Mathematica [C] time = 1.09, size = 222, normalized size = 1.29

$$-\frac{ia^2 2^{n-2}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\left(\frac{4e^{i(e+fx)}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{n+3}{2};-e^{2i(e+fx)}\right)}{n+1}+\frac{\left(1+e^{2i(e+fx)}\right){}_2F_1\left(1,1-\frac{n}{2};\frac{n+2}{2};-e^{2}\right.}{n}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + n)*a^2*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + Cos
[e + f*x])^2*((4*E^(I*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (3 + n)/2,
 -E^((2*I)*(e + f*x))])/(1 + n) + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric
2F1[1, 1 - n/2, (2 + n)/2, -E^((2*I)*(e + f*x))])/n + (4*E^((2*I)*(e + f*x)
)*Hypergeometric2F1[1, -1/2*n, (4 + n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((
2*I)*(e + f*x)))*(2 + n)))*Sec[(e + f*x)/2]^4)/f
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^2\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

maple [F] time = 5.17, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^2 \sec\left(fx+e\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2 \left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sec\left(e+fx\right)\sec^n\left(e+fx\right)dx+\int \sec^2\left(e+fx\right)\sec^n\left(e+fx\right)dx+\int \sec^n\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**2,x)
[Out] a**2*(Integral(2*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**
2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x))
```

3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*s
in(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+a*hypergeom([1/2, -1/2*n],[1-1/2*n],
cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3787, 3772, 2643}

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f
*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeom
etric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])
/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx &= a\int \sec^n(e+fx)\,dx+a\int \sec^{1+n}(e+fx)\,dx\\ &= \left(a\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-1-n}(e+fx)\,dx+\left(a\cos^n(e+fx)\sec\right.\\ &= -\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}+\frac{a\,{}_2F_1\left(\frac{1}{2}\right.}{}\end{aligned}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 0.80

$$\frac{a\sqrt{-\tan^2(e+fx)}\csc(e+fx)\sec^{n-1}(e+fx)\left((n+1)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e+fx)\right)+n\sec(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2}\right.\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
[Out] (a*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + n)*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Sec[e + f*x]^2] + n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sec[e + f*x]^2]*Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

maple [F] time = 2.50, size = 0, normalized size = 0.00

$$\int\left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int \sec^{n}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=174

$$\frac{(1-n)\sin(e+fx)\sec^{n-2}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};c\right.}{af\sqrt{\sin^2(e+fx)}}$$

```
[Out] sec(f*x+e)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))+(1-n)*hypergeom([1/2, 1-1/2*n],[
2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-2+n)*sin(f*x+e)/a/f/(2-n)/(sin(f*x+e)^2
)^(1/2)-hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1
+n)*sin(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3820, 3787, 3772, 2643}

$$\frac{(1-n)\sin(e+fx)\sec^{n-2}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};c\right.}{af\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) + ((1 - n)*Hypergeom
etric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-2 + n)*S
in[e + f*x])/(a*f*(2 - n)*Sqrt[Sin[e + f*x]^2]) - (Hypergeometric2F1[1/2, (
1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(a
*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(1-n)\int \sec^{-1+n}(e+fx)(a-a\sec(e+fx))\,dx}{a^2}$$

$$= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{(1-n)\int \sec^{-1+n}(e+fx)\,dx}{a} + \frac{(1-n)\int \sec^n(e+fx)\,dx}{a}$$

$$= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left((1-n)\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{1-n}(e+fx)\,dx}{a} + \cdots$$

$$= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{(1-n)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)\sec^{-2+n}(e+fx)\sin(e \cdots}{af(2-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]), x]
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(fx+e\right)^n}{a\sec\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^n}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

maple [F] time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\sec^n\left(fx+e\right)}{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{a+\frac{a}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{n}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e)),x)
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1), x)/a
```

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{2(2-n)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2}\right.}{3a^2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] -2/3*(2-n)*sec(f*x+e)^(1+n)*sin(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*sec(f*x+e)^
(1+n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^2-1/3*(3-2*n)*hypergeom([1/2, 1/2-1/2*n
],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/a^2/f/(sin(f*x+e)^
2)^(1/2)+2/3*(2-n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f*x+
e)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3817, 4020, 3787, 3772, 2643}

$$-\frac{(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{2(2-n)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2}\right.}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
 - (Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - ((3 -
 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) + (2*(2 - n)*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
 f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \frac{\sec^n(e+fx)(a(-3+n)-a(-1+n)\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \sec^n(e+ \ldots}{}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{((3-2n)(1 \ldots}{}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left((3-2n)(1 \ldots\right.}{}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3-2n)\,{}_2F \ldots}{}$$

Mathematica [F] time = 10.14, size = 0, normalized size = 0.00

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2, x]
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^n}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{n}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2
```

3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{2\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\,\sec^{n+1}(e+f}{f(2n+3)}$$

```
[Out] 2*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(1+sec(f*x+e))^(1/2)+
2*sec(f*x+e)^(1+n)*sin(f*x+e)*(1+sec(f*x+e))^(1/2)/f/(3+2*n)+2*(16*n^2+24*n
+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(4*n^2+8*n+3)/(1+
sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3814, 4016, 3806, 65}

$$\frac{2\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\,\sec^{n+1}(e+f}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt
[1 + Sec[e + f*x]]) + (2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e
+ f*x])/(f*(3 + 2*n)) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n
, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[1 + Sec[
e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)} + \frac{2\int \sec^n(e+fx)\sqrt{1+s}}{}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+f}}{f(3+2n)}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+f}}{f(3+2n)}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+f}}{f(3+2n)}$$

Mathematica [C] time = 58.40, size = 398, normalized size = 2.46

$$-\frac{i2^{n-\frac{5}{2}}e^{-\frac{1}{2}i(2n+3)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{3}{2}}\sec^5\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^{5/2}\left(\frac{10e^{i(n+2)(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{n+4}{2};-e^{2i(e+fx)}\right)}{n+2}+\frac{5}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*((
10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E^(
(2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F1[1
, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f*x))
*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n + (5*E^
(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((2*I)*(
e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1, 1 - n/2
, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e + f*x))*H
ypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[
(e + f*x)/2]^5*(1 + Sec[e + f*x])^(5/2))/(E^((I/2)*(3 + 2*n)*(e + f*x))*f*S
ec[e + f*x]^(5/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sec\left(fx+e\right)^2+2\,\sec\left(fx+e\right)+1\right)\sec\left(fx+e\right)^n\,\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((sec(f*x + e)^2 + 2*sec(f*x + e) + 1)*sec(f*x + e)^n*sqrt(sec(f*x
+ e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(1+\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^{5/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] 2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(1+4*n)*hype
rgeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1
/2)
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3814, 21, 3806, 65}

$$\frac{2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
+ (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e +
 f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{\sec^n(e+fx)\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}dx}{1+2n}$$
$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int \sec^n(e+fx)\sqrt{1+\sec(e+ }{1+2n}$$
$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{((1+4n)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{1-x}}\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+}$$
$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+\right.}{f(1+2n)\sqrt{1+\sec(e+}$$

Mathematica [A] time = 0.46, size = 83, normalized size = 0.85

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\,\sec^n(e+fx)\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)-\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2])/(f*n)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^{3/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec{\left(e+fx \right)}+1\right)^{\frac{3}{2}}\sec^{n}{\left(e+fx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(3/2),x)
[Out] Integral((sec(e + f*x) + 1)**(3/2)*sec(e + f*x)**n, x)
```

3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{2\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] 2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+sec(f*x+e))^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3806, 65}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx &= -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= \frac{2\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 1.00

$$\frac{2\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec\left(e+fx\right)+1}\,\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**n, x)
```

3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1+sec
(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 16.09, size = 2938, normalized size = 49.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*(3*AppellF1[1/2, -1/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(
1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appe
llF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(S
ec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f
*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appel
lF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e +
 f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*
(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(
Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan
```

```
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (
(-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2
*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2
+ n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*App
ellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n,
1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(
e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e
 + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]
) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\sqrt{1+\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

```
[Out] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/sqrt(sec(e + f*x) + 1), x)
```

3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1
+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 17.21, size = 2990, normalized size = 48.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))
/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)
*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])
^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Ap
pellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(
Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e +
f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Ap
pellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2
, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
```

```
/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2
+ n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appe
llF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1
/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3
/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 -
n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/2, -
1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2 + n
, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)
)))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/2 +
n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)/2]*
Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*
x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}}{\sec\left(fx+e\right)^{2}+2\,\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f*x
+ e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(f x + e\right)}{\left(1 + \sec\left(f x + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(f x + e\right)^{n}}{\left(\sec\left(f x + e\right) + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{{\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e + f x\right)}{\left(\sec\left(e + f x\right) + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(3/2),x)
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.299 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] 2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyperge
om([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1-sec
(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 21, 3806, 64}

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) - (
(1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{(-\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}}{1+2n}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int(-\sec(e+fx))^n\sqrt{1+\sec}}{1+2n}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{((1+4n)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(-x)}{\sqrt{1}}\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{}}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)}{fn(1+2n)\sqrt{1-\sec(e+f}}$$

Mathematica [A] time = 0.44, size = 85, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\,(-\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)-\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
f*x)/2])/(f*n)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-\sec\left(fx+e\right)\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^n\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(e+fx)}+1\right)^{3/2} \left(-\frac{1}{\cos(e+fx)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n (\sec(e+fx)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

3.300 $\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -hypergeom([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1-sec
(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 64}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e +
 f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.05

$$\frac{2\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
ec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x}\,dx,x,1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.24, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*
(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -
1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*
(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(
e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]
*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[
e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1
[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sq
rt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Ap
pellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) +
 Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 +
 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*A
ppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2,
 -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n
)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
```

```
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f
*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*
n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(
-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e +
 f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3
*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{1+\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

```
[Out] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^n}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x^2}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.28, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]

[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^
(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
+ Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*S
ec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e
 + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) +
 (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/
2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
+ n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3
*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2
, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2,
 -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) +
((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*
(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/
2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*
```

```
AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
+ n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -
1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^
2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*S
ec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}}{\sec\left(fx+e\right)^{2}+2\,\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f
*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^n}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] 2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyperg
eom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1-s
ec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3814, 21, 3806, 64}

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int \frac{(d\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+f}{\sqrt{1+\sec(e+fx)}}}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int (d\sec(e+fx))^n\sqrt{1+s}}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(d(1+4n)\tan(e+fx))\,\text{Subst}\left(\int\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+f\right.}{fn(1+2n)\sqrt{1-\sec(e+}}$$

Mathematica [A] time = 0.41, size = 85, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\,(d\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx)\right)\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
 f*x)/2])/(f*n)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

3.304 $\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -hypergeom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1-se
c(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 64}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx &= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int\frac{(dx)^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 1.05

$$\frac{2\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 - n
)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx &= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\
&= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 6.22, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 +
Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
 -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*S
qrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*A
ppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2
+ n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
+ Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1
+ 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*
AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2
, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*
n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e +
f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
```

```
f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]
*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e
+ f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{1+\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n}{\sqrt{\frac{1}{\cos\left(e+f\,x\right)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
[Out] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^n}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx &= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.24, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)
^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/
2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*
Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(
e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
+ (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1
/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/
3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
 n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) +
 ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2
*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3
/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)
*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
```

```
^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)
^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*
Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}}{\sec\left(fx+e\right)^{2}+2\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(
f*x + e) + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n}{\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
[Out] int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^n}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=177

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^3(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+$$

```
[Out] 2*a^3*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(a+a*sec(f*x+e))^
(1/2)+2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)*(a+a*sec(f*x+e))^(1/2)/f/(3+2*n)+2*
a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(
4*n^2+8*n+3)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 4016, 3806, 65}

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\,\sec^{n+1}}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*
Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e +
f*x]]*Sin[e + f*x])/(f*(3 + 2*n)) + (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometr
ic2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*
n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\,\sin(e+fx)}{f(3+2n)} + \frac{(2a)\int \sec^n(e+f}{}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\,\mathrm{se}}}{f(3+2n}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\,\mathrm{se}}}{f(3+2n}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\,\mathrm{se}}}{f(3+2n}$$

Mathematica [C] time = 8.12, size = 400, normalized size = 2.26

$$-\frac{i2^{n-\frac{5}{2}}e^{-\frac{1}{2}i(2n+3)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{3}{2}}\sec^5\left(\frac{1}{2}(e+fx)\right)(a(\sec(e+fx)+1))^{5/2}\left(\frac{10e^{i(n+2)(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-n-1);\frac{n+4}{2};-e^{2i(e+fx)}\right)}{n+2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*((
10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E^(
(2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F1[1
, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f*x))
*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n + (5*E^
(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((2*I)*(
e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1, 1 - n/2
, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e + f*x))*H
ypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[
(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2))/(E^((I/2)*(3 + 2*n)*(e + f*x))
*f*Sec[e + f*x]^(5/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(a+a*se
c(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 21, 3806, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x
]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{\sec^n(e+fx)\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec}{\sqrt{a+a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{a+a}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\tan(e+fx)\right)\text{Subst}\left(\int\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\text{se}\right.}{f(1+2n)\sqrt{a+a\,\text{sec}}}$$

Mathematica [A] time = 0.43, size = 86, normalized size = 0.80

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,\sec^n(e+fx)\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e
 + f*x)/2])/(f*n)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sec^n\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)**n, x)
```

3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=48

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 65}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 1.06

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e +
 f*x])]*Tan[(e + f*x)/2])/f
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

maple [F] time = 1.23, size = 0, normalized size = 0.00

$$\int\left(\sec^{n}\left(fx+e\right)\right)\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)**n, x)
```

3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

[Out] AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)

Rubi [A] time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.24, size = 2964, normalized size = 48.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2])/(f
*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2,
 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e
+ f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqr
t[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1
 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2
)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*Ap
pellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*A
ppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
```

```
*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((
2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*(
(1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + n)*AppellF1[3/2,
1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*Ap
pellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 +
n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*A
ppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])
/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1
[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]
*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Se
c[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1
/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)
/2]^2)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/a/f/
(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.26, size = 2992, normalized size = 44.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]
^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1
[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
+ f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2
+ n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f
*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(
3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(
e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2,
 -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n
)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
 f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1
```

```
[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -
3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*
AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
 f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^
2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3
*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*Appell
F1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(
2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/
2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/
2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)
/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e
+ f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^
2*sec(f*x + e) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(fx+e\right)}{\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{n}\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral(sec(e + f*x)**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.312 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\mathrm{se}}{f(2n+1)\sqrt{a\sec(e}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(
f*x+e)^(1-n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e)
)^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\mathrm{se}}{f(2n+1)\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec
[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec
[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{(-\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2\right.\right.}{\sqrt{a+a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int (-\sec(e+fx))^n\sqrt{a}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^3(1+4n)\tan(e+fx)\right)\text{Subst}\Big(}{f(1+2n)\sqrt{a-a\sec(e+f}}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)(-\sec(e+fx))^n\sec^1\right.}{f(1+2n)\sqrt{}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-}}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.38, size = 88, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,(-\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan
[(e + f*x)/2])/(f*n)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}} \left(-\sec \left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos \left(e+f\,x\right)}\right)}^{3/2}\,{\left(-\frac{1}{\cos \left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(e+fx\right)\right)^{n} \left(a\left(\sec \left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.313 $\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\, dx$

Optimal. Leaf size=70

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\, {}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x+e)^(1-
n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\, {}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n
*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\, dx &= \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a-ax}}\, dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{\left(a^2(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\, dx,\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,\text{si}}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 71, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,(-\sec(e+fx))^n\sec^{-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
qrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^n\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*sqrt(a*(sec(e + f*x) + 1)), x)
```

3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/
n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.23, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appell
F1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-
1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3
*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1
[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
+ f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*A
ppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[
e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1
/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec
[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
```

```
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1
/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-
1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n
)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*Appell
F1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2
 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e +
 f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/
2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*A
ppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e
+ f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1
[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1
+ 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/a/
f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}\,x^2}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.22, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]
^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*
(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Ta
n[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF
1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*
(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e +
f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF1
[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[
```

```
1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-
1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e +
f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2
]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*
(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 -
 n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2,
1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
+ f*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]
^2)^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]
^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3
/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2
*a^2*sec(f*x + e) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.316 $\int (d\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec}{f(2n+1)\sqrt{a\sec(e}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*
sec(f*x+e))^n*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(d*sec(f*x+
e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec}{f(2n+1)\sqrt{a\sec(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e
 + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
+ a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{(d\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right.\right.}{\sqrt{a+a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int (d\sec(e+fx))^n\sqrt{a}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3d(1+4n)\tan(e+fx)\right)\text{Subst}}{f(1+2n)\sqrt{a-a\sec(e+f}}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\sec^{1-n}(e+fx)(d\sec(\right.}{f(1+2n)\sqrt{}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.34, size = 88, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\,(d\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\,{}_2F_1\left(\frac{1}{2},n+\frac{3}{2};\frac{3}{2};2\sin^2\left(\frac{1}{2}(e+fx\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*n)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(d\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(d*sec(e + f*x))**n, x)
```

3.317 $\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec(f*x+e)
)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
 n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx &= -\frac{\left(a^2d\tan(e+fx)\right)\operatorname{Subst}\left(\int\frac{(dx)^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{\left(a^2\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)\right)\operatorname{Subst}\left(\int\frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sec^{-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(d*Sec[e + f*x])^n*
Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^n\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(d\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(d*sec(e + f*x))**n, x)
```

3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.21, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1
+ n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appell
F1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec
[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF
1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*
AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f
*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec
[e + f*x]]*Tan[(e + f*x)/2]*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2]) + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[
1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-
1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2
 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Se
c[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*
x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(
```

```
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-
1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(
-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 +
n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*Appel
lF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1
- n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^
2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e
+ f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e
 + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e +
 f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2])
+ Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1
 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{n}}{\sqrt{a \sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec\left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/a
/f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}\,(1+x)^2}\,dx, x, \sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.22, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2
]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2
*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + T
an[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e
+ f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Appell
F1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2
*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*
(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-3/2 + n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1
```

```
[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(
-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*
(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
]) + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/
2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3
*(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1
- n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2,
 1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3
/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2
]^2)^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2
]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n,
2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[
3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2)))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\,\left(d\sec\left(fx+e\right)\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
2*a^2*sec(f*x + e) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n}{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.320 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=178

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^3(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+$$

```
[Out] 2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f
/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^3*(7+4*n)*(-sec(f*x+e))^n*tan(f*x
+e)/f/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*(a-a*sec(f
*x+e))^(1/2)*tan(f*x+e)/f/(3+2*n)
```

Rubi [A] time = 0.33, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3814, 4016, 3806, 65}

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}\,(-\sec(}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^
3*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a -
 a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e
 + f*x])/(f*(3 + 2*n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
```

```
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,\tan(e+fx)}{f(3+2n)} - \frac{(2a)\int(-\sec(}{}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-}}{f(3}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-}}{f(3}$$

$$= \frac{2a^3\left(3+24n+16n^2\right)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f\left(3+8n+4n^2\right)\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 24.65, size = 429, normalized size = 2.41

$$\frac{2^{n-\frac{5}{2}}e^{-i\left(n-\frac{1}{2}\right)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc^5\left(\frac{e}{2}+\frac{fx}{2}\right)(a-a\sec(e+fx))^{5/2}\left(\frac{e^{in(e+fx)}\,{}_2F_1\left(1,\frac{1}{2}(-n-3);\frac{n+2}{2};-e^{2i(e+fx)}\right)}{n}+\frac{10e^{i(n+2)(e+fx}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(-1/2 + n)*Csc[e/
2 + (f*x)/2]^5*((E^(I*n*(e + f*x))*Hypergeometric2F1[1, (-3 - n)/2, (2 + n)
/2, -E^((2*I)*(e + f*x))])/n + (10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F
1[1, (-1 - n)/2, (4 + n)/2, -E^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n
)*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x)
)])/(4 + n) - (5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3
+ n)/2, -E^((2*I)*(e + f*x))])/(1 + n) - (E^(I*(5 + n)*(e + f*x))*Hypergeom
etric2F1[1, 1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) - (10*E^(I*(
3 + n)*(e + f*x))*Hypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*
x))])/(3 + n))*(-Sec[e + f*x])^n*Sec[e + f*x]^(-5/2 - n)*(a - a*Sec[e + f*x
])^(5/2))/(E^(I*(-1/2 + n)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec\left(fx+e\right)^2-2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{-a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 - 2*a^2*sec(f*x + e) + a^2)*sqrt(-a*sec(f*x +
e) + a)*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(f x+e\right)\right)^{n} \left(a-a \sec \left(f x+e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(-a \sec \left(f x+e\right)+a\right)}^{\frac{5}{2}} \left(-\sec \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.321 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(1+2*n)
/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec
(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3814, 21, 3806, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(-\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2n\right)+a(}{\sqrt{a-a\sec(e+f}}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int(-\sec(e+fx))^n}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)\tan(e+fx)\right)\text{Sub}}{f(1+2n)\sqrt{a-a\sec(e}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(}{f(1}$$

Mathematica [C] time = 13.72, size = 346, normalized size = 3.20

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}\left(3n\left(n^2+4n+3\right)e^{i(n+2)(e+fx)}\,{}_2F_1\left(1\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -1/2*n, (3 + n)/2, -E^((2*I)*(e + f*x))]))*(-Sec[e
 + f*x])^n*Sec[e + f*x]^(-3/2 - n)*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(1
 + 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3 + n)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sec\left(fx+e\right)-a\right)\sqrt{-a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.322 $\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3806, 65}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a - a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx &= \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 72.78, size = 213, normalized size = 4.53

$$\frac{2^{n-\frac{1}{2}}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{a-a\sec(e+fx)}\left((n+1)e^{ifnx}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{n+2}{2};-e^{2i(e+fx)}\right)-ne\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hypergeomet
ric2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1 + n)
*x))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*(-Se
c[e + f*x])^n*Sec[e + f*x]^(-1/2 - n)*Sqrt[a - a*Sec[e + f*x]])/(f*n*(1 + n
))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a-\frac{a}{\cos\left(e+fx\right)}}\,\left(-\frac{1}{\cos\left(e+fx\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(e+fx\right)\right)^{n} \sqrt{-a \left(\sec \left(e+fx\right)-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
[Out] Integral((-sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] AppellF1(1/2,1-n,1,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/f/(a-a*s
ec(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1-\sec(e+fx)}}\,dx}{\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2}; 1-n, 1; \frac{3}{2}; 1+\sec(e+fx), \frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
[Out] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]], x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a\sec\left(fx+e\right)+a}\,\left(-\sec\left(fx+e\right)\right)^n}{a\sec\left(fx+e\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a*sec(f*x + e) - a),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^n}{\sqrt{-a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^n}{\sqrt{a-a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{-a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 1/2*AppellF1(1/2,1-n,2,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/a/f/
(a-a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1-\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{(1-x^2)^{-1+n}}{(2-x^2)^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}$$
$$= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}$$

Mathematica [F] time = 75.59, size = 0, normalized size = 0.00

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
[Out] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2), x]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^n}{a^2\sec\left(fx+e\right)^2-2\,a^2\sec\left(fx+e\right)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 -
2*a^2*sec(f*x + e) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
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step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x
/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nos
tep^2-1)]Evaluation time: 1.61Unable to divide, perhaps due to rounding err
or%%%{1,[0,4,1,0]%%%}+%%%{2,[0,2,1,1]%%%}+%%%{1,[0,0,1,2]%%%} / %%%{1,[0,0,
0,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(a-a\sec \left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(-a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n}{\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2),x)
[Out] int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^n}{\left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(3/2),x)
[Out] Integral((-sec(e + f*x))**n/(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec}{f(2n+1)\sqrt{a-a\sec}}$$

```
[Out] 2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/
f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec}{f(2n+1)\sqrt{a-a\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
 - a*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{\sec^n(e+fx)\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e+\ldots}{\sqrt{a-a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{a-a\sec\ldots}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\tan(e+fx)\right)\text{Subst}\left(\int \frac{x\ldots}{\sqrt{\ldots}}\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{\ldots}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)(-\sec(e+fx))^{-n}\sec^{1+n}(\ldots\right.}{f(1+2n)\sqrt{a-\ldots}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(\ldots\right.}{f(1+\ldots}$$

Mathematica [C] time = 2.10, size = 332, normalized size = 2.55

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}\left(3n\left(n^2+4n+3\right)e^{i(n+2)(e+fx)}\,{}_2F_1\left(1,\frac{1-\ldots}{2}\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -1/2*n, (3 + n)/2, -E^((2*I)*(e + f*x))]))*(a - a*
Sec[e + f*x])^(3/2))/(E^((I/2)*(1 + 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3
+ n)*Sec[e + f*x]^(3/2)))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sec\left(fx+e\right)-a\right)\sqrt{-a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(3/2),x)
[Out] Integral((-a*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)**n, x)
```

3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/f/
((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1 +
 n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\left(a^2(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)\right)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx,x,\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 0.40, size = 185, normalized size = 2.68

$$-\frac{2^n e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n \cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}\left(ne^{i(e+f(n+1)x)}\,{}_2F_1\left(1,1-\frac{n}{2};\frac{n+3}{2};\cdot\right.\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] -((2^n*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*
x))))^n*Cos[e + f*x]*Csc[(e + f*x)/2]*(-(E^(I*f*n*x)*(1 + n)*Hypergeometric
2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))]) + E^(I*(e + f*(1 + n)*x
))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*Sqrt[a
 - a*Sec[e + f*x]])/(f*n*(1 + n)))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int\left(\sec^n\left(fx+e\right)\right)\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec\left(fx+e\right)+a}\,\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sec(e + f*x) - 1))*sec(e + f*x)**n, x)
```

3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d}{f(2n+1)\sqrt{a-a}}$$

```
[Out] 2*a^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(1
+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d}{f(2n+1)\sqrt{a-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x
]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]
*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a - a
*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(d\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2\right.\right.}{\sqrt{a-a\sec(e+fx)}}}{1+2n}$$
$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int (d\sec(e+fx))^n\sqrt{a}}{1+2n}$$
$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{\left(a^3d(1+4n)\tan(e+fx)\right)\text{Subst}}{f(1+2n)\sqrt{a-a\sec(e+f}}$$
$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)(-\sec(e+fx))^{-n}(d\,\text{se}\right.}{f(1+2n)}$$
$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\text{s}\right.}{f(}$$

Mathematica [C] time = 1.16, size = 346, normalized size = 2.66

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}\left(3n\left(n^2+4n+3\right)e^{i(n+2)(e+fx)}\,{}_2F_1\left(1,\frac{1-}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -1/2*n, (3 + n)/2, -E^((2*I)*(e + f*x))]))*Sec[e +
 f*x]^(-3/2 - n)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(
1 + 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3 + n)))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sec\left(fx+e\right)-a\right)\sqrt{-a\sec\left(fx+e\right)+a}\,\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x + e\right)\right)^{n} \left(a - a \sec \left(f x + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a \sec \left(f x + e\right) + a\right)^{\frac{3}{2}} \left(d \sec \left(f x + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{n} \left(- a \left(\sec{\left(e + f x \right)} - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.328 $\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] 2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/
((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3806, 67, 65}

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx &= -\frac{\left(a^2d\tan(e+fx)\right)\text{Subst}\left(\int\frac{(dx)^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\left(a^2(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx)\right)\text{Subst}\left(\int\frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n}{f\sqrt{a-a\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.58, size = 213, normalized size = 3.09

$$\frac{2^{n-\frac{1}{2}}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{a-a\sec(e+fx)}\left((n+1)e^{ifnx}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{n+2}{2};-e^{2i(e+fx)}\right)-ne\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hypergeomet
ric2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1 + n)
*x))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*Sec[
e + f*x]^(-1/2 - n)*(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*(1 +
n))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

sympy **[F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=72

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f
*x+e)/f/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,x,1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e-}{f\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 14.37, size = 2246, normalized size = 31.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Appell
F1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-
1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n,
2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2
, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*Appel
lF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(
Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[
(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
+ 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2
- n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))
)/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n
)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos
[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)))
```

<u>fricas</u> [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

maple [F] time = 2.44, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(1+\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec\left(e+fx\right)+1\right)^{m}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**m,x)
[Out] Integral((sec(e + f*x) + 1)**m*sec(e + f*x)**n, x)
```

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1-sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec[e
 + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rubi steps

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx,x,1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1-\text{se}}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 2.34, size = 255, normalized size = 2.87

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+fx}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2};m+n,2-n;m+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*Sec[e + f*x]^n*Sin[e + f*x])/(f*(1 +
 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/2 +
 m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m, 1
+ m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

maple [F] time = 2.66, size = 0, normalized size = 0.00

$$\int\left(1-\sec\left(fx+e\right)\right)^{m}\left(\sec^{n}\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

```
[Out] int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\sec\left(e+fx\right)\right)^m \sec^n\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*sec(f*x+e)**n,x)
[Out] Integral((1 - sec(e + f*x))**m*sec(e + f*x)**n, x)
```

3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=88

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+se
c(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec^n(e+fx)(1+\sec(e+ \ldots$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1-x)}{\ldots}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec \ldots}{f}$$

Mathematica [B] time = 6.24, size = 2248, normalized size = 25.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*A
ppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
 m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*
Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3
*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
```

```
)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m
+ n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
 + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

maple [F] time = 2.67, size = 0, normalized size = 0.00

$$\int\left(\sec^{n}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}\,\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**n, x)
```

3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(a-a*sec(f*x+
e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3828, 3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m \sec^n(e+$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m \tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1-x)^{-}}{}\right.}{f\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(a-}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-a\sec\left(fx+e\right)+a\right)^m \sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m \sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

maple [F] time = 2.66, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m \sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{m}\sec^{n}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((-a*(sec(e + f*x) - 1))**m*sec(e + f*x)**n, x)
```

3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1+sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx,x,1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.23, size = 2248, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*
((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*Appe
llF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
+ n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*Ap
pellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*T
an[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*
(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
+ m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m +
 n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
+ n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2
, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*App
ellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

maple [F] time = 2.36, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(1+\sec\left(fx+e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}+1\right)}^m\,{\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(e+fx\right)\right)^{n} \left(\sec\left(e+fx\right)+1\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
[Out] Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

3.334 $\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$

Optimal. Leaf size=70

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f
*x+e)/f/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,x,1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan}{f\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 0.31, size = 257, normalized size = 3.67

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+f}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2};m+n,2-n;m+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n*Sin[e + f*x])/(f*(
1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/
2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m,
 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

maple [F] time = 2.49, size = 0, normalized size = 0.00

$$\int\left(1-\sec\left(fx+e\right)\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)^{m}\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec \left(e + fx\right)\right)^{n} \left(1 - \sec \left(e + fx\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(-sec(f*x+e))**n,x)
[Out] Integral((-sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

3.335 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+
e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3828, 3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (-\sec(e+fx))^n(1+$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx)\right)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 6.22, size = 2250, normalized size = 25.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^
n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2,
 m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*
AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n
)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(
1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m +
n)*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*(
(-3*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2
, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
```

```
f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m +
 n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
])/5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*
(m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
+ m + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]
) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1
[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)
*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

maple [F] time = 2.88, size = 0, normalized size = 0.00

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**m, x)
```

3.336 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=87

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+$$
$$f$$

```
[Out] 2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-se
c(f*x+e))^(-1/2-m)*(a-a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3828, 3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+$$
$$f$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m(-$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(\int}{f\sqrt{1+\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1}{f}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a\sec\left(fx+e\right)+a\right)^m\left(-\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

maple [F] time = 2.89, size = 0, normalized size = 0.00

$$\int \left(-\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- \sec{\left(e + f x \right)}\right)^{n} \left(- a \left(\sec{\left(e + f x \right)} - 1\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=79

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+
e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx &= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.21, size = 2248, normalized size = 28.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 - n)*A
ppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 +
 m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*
Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3
*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m
+ n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
 + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(fx+e\right)\right)^{n} \left(\sec \left(fx+e\right)+1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

maple [F] time = 2.41, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(fx+e\right)\right)^{n} \left(\sec \left(fx+e\right)+1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos \left(e+f\,x\right)}+1\right)}^m\,{\left(\frac{d}{\cos \left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n,x)
[Out] int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(e+fx\right)\right)^{n} \left(\sec \left(e+fx\right)+1\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
[Out] Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

3.338 $\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx$

Optimal. Leaf size=79

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+
e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx &= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(1-x)^{-\frac{1}{2}+m}(dx)^{-1+n}}{\sqrt{1+x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\\ &= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n \tan}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 0.28, size = 257, normalized size = 3.25

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+fx))}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2};m+n,2-n;m+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m\right.\right.}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]

[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d \sec\left(fx+e\right)\right)^{n} \left(-\sec\left(fx+e\right)+1\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec\left(fx+e\right)\right)^{n} \left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)

maple [F] time = 2.38, size = 0, normalized size = 0.00

$$\int \left(1-\sec\left(fx+e\right)\right)^{m} \left(d \sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)

[Out] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec\left(fx+e\right)\right)^{n} \left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)

```
[Out] int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(1-\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(d*sec(f*x+e))**n,x)
[Out] Integral((d*sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=95

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

```
[Out] -AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*(1+sec(f
*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^n(1+\sec$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\mathrm{Subst}\Big(\int}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n(}{fn\sqrt{1-\sec(e+}}$$

Mathematica [B] time = 6.22, size = 2250, normalized size = 23.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(
3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)
^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2
, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)
*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-1/3*((1 -
n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^
(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m +
 n)*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2] + 3*(-1/3*((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((m + n)*A
ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*
((-3*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/
2, 1 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
```

```
f*x)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m
+ n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2])/5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)
*(m + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
 + m + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2
]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

maple [F] time = 2.43, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\left(d\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(d*sec(e + f*x))**n, x)
```

3.340 $\int (d\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx$

Optimal. Leaf size=96

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\text{se}\right.}{fn\sqrt{\sec(e+fx)+1}}$$

```
[Out] -AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(1-sec(f*x+e))^(-1/2-m)*(
d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\text{se}\right.}{fn\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[e
 + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 + Sec[e + f*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m(d\sec$$

$$= -\frac{\left(d(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(\int}{f\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(1-\sec(e+fx))}{fn\sqrt{1+\sec(e+}}$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
[Out] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-a\sec\left(fx+e\right)+a\right)^m\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

maple [F] time = 2.50, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\sec\left(fx+e\right)+a\right)^m\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
[Out] int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{n} \left(- a \left(\sec{\left(e + f x \right)} - 1\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
[Out] Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=211

$$\frac{2^{m+\frac{1}{2}}m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)(m+3)}$$

```
[Out] (4+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^3+6*m^2+11*m+6)+sec(f*x+e)^2*(a+a*
sec(f*x+e))^m*tan(f*x+e)/f/(3+m)+2^(1/2+m)*m*(m^2+3*m+5)*hypergeom([1/2, 1/
2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*t
an(f*x+e)/f/(m^3+6*m^2+11*m+6)+m*(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(m^2
+5*m+6)
```

Rubi [A] time = 0.35, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3824, 4010, 4001, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((4 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)*(2 + m)*(3 + m)) +
 (Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + (2^(1/2
 + m)*m*(5 + 3*m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f
*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/
(f*(1 + m)*(2 + m)*(3 + m)) + (m*(a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])
/(a*f*(6 + 5*m + m^2))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{\int \sec^2(e+fx)(a+a}{}$$

$$= \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{m(a+a\sec(e+fx)}{af\left(6+5m\right.}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e}{f(3+m}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e}{f(3+m}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e}{f(3+m}$$

$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e}{f(3+m}$$

Mathematica [A] time = 1.36, size = 154, normalized size = 0.73

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}m\left(m^2+3m+5\right)\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(e\right.\right.}{f(m+2)(m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*(5 +
 3*m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (
```

```
1 + Sec[e + f*x])^(1/2 + m)*(4 + m + m^2 + m*(1 + 2*m)*Sec[e + f*x] + (2 +
5*m + 2*m^2)*Sec[e + f*x]^2))*Tan[e + f*x])/(f*(2 + m)*(3 + m)*(1 + 2*m))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int\left(\sec^{4}\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}}{{\cos\left(e+f\,x\right)}^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^4,x)
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**4, x)
```

3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=155

$$\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)}$$

```
[Out] -(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+2^(1/2+m)*(m^2+m+1)*hypergeom(
[1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x
+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(2+m)
```

Rubi [A] time = 0.19, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3800, 4001, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(2 + 3*m + m^2))) + (2^(1/2 + m)
*(1 + m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m
)*(2 + m)) + ((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} + \frac{\int \sec(e+fx)(a(1+m)-a\sec(e \ldots}{a(2+ \ldots}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx \ldots}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx \ldots}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx \ldots}{af(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{2^{\frac{1}{2}+m}\left(1+m+m^2\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m; \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.63, size = 123, normalized size = 0.79

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}\left(m^2+m+1\right)\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(e+fx) \ldots\right.\right.}{f(m+2)(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*(1 + m
 + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 +
Sec[e + f*x])^(1/2 + m)*(-1 + m + (1 + 2*m)*Sec[e + f*x]))*Tan[e + f*x])/(f
*(2 + m)*(1 + 2*m))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^3,x)
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^m\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**3, x)
```

3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=107

$$\frac{2^{m+\frac{1}{2}}m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}+\frac{\tan(e+fx)(}{f}$$

```
[Out] (a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)+2^(1/2+m)*m*hypergeom([1/2, 1/2-m],[3
/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+
e)/f/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3798, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}+\frac{\tan(e+fx)(}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hypergeo
metric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{m\int \sec(e+fx)(a+a\sec(e+fx)}{1+m}$$
$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{\left(m(1+\sec(e+fx))^{-m}(a+a\sec\right.}{}$$
$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} - \frac{\left(m(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\mathrm{s}\right.}{}$$
$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{2^{\frac{1}{2}+m}m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\mathrm{se}\right.}{}$$

Mathematica [A] time = 0.22, size = 95, normalized size = 0.89

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)+(\sec(\right.}{2fm+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*Hype
rgeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 + Sec[e + f*x]
)^(1/2 + m))*Tan[e + f*x])/(f + 2*f*m)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{m}}{\cos\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec \left(e+fx\right)+1\right)\right)^{m} \sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**2, x)
```

3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m \, dx$

Optimal. Leaf size=73

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

```
[Out] 2^(1/2+m)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(
-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec(e+fx)(1+\sec(e+fx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x}{\sqrt{1}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\text{se}}{f}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 1.00

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])/f
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/cos(e + f*x),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^m \sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x), x)
```

3.345 $\int (a + a\sec(e + fx))^m\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] AppellF1(1/2+m,1,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e)
)^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3779, 3778, 136}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[e
 + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e +
 f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\int (a + a\sec(e + fx))^m\,dx = \left((1 + \sec(e + fx))^{-m}(a + a\sec(e + fx))^m\right)\int (1 + \sec(e + fx))^m\,dx$$

$$= -\frac{\left((1 + \sec(e + fx))^{-\frac{1}{2}-m}(a + a\sec(e + fx))^m\tan(e + fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}\,x}\,dx,\right.}{f\sqrt{1 - \sec(e + fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2} + m; \frac{1}{2}, 1; \frac{3}{2} + m; \frac{1}{2}(1 + \sec(e + fx)), 1 + \sec(e + fx)\right)(a + a\sec(e + fx)}{f(1 + 2m)\sqrt{1 - \sec(e + fx)}}$$

Mathematica [B] time = 6.80, size = 711, normalized size = 8.57

$$\frac{}{f\left(45\cos^2\left(\frac{1}{2}(e + fx)\right)(-2m\cos(e + fx) + \cos(2(e + fx)) + 2m + 1)F_1\left(\frac{1}{2}; m, 1; \frac{3}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right), -\tan^2\left(\frac{1}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (30*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(
e + f*x)/2]^2*Cos[e + f*x]*(a*(1 + Sec[e + f*x]))^m*Sin[e + f*x]*(3*AppellF
1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/
2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1
+ m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
/(f*(45*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]^2
*Cos[(e + f*x)/2]^2*(1 + 2*m - 2*m*Cos[e + f*x] + Cos[2*(e + f*x)]) + 6*App
ellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x
)/2]^2*(-5*AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(1 + 2*m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) + 5*m*AppellF1[3/2,
 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 2*m - 2*(2 +
m)*Cos[e + f*x] + Cos[2*(e + f*x)]) - 48*(2*AppellF1[5/2, m, 3, 7/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[e + f*x]*Csc[e + f*x]*Sin[(
e + f*x)/2]^4) + 40*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])^2*Cos[e + f*x]*Sin[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx + e\right) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx + e\right) + a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(a + a\sec\left(fx + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m,x)
[Out] int((a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos\left(e + f\,x\right)}\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m,x)
[Out] int((a + a/cos(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(e + fx\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m,x)
[Out] Integral((a*sec(e + f*x) + a)**m, x)
```

3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] -AppellF1(1/2+m,2,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e
))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3828, 3827, 136}

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \cos(e+fx)(1+\sec(e+fx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x}{\sqrt{1}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\,s}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

<u>**Mathematica**</u> **[B]** time = 16.66, size = 3781, normalized size = 45.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1 + m)*Cos[(e + f*x)/2]^3*Cos[e + f*x]*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^m*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1
/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2,
m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m
, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (
2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appel
lF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appel
lF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3
/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2)/3)))/(f*(2^m*Cos[(e + f*x)/2]^4*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(
(-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - 3*2^m*Cos[(e + f*x)/2]^2*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^m*Sin[(e + f*x)/2]^2*((-3*AppellF1[1/2, m, 1, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2
*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appell
F1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appell
F1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/
2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)/3)) + 2^(1 + m)*Cos[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m
*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*AppellF1[1/2, m, 1, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sec[(e +
f*x)/2]^2*(-1/3*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*AppellF1[3/2, 1 + m, 1, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3))/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
```

```
]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2) + (2*((-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[
3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2])/3))/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) + (3*AppellF1[1/2, m, 1, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(-2*(AppellF1[3/2,
 m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 +
m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2] + 3*(-1/3*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*AppellF1[3/2, 1 +
 m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/3) - 2*Tan[(e + f*x)/2]^2*((-6*AppellF1[5/2, m, 3, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
 + (3*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 - m*((-3*AppellF1[5/2, 1 + m, 2,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 -
(2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*((-2*A
ppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*(-
2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*App
ellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*Tan[(e + f*x)/2]^2*(-2*((-9*AppellF1[5
/2, m, 4, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/5 + (3*m*AppellF1[5/2, 1 + m, 3, 7/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + m*((-6*Appe
llF1[5/2, 1 + m, 3, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 2, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5)))/3))/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)/3)^2) + 2^(1 + m)*m*Cos[(e + f*x)/2]^3*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(-1 + m)*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2
*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appell
F1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appell
F1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/
2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)/3))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)
/2]^2*Sec[e + f*x]*Tan[e + f*x])))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

maple [F] time = 1.41, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
[Out] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a/cos(e + f*x))^m,x)
[Out] int(cos(e + f*x)*(a + a/cos(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^m\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))**m,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*cos(e + f*x), x)
```

3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=98

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}}$$

```
[Out] -2/3*AppellF1(3/2,1/2-m,1/2,5/2,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^(3/2
)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3827, 133}

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^{3/2}(1+$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{2F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^{3/2}(}{3f\sqrt{1-\sec(e+}}$$

Mathematica [**B**] time = 15.01, size = 2529, normalized size = 25.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^m*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e
 + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^(1 + m)*AppellF1[
1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f
*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^2)^2*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 +
 m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^m)/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 +
 m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Sin[e +
f*x]*Tan[(e + f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m,
 -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 +
 m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1
[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)) - (3*2^(1 + m)*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2 +
 m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e + f*x)/2]^2)*(3*
AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -
1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/
2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2,
5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2] + 3*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)
```

```
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2
+ m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*((-3*Appe
llF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/
2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/5 + (3 + 2*m)*((3*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(5
/2 + m)*AppellF1[5/2, 7/2 + m, -1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Tan[(e + f*x)/2]^
2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*m*AppellF1[1/2,
3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f
*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m)*Tan[(e + f*x)/2]*(-(Cos[(e
+ f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*
Tan[e + f*x]))/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/
2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2))))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}d\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m*d*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.348 $\int \sqrt{d \sec(e+fx)} (a+a\sec(e+fx))^m dx$

Optimal. Leaf size=96

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] -2*AppellF1(1/2,1/2-m,1/2,3/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2-
m)*(a+a*sec(f*x+e))^m*(d*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3828, 3827, 133}

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*Se
c[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sqrt{d\sec(e+fx)}\,(1+\sec$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\Big(\int}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+fx)}\,(1+}{f\sqrt{1-\sec(e+fx}}$$

Mathematica [B] time = 14.64, size = 2225, normalized size = 23.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*
(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x)/2]^2]*(App
ellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((
AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)*((2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + m))/(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) - (
2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]^2)/(Sqrt[Se
c[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) + (2^(1 + m)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-1/6*(AppellF1[3/2
, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2]) + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
)/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - (2^(1
+ m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-1/6*(App
ellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3 - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 - (Tan[(e +
f*x)/2]^2*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*Appell
F1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/5 - (1 + 2*m)*((-3*AppellF1[5/2, 3/2 + m, 3/2
, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
```

```
f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/3))/(S
qrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)^2) + (2^(1 +
m)*(1/2 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*x)/2]*(
-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[
e + f*x]*Tan[e + f*x]))/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1
/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m
, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3
/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)/3))))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int\sqrt{d\sec\left(fx+e\right)}\,\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
[Out] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^m\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sqrt{d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(d*sec(e + f*x)), x)
```

3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] 2*AppellF1(-1/2,1/2-m,1/2,1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2-
m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3827, 133}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec[e
 + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}\,(dx)^{3/2}}\,dx,\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Mathematica [C] time = 14.95, size = 2424, normalized size = 25.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 +
m)*(a*(1 + Sec[e + f*x]))^m*(Cos[2*(e + f*x)]*((1 + Sec[e + f*x])^m/(2*Sqrt
[Sec[e + f*x]]) - (I/2)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m*Sin[e + f*x
]) + ((1 + Sec[e + f*x])^m/2 + (I/2)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])
/Sqrt[Sec[e + f*x]] + Sqrt[Sec[e + f*x]]*Sin[e + f*x]*((-1/2*I)*(1 + Sec[e
+ f*x])^m + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])/2))*Tan[(e + f*x)/2])/(
f*(Sec[(e + f*x)/2]^2)^(3/2)*Sqrt[d*Sec[e + f*x]]*(1 + Sec[e + f*x])^m*(-3*
AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((-3*2^m*AppellF1[1/2, -1/2 + m, 3/2,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f
*x])^(-1/2 + m))/(Sqrt[Sec[(e + f*x)/2]^2]*(-3*AppellF1[1/2, -1/2 + m, 3/2,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m,
 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/
2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)) + (9*2^m*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*
x)/2]^2)/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2
 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)) - (3*2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*
x)/2]*(-1/2*(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + m)*AppellF1[3/
2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/3))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2,
-1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[
3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*
m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-
1/2 + m)*Tan[(e + f*x)/2]*((3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2] - 3*(-1/2*(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + ((-1/2 + m)*AppellF1[3
```

```
/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(3*((-3*AppellF1[5/2, -1/
2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/2 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
) + (1 - 2*m)*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*Ap
pellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*App
ellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*(-1/2 + m)*AppellF1[1/2
, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(-3/2 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e
 + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/
((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d \sec\left(fx+e\right)}\left(a \sec\left(fx+e\right)+a\right)^{m}}{d \sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sec\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a \sec\left(fx+e\right)\right)^{m}}{\sqrt{d \sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sec\left(fx+e\right)+a\right)^{m}}{\sqrt{d \sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2),x)
[Out] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^m}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(1/2),x)
[Out] Integral((a*(sec(e + f*x) + 1))**m/sqrt(d*sec(e + f*x)), x)
```

3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/3*AppellF1(-3/2,1/2-m,1/2,-1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1
/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(d*sec(f*x+e))^(3/2)/(1-sec(f*x+e))^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3828, 3827, 133}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 - Se
c[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}\,(dx)^{5/2}}\,dx,\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e}{3f\sqrt{1-\sec(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

Mathematica [C] time = 19.42, size = 3349, normalized size = 34.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(1 + m)*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*(
a*(1 + Sec[e + f*x]))^m*((Cos[2*(e + f*x)]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e
+ f*x])^m)/4 + Cos[2*(e + f*x)]^2*Sqrt[Sec[e + f*x]]*((1 + Sec[e + f*x])^m/
2 + (I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]) + Cos[2*(e + f*x)]*Sqrt[Se
c[e + f*x]]*((1 + Sec[e + f*x])^m/4 + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)
]^2)/4) + Sqrt[Sec[e + f*x]]*((-1/4*I)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)
] + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]^2)/2 + (I/4)*(1 + Sec[e + f*x])^
m*Sin[2*(e + f*x)]^3))*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3
/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/(3*f*(d*Sec[e + f*x])^(3/2)
*(1 + Sec[e + f*x])^m*((2^m*Sec[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e +
f*x])^(1/2 + m)*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x
)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1
/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/
2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)
*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2))))/3 + (2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1
/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/
2]^2)^(1/2 + m)*Tan[(e + f*x)/2]) - (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2
+ m)*Tan[(e + f*x)/2]^2*((-3*AppellF1[5/2, -1/2 + m, 7/2, 7/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(-
1/2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) - (1/2 + m)*AppellF1[3/2, -
1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*S
ec[(e + f*x)/2]^2)^(-1/2 + m)*Tan[(e + f*x)/2]^2*(-(Sec[(e + f*x)/2]^2*Sin[
e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (9*AppellF1
[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e +
f*x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 +
m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

```
 + (27*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Cos[e + f*x]*Tan[(e + f*x)/2])/(2*(Sec[(e + f*x)/2]^2)^(3/2)*(-3*App
ellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (9*Cos[e + f*x]*((-5*AppellF1[3/2, -1/
2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))
/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/
2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (9
*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Cos[e + f*x]*((5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] - 3*((-5*
AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m,
 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(5*((-21*AppellF1[5/2, -1/2 + m, 9/2,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/10 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1 - 2
*m)*((-3*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(1/2 + m)*AppellF1[5/2,
 3/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2,
-1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[
3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*
m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)^2)))/3 + (2^(1 + m)*(1/2 + m)*(Cos[(e + f*x)/2]^2*Se
c[e + f*x])^(-1/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3
/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))*(-(Cos[(e + f*x)/2]*Sec[e +
f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/3))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}}{d^{2}\sec\left(fx+e\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d^2*sec(f*x + e)^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sec\left(fx+e\right)\right)^{m}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^m}{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m/(d*sec(e + f*x))**(3/2), x)
```

3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d
+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{5}{2}}(c+dx)\,dx + a\int \cos^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3a)\int \sqrt{c$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin}{5d}$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx}{21d}$$

Mathematica [C] time = 6.10, size = 490, normalized size = 4.41

$$a\left(-\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])/
(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[3
*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d*x]
)/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(cos(d*x +
c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

maple [A] time = 3.65, size = 270, normalized size = 2.43

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.18, size = 87, normalized size = 0.78

$$-\frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x
)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{3}{2}}(c+dx)\,dx + a\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int -$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+d}{3d}$$

Mathematica [C] time = 5.67, size = 232, normalized size = 2.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*
Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a\cos\left(dx+c\right)^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(cos(d*x +
c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 3.60, size = 219, normalized size = 2.52

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.78, size = 80, normalized size = 0.92

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))\,dx$$
$$= a\int \sqrt{\cos(c+dx)}\,dx + a\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

Mathematica [C] time = 5.21, size = 222, normalized size = 3.64

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c
 + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*S
in[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTa
n[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d
*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((a\cos(dx+c)\sec(dx+c) + a\cos(dx+c))\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(cos(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 3.41, size = 225, normalized size = 3.69

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.76, size = 53, normalized size = 0.87

$$\frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*
d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.354 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4225, 2748, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))\,dx &= \int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+a\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] time = 1.85, size = 155, normalized size = 4.43

$$\frac{a\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}}-2\sin(c)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + A
rcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
2*d)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 3.02, size = 150, normalized size = 4.29

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.20, size = 27, normalized size = 0.77

$$\frac{2\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)dx+\int \sqrt{\cos\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c + d*x
)), x))
```

3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 4.97, size = 209, normalized size = 3.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d
*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S
ec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x -
 ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 3.63, size = 146, normalized size = 2.56

$$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*a*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.03, size = 60, normalized size = 1.05

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1/
2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d
*x)), x))
```

3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - a\int \sqrt{\cos(c+dx)}\,d\\ &= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 6.13, size = 444, normalized size = 5.35

$$a\left(\frac{\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\,c}}\right.}{2d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec[
c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Sin[
c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

maple [B] time = 5.69, size = 369, normalized size = 4.45

$$\frac{2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x)

[Out] 2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)

mupad [B] time = 1.17, size = 87, normalized size = 1.05

$$\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(3/2),x)

[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(
-3/2), x))
```

3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a + a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{a + a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+ \ldots}$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3a \ldots$$
$$= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \ldots$$

Mathematica [C] time = 6.16, size = 477, normalized size = 4.30

$$a\left(\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c)\right)\ldots\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+\ldots}}}\ldots\right)}{10d}\ldots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Se
c[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]
^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin
[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 5.63, size = 384, normalized size = 3.46

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{40\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.28, size = 87, normalized size = 0.78

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4, 1/
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2
)+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+10/21*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)) + (10*a*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (6
*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
```

```
KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+a\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=a\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx+a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{7}(5a)\int \frac{1}{\cos^{\frac{5}{2}}(c-$$
$$=\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{1}{21}(5a$$
$$=-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+$$

Mathematica [C] time = 4.61, size = 294, normalized size = 2.18

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{126\sec(c)\cos^3(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\text{ta}\right.}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((189*Cos[c] + 85*Cos[d*x] - 85*Co
s[2*c + d*x] + 231*Cos[c + 2*d*x] + 21*Cos[3*c + 2*d*x] + 25*Cos[2*c + 3*d*
x] - 25*Cos[4*c + 3*d*x] + 63*Cos[3*c + 4*d*x])*Csc[c] - 200*Cos[c + d*x]^4
*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] -
 (126*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTa
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Ta
n[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(840*d*Cos
[c + d*x]^(7/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)

maple [B] time = 6.01, size = 437, normalized size = 3.24

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{112\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\right.\right.}}{84\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x)

[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/112*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+c
os(1/2*d*x+1/2*c)^2)^4-5/84*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+44/105*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+c
os(1/2*d*x+1/2*c)^2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*co
s(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)

mupad [B] time = 1.38, size = 87, normalized size = 0.64

$$\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))/cos(c + d*x)^(7/2),x)

[Out] (2*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
+ d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-7/4, 1
/2], -3/4, cos(c + d*x)^2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=147

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\,\mathrm{s}}{}$$

```
[Out] 32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos\right.$$

$$= \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\left(10\right.$$

$$= \frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4a}{}$$

$$= \frac{20a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4a}{}$$

$$= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}\,\mathrm{si}}{21d}$$

Mathematica [C] time = 6.12, size = 548, normalized size = 3.73

$$-\frac{4\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos\right.}{\sqrt{\tan^2(c)+1}\,\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\,\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}}\right.}{15d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-8*Cot[c])
/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d) +
(Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c]*S
in[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*x])/
(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*Cos[c + d*x]^2*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2
]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (4*Cos[c +
 d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
```

```
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos(dx+c)^4\sec(dx+c)^2 + 2\,a^2\cos(dx+c)^4\sec(dx+c) + a^2\cos(dx+c)^4\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

maple [A] time = 3.62, size = 260, normalized size = 1.77

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+608\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.18, size = 136, normalized size = 0.93

$$-\frac{2\,a^2\cos(c+d\,x)^{7/2}\sin(c+d\,x)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{4\,a^2\cos(c+d\,x)^{9/2}\sin(c+d\,x)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4,
 cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=121

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin}{}$$

```
[Out] 12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a^2*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4264, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos\right.$$
$$= \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6a\right.$$
$$= \frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2 c}{}$$
$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+}{}$$
$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\sin(}{7d}$$

Mathematica [C] time = 6.12, size = 516, normalized size = 4.26

$$-\frac{3\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-3*Cot[c])
/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[
3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*d*x
])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*Cos[c + d*x]^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (3*Cos[c
+ d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
```

```
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

maple [A] time = 3.44, size = 272, normalized size = 2.25

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.04, size = 129, normalized size = 1.07

$$\frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d}-\frac{4\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\,\cos\left(c+d\,x\right.\right.}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=95

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)/d+4/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos\right.$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a\right.$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a\right.$$

$$= \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2\sqrt{\cos(c+dx)}\sin(}{3d}$$

Mathematica [C] time = 5.93, size = 235, normalized size = 2.47

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-24\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x
)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan
[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos(dx+c)^2\sec(dx+c)^2 + 2a^2\cos(dx+c)^2\sec(dx+c) + a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

maple [A] time = 3.75, size = 250, normalized size = 2.63

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.92, size = 104, normalized size = 1.09

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a^2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}-\frac{2\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2639, 4045, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}\left(4a^2\sqrt{\cos\right.$$

$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)\int \frac{}{\sqrt{\cos}}$$

$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+a}{3d}$$

Mathematica [C] time = 5.56, size = 224, normalized size = 3.34

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*C
os[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[
c + d*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\cos(dx+c)\sec(dx+c)^2 + 2\,a^2\cos(dx+c)\sec(dx+c) + a^2\cos(dx+c)\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

maple [B] time = 3.68, size = 228, normalized size = 3.40

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\cdots\right.\right.}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.88, size = 59, normalized size = 0.88

$$\frac{4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.363 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=44

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3771, 2641, 4043}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \left(2a^2\sqrt{\mathrm{co}\right.$$
$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.16, size = 39, normalized size = 0.89

$$\frac{2a^2\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*(2*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Sqrt[Cos[c + d*x]]))/d
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

maple [A] time = 3.41, size = 104, normalized size = 2.36

$$-\frac{4a^2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4*a^2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.21, size = 82, normalized size = 1.86

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sqrt{\cos(c+dx)}\,\sec(c+dx)\,dx+\int \sqrt{\cos(c+dx)}\,\sec^2(c+dx)\,dx+\int \sqrt{\cos(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c
+ d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x)), x))
```

3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3
/2)+4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos\right.$$

$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c}$$

$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - \left(2a^2\right)\int \sqrt{\cos(c}$$

$$= -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.15, size = 470, normalized size = 5.16

$$\frac{\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(Sin
[c] + 6*Sin[d*x]))/(6*d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec
[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (Cos[c + d*x]^2*Csc[c]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

maple [B] time = 5.79, size = 371, normalized size = 4.08

$$\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(4\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{d}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.27, size = 109, normalized size = 1.20

$$\frac{2a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\;\frac{1}{4};\;\cos(c+\right.}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x
)**2/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x+c)
^(5/2)+4/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos\right.$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c}$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,c$$

$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}\left(8\right.$$

$$= -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] time = 6.20, size = 503, normalized size = 4.16

$$\frac{2\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((4*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 10*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 12
*Sin[d*x]))/(15*d)) - (Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x
])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (2*Cos[c + d*x]^2*Csc[c]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
```

```
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

maple [B] time = 6.00, size = 386, normalized size = 3.19

$$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cos\cdots}}{30\sqrt{-2(\sin\cdots}}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-4/5*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.38, size = 114, normalized size = 0.94

$$\frac{6\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,{\cos\left(c+d\,x\right)}^2\right)+20\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,{\cos\left(c+d\,x\right)}^2\right)}{15\,d\,{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*c
os(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*a
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{2\sec\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{\sec^{2}\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(2*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*
x)**2/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12a^2\sin(c+}{5d\sqrt{\cos(c+}}$$

```
[Out] -12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^2*sin(d*x+c)/d/cos(d*x+c)
^(7/2)+4/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+8/7*a^2*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+12/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12a^2\sin(c+}{5d\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(3/2
)) + (12*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos}\right.$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(4a^2\right.$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(4a^2\right)$$

$$= -\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [C] time = 6.24, size = 531, normalized size = 3.61

$$\frac{3\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[c + d*
x]^2*(7*Sin[c] + 10*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^3*(5*Sin[c] +
14*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]*(10*Sin[c] + 21*Sin[d*x]))/(35*
d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(7*d*Sqrt[1 + Cot[c]^2]) + (3*Cos[c + d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(
a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
```

```
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

maple [B] time = 6.89, size = 439, normalized size = 2.99

$$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{224\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{d}{2}\right.\right.}}{14\left(-\frac{1}{2}+\cos^2\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/224*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-1/14*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+31/70*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*c
os(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

mupad [B] time = 1.46, size = 114, normalized size = 0.78

$$\frac{30\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)+84\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right.\right.}{105\,d\,{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{1-{\cos\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)

[Out] (30*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 70*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)

[Out] Timed out

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=147

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a}{}$$

```
[Out] 68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/2)*s
in(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4264, 3791, 3769, 3771, 2639, 2641}

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3}{\sec^{\frac{5}{2}}(}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{6a^3\cos}{}$$

$$= \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{6a^3\mathrm{c}}{}$$

$$= \frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c}{21d}$$

$$= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c}{21d}$$

Mathematica [C] time = 6.13, size = 548, normalized size = 3.73

$$-\frac{17\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-17*Cot[c]
)/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d)
+ (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*Cos
[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*Sin[
3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*Cos[c + d*x]^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(17*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
```

```
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(60*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos(dx+c)^4\sec(dx+c)^3 + 3a^3\cos(dx+c)^4\sec(dx+c)^2 + 3a^3\cos(dx+c)^4\sec(dx+c) + a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4)*sqrt(cos(d
*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

maple [A] time = 3.89, size = 260, normalized size = 1.77

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.15, size = 206, normalized size = 1.40

$$\frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{3\,d}-\frac{2\left(\frac{33\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}-\frac{5\,a^3\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)}{\sqrt{\sin(c+d\,x)^2}}\right)\,{}_2\mathrm{F}}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2
) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyperg
eom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9/2)*
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x
)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/
4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a}{}$$

```
[Out] 28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*sin(d
*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4264, 3791, 3769, 3771, 2641, 2639}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3}{\sec^{\frac{3}{2}}(}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^3\cos}{}$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c}{21d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(}{21d}$$

Mathematica [C] time = 6.13, size = 516, normalized size = 4.26

$$-\frac{7\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(d\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{c}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-7*Cot[c])
/(10*d) + (107*Cos[d*x]*Sin[c])/(336*d) + (3*Cos[2*d*x]*Sin[2*c])/(40*d) +
(Cos[3*d*x]*Sin[3*c])/(112*d) + (107*Cos[c]*Sin[d*x])/(336*d) + (3*Cos[2*c]
*Sin[2*d*x])/(40*d) + (Cos[3*c]*Sin[3*d*x])/(112*d)) - (13*Cos[c + d*x]^3*C
sc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]
) - (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((
HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[
c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
))/(20*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3)*sqrt(cos(d
*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

maple [A] time = 4.00, size = 272, normalized size = 2.25

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.03, size = 143, normalized size = 1.18

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}-\frac{6\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\right.}{7\,d\,\sqrt{\sin\left(c+d\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3
*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c + d*
x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)
) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(
c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

```
[Out] 36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3769, 3771, 2639, 2641}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3}{\sqrt{\sec(}}\right.$$
$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\right.$$
$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+a^3\int \frac{}{\sqrt{}}$$
$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+}{d}$$
$$= \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+}{d}$$

Mathematica [C] time = 6.11, size = 233, normalized size = 2.56

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos(dx+c)^2\sec(dx+c)^3+3\,a^3\cos(dx+c)^2\sec(dx+c)^2+3\,a^3\cos(dx+c)^2\sec(dx+c)+a^3\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^2*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2)*sqrt(cos(d
*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

maple [A] time = 3.30, size = 250, normalized size = 2.75

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 0.97, size = 104, normalized size = 1.14

$$\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}-\frac{2\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\right.}{7\,d\,\sqrt{\sin(c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2
)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos(c+d\right.$$

$$= \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\left(3a^3\right)\int \frac{1}{\sqrt{\cos(c+}}$$

$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{}}{}$$

$$= \frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3}{}$$

Mathematica [C] time = 4.91, size = 240, normalized size = 2.64

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-3*Cos[d*x]*Csc[c] - 9*Cos[2*
c + d*x]*Csc[c] + 9*Cos[c - d*x - ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] + 3
*Cos[c + d*x + ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt
[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Sin[
2*(c + d*x)] - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Ar
cTan[Tan[c]]]^2]))/(24*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)+a^3\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)
^2 + 3*a^3*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))*sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

maple [A] time = 4.09, size = 172, normalized size = 1.89

$$-\frac{4a^3\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.02, size = 104, normalized size = 1.14

$$\frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}+\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c\right.}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*
x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(
c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.371 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(
3/2)+6*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3771, 2639, 2641, 3768}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\sec(c+dx)} + \right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a^3\sqrt{\cos(c+dx\right.$$

$$= \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + a^3\int \sqrt{\cos(c+dx)}\,dx + \left(3a^3\right)\int$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin}{d\sqrt{\cos}}$$

$$= -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3}{d\sqrt{}}$$

Mathematica [C] time = 6.19, size = 479, normalized size = 5.26

$$\frac{\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)}}\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(-1/8*((-5 +
 Cos[2*c])*Csc[c]*Sec[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Se
c[c]*Sec[c + d*x]*(Sin[c] + 9*Sin[d*x]))/(12*d)) - (5*Cos[c + d*x]^3*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (C
os[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec(dx+c)^3 + 3\,a^3\sec(dx+c)^2 + 3\,a^3\sec(dx+c) + a^3\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

maple [B] time = 5.92, size = 371, normalized size = 4.08

$$4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(10\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.64, size = 126, normalized size = 1.38

$$\frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{6\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{I}}{3\,d\cos(c+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/d
 + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-3/
```

```
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sqrt{\cos(c+dx)}\sec(c+dx)\,dx+\int 3\sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\sec^3(c+dx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(3*sqrt(cos(
c + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x))*sec(c + d*x)**3
, x) + Integral(sqrt(cos(c + d*x)), x))
```

3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
5/2)+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3771, 2641, 3768, 2639}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + 3a^3\sec^{\frac{5}{2}}(c+$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}$$

$$= \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + a^3\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}($$

$$= \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + a^3\int$$

$$= -\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} -$$

Mathematica [C] time = 6.21, size = 501, normalized size = 4.28

$$\frac{9\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((9*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d
*x]^2*(Sin[c] + 5*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 18*S
in[d*x]))/(20*d)) - (Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (9*Cos[c + d*x]^3*Csc[c]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

maple [B] time = 5.70, size = 386, normalized size = 3.30

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\cos\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.58, size = 154, normalized size = 1.32

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+\cdots\right.\right.}{d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{3\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{\sec^3(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(3*sec(c + d
*x)**2/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x)**3/sqrt(cos(c + d*x))
, x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d*x+
c)^(7/2)+6/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+28/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3768, 3771, 2639, 2641}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx+\left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{1}{7}\left(5a^3\sqrt{c}\right.$$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+a^3\int$$

$$= -\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+$$

$$= -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [C] time = 6.24, size = 531, normalized size = 3.61

$$\frac{7\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((7*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d
*x]^3*(5*Sin[c] + 21*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*Sin[c]
 + 130*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(65*Sin[c] + 147*Sin[d*x])
)/(210*d)) - (13*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) + (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*
x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
```

```
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]]))/(20*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^3 \sec(dx+c)^3 + 3\,a^3 \sec(dx+c)^2 + 3\,a^3 \sec(dx+c) + a^3}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

maple [B] time = 6.61, size = 439, normalized size = 2.99

$$-\frac{16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-\frac{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{160\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{10\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-3/160*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-
2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+53/105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/20*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-13/
168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 1.64, size = 145, normalized size = 0.99

$$\frac{\frac{2 a^3 \sin(c+d x)\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};\cos(c+d x)^2\right)}{7}+\frac{6 a^3 \cos(c+d x)\sin(c+d x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};\cos(c+d x)^2\right)}{5}+2 a^3 \cos\left(c+d x\right)^2 \sin\left(c+d x\right)\,{}_2}{d \cos\left(c+d x\right)^{7/2}\sqrt{1-\cos\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*a
^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/
5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(
c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{3\sec\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{3\sec^2\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{\sec^3\left(c+dx\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(3*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(3*sec(c +
d*x)**2/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**3/cos(c + d*x)**(3
/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)}{d(a\sec(c}$$

```
[Out] 21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-
cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c)^(1
/2)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4264, 3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)}{d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{\left(7\sqrt{\cos(\right.}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

$$= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)}{5ad}$$

Mathematica [C] time = 1.77, size = 314, normalized size = 2.45

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{-20\sin(c)\cos(dx)+6\sin(2c)\cos(2dx)-20\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)-30\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)-96\cot(c)-30\csc}{d\sqrt{\cos(c+dx)}}\right.}{15a(\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
 + d*x)))/E^(I*(c + d*x))]) + (-96*Cot[c] - 30*Csc[c] - 20*Cos[d*x]*Sin[c]
+ 6*Cos[2*d*x]*Sin[2*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 20*Co
s[c]*Sin[d*x] + 6*Cos[2*c]*Sin[2*d*x])/(d*Sqrt[Cos[c + d*x]])))/(15*a*(1 +
Sec[c + d*x]))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)

maple [A] time = 3.64, size = 229, normalized size = 1.79

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(25\,\mathrm{Ellipt}\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)

[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\sec(c+dx)+a)}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d-si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\left(5\sqrt{\cos\right.}}{}$$

$$= \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{\left(5\sqrt{\right.}}{}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{5\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}}{d(a+a\sec}$$

Mathematica [C] time = 2.15, size = 292, normalized size = 2.92

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)+4\cos(c)\sin(dx)+6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+12\cot(c)+6\csc(c)}{d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx}}\right.}{}\right.}{3a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*
Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]) + (12*Cot[c] + 6*Csc[c] + 4*Cos[d*x]*Sin[c] + 6*S
ec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 4*Cos[c]*Sin[d*x])/(d*Sqrt[Cos[c +
d*x]])))/(3*a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)

maple [A] time = 3.93, size = 215, normalized size = 2.15

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(5\,\mathrm{EllipticF}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)

[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1
/2)
```

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3819, 3787, 3771, 2639, 2641}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - Sin
[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,d}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a}$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

Mathematica [C] time = 1.74, size = 270, normalized size = 3.75

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\cot(c)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+\left(-1+e\right.\right.}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i}}}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec
[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) - (2*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*
Sin[(d*x)/2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.32, size = 199, normalized size = 2.76

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(\mathrm{EllipticF}\left(c\cdots\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{si}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))/a/d+sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1/
2)
```

Rubi [A] time = 0.14, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3820, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + Sin[
c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a^2}$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\int \sqrt{\cos(c+}}{2a}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c-}$$

Mathematica [C] time = 1.09, size = 262, normalized size = 3.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+\left(-1+e^{2ic}\right)e^{i(c+}\right.}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i(c+dx)}}\left(1+\right.}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^
((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(
d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)\sec(dx+c)+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

maple [A] time = 3.35, size = 198, normalized size = 2.83

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{EllipticF}\left(\cos\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)

[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)

[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a

3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2
)
```

Rubi [A] time = 0.14, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3818, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - Sin[c +
 d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\cos(c+dx)}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

Mathematica [C] time = 1.11, size = 263, normalized size = 3.76

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{2}e^{-i(c+dx)}\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-\left(-1+e^{2ic}\right)e^{i(c+dx)}\right.}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i(c+dx)}\left(1+e^{2}\right.}}}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(
(2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(d
*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2\sec(dx+c)+a\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 3.39, size = 200, normalized size = 2.86

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \text{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x)/a
```

3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=96

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] -3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x
+c))+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{2a}$$

$$= -\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{}{d\cos^{\frac{3}{2}}(c}$$

Mathematica [C] time = 1.75, size = 303, normalized size = 3.16

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+d}}\right.}{}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c +
 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Cos[c + d*x]^(3/2)) -
((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]
)))/(a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

maple [A] time = 4.32, size = 253, normalized size = 2.64

$$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)

[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=124

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-sin
(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))-3*sin(d*x+c)/a/d/cos(d*x+c)^(1/
2)
```

Rubi [A] time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4264, 3818, 3787, 3768, 3771, 2639, 2641}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c}{ad\sqrt{\cos}$$

Mathematica [C] time = 3.75, size = 338, normalized size = 2.73

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(10\cos\left(\frac{1}{2}(c-dx)\right)+8\cos\left(\frac{1}{2}(3c+dx)\right)+4\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+9\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec\left(\frac{1}{2}(c+dx\right.}{4d\cos^{\frac{5}{2}}(c+dx)}\right.}{3a(\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(-1/4*((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*
Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c
/2]*Sec[c/2]*Sec[(c + d*x)/2])/(d*Cos[c + d*x]^(5/2)) + ((2*I)*Sqrt[2]*(9*(
1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a*(1 + Sec
[c + d*x]))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4\sec(dx+c)+a\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^4*sec(d*x + c) + a*cos(d*x + c)
^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 6.56, size = 413, normalized size = 3.33

$$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticF}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*x
+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{7/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d}-\frac{3\sin(c+dx)}{a^2d(\sec(c}$$

```
[Out] 56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+c)/
a^2/d-3*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^(3/
2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d}-\frac{3\sin(c+dx)}{a^2d(\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a^2*d*(1
+ Sec[c + d*x])) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx}}{3a^2}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(15\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(}{a^2d(1+\sec(c+}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(}{a^2d(1+\sec(c+}$$

$$= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + 56\cos^{\frac{3}{2}}(c$$

Mathematica [C] time = 2.59, size = 366, normalized size = 2.29

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\left(-40\sin(c)\cos(dx)+6\sin(2c)\cos(2dx)-40\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)+5\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)+5\tan\left(\frac{c}{2}\right)\sec^2\left(\right.\right.}{3d\cos^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(56*(1 + E^((2*I)*(c + d*x))) + 56*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (2*(-216*Cot[c] - 120*Csc[c] - 40*Cos[d*x]*
Sin[c] + 6*Cos[2*d*x]*Sin[2*c] - 120*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]
 + 5*Sec[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 40*Cos[c]*Sin[d*x] + 6*Cos[
2*c]*Sin[2*d*x] + 5*Sec[(c + d*x)/2]^2*Tan[c/2]))/(3*d*Cos[c + d*x]^(3/2)))
)/(5*a^2*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 3.99, size = 283, normalized size = 1.77

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
```

```
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)*cos(d*x+c)^(1/2)/
a^2/d-7/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*c
os(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+d}}{3a^2}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(}\right.}{}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c}{3d(a+a\sec(c+d}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos}}{3a^2}$$

Mathematica [C] time = 1.80, size = 341, normalized size = 2.47

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin(c)\cos(dx)+8\cos(c)\sin(dx)-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)-2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+36\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+d\right.}{d\cos^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]) + (48*Cot[c] + 36*Csc[c] + 8*Cos[d*x]*Sin[c]
 + 36*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2*Sec[c/2]*Sec[(c + d*x)/2]^
3*Sin[(d*x)/2] + 8*Cos[c]*Sin[d*x] - 2*Sec[(c + d*x)/2]^2*Tan[c/2])/(d*Cos[
c + d*x]^(3/2))))/(3*a^2*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 3.86, size = 270, normalized size = 1.96

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2(\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/
2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2
*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*
d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

```
[Out] 4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))
/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - S
in[c + d*x]/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+}{\sqrt{\sec(c+dx}}}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{(}{}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{(5}{}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{5}{}$$

$$= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{}{3a}$$

Mathematica [C] time = 6.19, size = 374, normalized size = 3.34

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{8\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{8\cot\left(\frac{c}{2}\right)}{d}\right)}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}+\frac{4i\sqrt{2}\,e^{-i(c+dx)}\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (((4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12*
(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/
2]^4*((-8*Cot[c/2])/d - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (2*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 3.76, size = 257, normalized size = 2.29

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{Ell}\cdots\right.}{6a^2\sqrt{-2\left(\mathrm{si}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2
*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d
*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a
+a*sec(d*x+c))^2+sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c +
d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(}$$

Mathematica [C] time = 6.18, size = 656, normalized size = 6.02

$$-\frac{4\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\,\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\right.}}{3d\sqrt{\cot^2(c)+1}\,(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-1/2*I)*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*((2*E^((2*I
)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*
Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
```

```
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d
*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec[c
 + d*x])^2 - (4*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*((4*Csc
[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) - (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*
d)))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*
x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

maple [A] time = 3.97, size = 257, normalized size = 2.36

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Ell}\cdots}{6a^{2}\sqrt{-2\left(\sin\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/
2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x
+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos{\left(c+dx \right)}}\sec^{2}{\left(c+dx \right)}+2\sqrt{\cos{\left(c+dx \right)}}\sec{\left(c+dx \right)}+\sqrt{\cos{\left(c+dx \right)}}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c
 + d*x) + sqrt(cos(c + d*x))), x)/a**2
```

3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=57

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] 1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3815, 21, 3771, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{3a}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{6a^2}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}$$

$$= \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

Mathematica [A] time = 0.24, size = 63, normalized size = 1.11

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
[Out] (4*Cos[(c + d*x)/2]^4*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2\sec(dx+c)^2+2a^2\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 3.72, size = 188, normalized size = 3.30

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}{\left(c+dx \right)}\sec^{2}{\left(c+dx \right)}+2\cos^{\frac{3}{2}}{\left(c+dx \right)}\sec{\left(c+dx \right)}+\cos^{\frac{3}{2}}{\left(c+dx \right)}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x)**2 + 2*cos(c + d*x)**(3/2)*sec
(c + d*x) + cos(c + d*x)**(3/2)), x)/a**2
```

3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+
a*sec(d*x+c))^2-sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3816, 4019, 3787, 3771, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x
]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a^2}$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}\,(1+\sec(c+d$$

Mathematica [C] time = 5.12, size = 312, normalized size = 2.86

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+}}\right.}{}\right.}{3a^2(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-1/2*((7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*C
os[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(d*Cos[c + d*x]^(3
/2)) + ((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*S
```

```
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 3.36, size = 257, normalized size = 2.36

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{E}}{6a^2\cos\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{\text{si}}{3d\cos^{\frac{5}{2}}(c+d}$$

```
[Out] -4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3
/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2+4*s
in(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{\text{si}}{3d\cos^{\frac{5}{2}}(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+d}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+d}$$

$$= \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{s}{3d\cos^{\frac{5}{2}}(c+}$$

$$= -\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c}$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{}{3a^2d\cos}$$

Mathematica [C] time = 6.26, size = 393, normalized size = 2.89

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{8\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{8\sec(c)\sin(dx)\sec(c+dx)}{d}+\frac{8\text{co}}{}\right.}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (((-4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12
*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)
/2]^4*((8*Cot[c/2]*Sec[c])/d + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])
/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (8*Sec[c]*Sec[c
 + d*x]*Sin[d*x])/d + (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4\sec(dx+c)^2+2a^2\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^4*sec(d*x + c) + a^2*cos(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 4.15, size = 405, normalized size = 2.98

$$-\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(5\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(\sec(c+dx)}$$

```
[Out] 7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(
3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(5/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/
cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(5/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+d}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+d}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+}$$

$$= \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(}{a^2d\sqrt{\mathrm{co}}}$$

Mathematica [C] time = 2.14, size = 372, normalized size = 2.30

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(82\cos\left(\frac{1}{2}(c-dx)\right)+65\cos\left(\frac{1}{2}(3c+dx)\right)+68\cos\left(\frac{1}{2}(c+3dx)\right)+37\cos\left(\frac{1}{2}(5c+3dx)\right)+53\cos\left(\frac{1}{2}(3c+5dx)\right)+10\cos\right.}{8d\cos^{\frac{7}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-1/8*((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 6
8*Cos[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] +
10*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c
 + d*x)/2]^3)/(d*Cos[c + d*x]^(7/2)) + ((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c
+ d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^
((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*S
qrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])
^2)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5\sec(dx+c)^2+2a^2\cos(dx+c)^5\sec(dx+c)+a^2\cos(dx+c)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^5*sec(d*x + c) + a^2*cos(d*x + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

maple [B] time = 6.66, size = 413, normalized size = 2.55

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
```

```
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2
)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{63\sin(}{10d\,(a}$$

```
[Out] 231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin(d*
x+c)/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/5*cos(d*x
+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-63/10*cos(d*x+c)^(3/2)*sin(d*x+
c)/d/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.43, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{63\sin(}{10d\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a +
a*Sec[c + d*x])^2) - (63*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx)}}{5a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c}{10d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c}{10d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(}{5d(a+a\sec(c+}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(}{5d(a+a\sec(c+}$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + 77\cos$$

Mathematica [C] time = 2.57, size = 391, normalized size = 1.89

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\frac{1}{16}\sec\left(\frac{c}{2}\right)\left(770\sin\left(c+\frac{dx}{2}\right)-840\sin\left(c+\frac{3dx}{2}\right)+150\sin\left(2c+\frac{3dx}{2}\right)-238\sin\left(2c+\frac{5dx}{2}\right)-40\sin\left(3c+\frac{5dx}{2}\right)-5\sin\left(3c+\frac{7dx}{2}\right)-5\sin\left(\right.\right.}{\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((42*I)*Sqrt[2]*(11*(1 + E^((2*I)*(c + d*x))) + 11*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[c + d*x]^3)/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (-264*Cot[c] - 198*Csc[c] + (Sec[c/2]*Sec[(
c + d*x)/2]^5*(-1210*Sin[(d*x)/2] + 770*Sin[c + (d*x)/2] - 840*Sin[c + (3*d
*x)/2] + 150*Sin[2*c + (3*d*x)/2] - 238*Sin[2*c + (5*d*x)/2] - 40*Sin[3*c +
 (5*d*x)/2] - 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + Sin[4*c + (
9*d*x)/2] + Sin[5*c + (9*d*x)/2]))/16)/Cos[c + d*x]^(5/2)))/(5*a^3*d*(1 + S
ec[c + d*x])^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 3.72, size = 296, normalized size = 1.43

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d
*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2
*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\cos(c+dx)}}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+d}{3ad(a\,\mathrm{se}}$$

```
[Out] -119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)*cos(d*x+c)^(
1/2)/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-2/3*sin(d*x
+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\cos(c+dx)}}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+d}{3ad(a\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - (119*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx)}\sin(c-}{30d\left(a^3+a^3\sec(c+d\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx)}\sin(c-}{30d\left(a^3+a^3\sec(c+d\right.}$$

$$= \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+d}{3ad(a+a\sec(c+dx))^2}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} -$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c}{5d(a-}$$

Mathematica [C] time = 2.05, size = 375, normalized size = 2.07

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\frac{1}{4}\sec\left(\frac{c}{2}\right)\left(-709\sin\left(c+\frac{dx}{2}\right)+715\sin\left(c+\frac{3dx}{2}\right)-170\sin\left(2c+\frac{3dx}{2}\right)+202\sin\left(2c+\frac{5dx}{2}\right)+25\sin\left(3c+\frac{5dx}{2}\right)+5\sin\left(3c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{7c}{2}\right.\right.}{3d\cos^{\frac{5}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]) + (720*Cot[c] + 708*Csc[c] + (Sec[c/2]*Sec
[(c + d*x)/2]^5*(1061*Sin[(d*x)/2] - 709*Sin[c + (d*x)/2] + 715*Sin[c + (3*
d*x)/2] - 170*Sin[2*c + (3*d*x)/2] + 202*Sin[2*c + (5*d*x)/2] + 25*Sin[3*c
+ (5*d*x)/2] + 5*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (7*d*x)/2]))/4)/(3*d*Co
s[c + d*x]^(5/2))))/(5*a^3*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^3\sec(dx+c)^3+3a^3\sec(dx+c)^2+3a^3\sec(dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 3.64, size = 283, normalized size = 1.56

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\sec(c+$$

```
[Out] 49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+
c))^3/cos(d*x+c)^(1/2)-8/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1
/2)-13/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\sec(c+dx)+a^3)}-\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - Sin[c + d*x]/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3) -
(8*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) - (13*S
in[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+\frac{5}{2}a}{\sqrt{\sec(c+dx)}\,(a}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{(}{}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{}{6a}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{}{6a}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2} - \frac{}{6a}$$

$$= \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{}{15}$$

Mathematica [C] time = 1.73, size = 357, normalized size = 2.30

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(806\cos\left(\frac{1}{2}(c-dx)\right)+664\cos\left(\frac{1}{2}(3c+dx)\right)+470\cos\left(\frac{1}{2}(c+3dx)\right)+265\cos\left(\frac{1}{2}(5c+3dx)\right)+117\cos\left(\frac{1}{2}(3c+5dx)\right)+30\,c\right.}{8d\cos^{\frac{5}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/8*((806*Cos[(c - d*x)/2] + 664*Cos[(3*c + d*x)/2] +
 470*Cos[(c + 3*d*x)/2] + 265*Cos[(5*c + 3*d*x)/2] + 117*Cos[(3*c + 5*d*x)/
```

```
2] + 30*Cos[(7*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(d*Cos[
c + d*x]^(5/2)) + ((4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

maple [A] time = 3.76, size = 270, normalized size = 1.74

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(348\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{\cos{\left(c+dx \right)}}}{\sec^{3}{\left(c+dx \right)}+3\sec^{2}{\left(c+dx \right)}+3\sec{\left(c+dx \right)}+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a}$$

```
[Out] -9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+1/2*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (2*Si
n[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x
]/(2*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{5}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}\,(a+a\sec(c}$$

$$= -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

Mathematica [C] time = 6.26, size = 721, normalized size = 4.65

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\right.}}{d\sqrt{\cot^2(c)+1}\,(a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (((-9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((
2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
(2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I
)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Se
c[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((36*C
sc[c])/(5*d) + (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (12*Se
c[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*
x)/2]^5*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) + (2
*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)\sec\left(dx+c\right)+a^3\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a^3*cos(d*
x + c)*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^3\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

maple [A] time = 3.61, size = 270, normalized size = 1.74

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x
+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{\cos{\left(c+dx \right)}}\sec^{3}{\left(c+dx \right)}+3\sqrt{\cos{\left(c+dx \right)}}\sec^{2}{\left(c+dx \right)}+3\sqrt{\cos{\left(c+dx \right)}}\sec{\left(c+dx \right)}+\sqrt{\cos{\left(c+dx \right)}}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c + d*x))*sec(c
 + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a*
*3
```

3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] -1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/
2)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 + Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - Sin[c + d
*x]/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6*d*
Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
```

```
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{5a^2}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

Mathematica [C] time = 1.57, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(14\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+20\cos\left(\frac{1}{2}(c+3dx)\right)-5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}(c+\cdots\right.}{8d\cos^{\frac{5}{2}}(c+dx)}\cdots}{15a^3(\mathrm{se}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((14*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*x)/2] + 20*Cos
[(c + 3*d*x)/2] - 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[c/2]
*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d*Cos[c + d*x]^(5/2)) - ((4*I)*Sqrt[2]*(3*
(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1
 + Sec[c + d*x])^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^2\sec(dx+c)^3+3a^3\cos(dx+c)^2\sec(dx+c)^2+3a^3\cos(dx+c)^2\sec(dx+c)+a^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^2*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x
 + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 4.18, size = 270, normalized size = 1.74

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
```

```
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)$$

```
[Out] 1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2
)/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
- Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - (4*Sin[c +
 d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6
*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
```

```
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}$$

Mathematica [C] time = 1.60, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(4\cos\left(\frac{1}{2}(c-dx)\right)+26\cos\left(\frac{1}{2}(3c+dx)\right)+10\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}(c+\right.}{8d\cos^{\frac{5}{2}}(c+dx)}\right.}{15a^3(\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/8*((4*Cos[(c - d*x)/2] + 26*Cos[(3*c + d*x)/2] + 10
*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[
c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(d*Cos[c + d*x]^(5/2)) + ((4*I)*Sqrt[2]*(
3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c +
 d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1
 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]))))/(15*a^3*
(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3\sec(dx+c)^3+3a^3\cos(dx+c)^3\sec(dx+c)^2+3a^3\cos(dx+c)^3\sec(dx+c)+a^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^3*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x
 + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 3.56, size = 270, normalized size = 1.74

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{I}\cdots\right.}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
```

```
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)\cdots}$$

```
[Out] 9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(5/2
)/(a+a*sec(d*x+c))^3-2/5*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
-9/10*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3816, 4019, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3) - (2*Sin
[c + d*x])/(5*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - (9*Sin[c + d
*x])/(10*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
```

```
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+d}$$

Mathematica [C] time = 6.25, size = 721, normalized size = 4.65

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\text{ta}\right.}}{d\sqrt{\cot^2(c)+1}\,(a\,\text{se}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]

[Out] (((9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((-36*Csc[c])/(5*d) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3)

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^4\sec(dx+c)^3+3\,a^3\cos(dx+c)^4\sec(dx+c)^2+3\,a^3\cos(dx+c)^4\sec(dx+c)+a^3 \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)

maple [A] time = 3.79, size = 268, normalized size = 1.73

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1} \cdots}{20 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)

[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El

```
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{\ldots}{15ad\,\mathrm{c}\ldots}$$

```
[Out] -49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(
7/2)/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c
))^2-13/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))+49/10*sin(d*x+
c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{\ldots}{15ad\,\mathrm{c}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Cos[c
+ d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec}$$

$$= \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{}{15ad\,c}$$

$$= -\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx}{5d\cos^{\frac{7}{2}}(c+dx)(a+a}$$

$$= -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} -$$

Mathematica [C] time = 1.91, size = 372, normalized size = 2.06

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\left(\frac{1}{2}(5c+3dx)\right)+670\cos\left(\frac{1}{2}(3c+5dx)\right)+\right.}{16d\cos^{\frac{7}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 124
3*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2]
+ 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2]^5)/(16*d*Cos[c + d*x]^(7/2)) - ((4*I)*Sqrt[2]*(147*(1 + E^((2
*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1 + Sec[
c + d*x])^3)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^5\sec(dx+c)^3 + 3a^3\cos(dx+c)^5\sec(dx+c)^2 + 3a^3\cos(dx+c)^5\sec(dx+c) + a^3\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^5*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^5*sec(d*x + c) + a^3*cos(d*x
 + c)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

maple [B] time = 4.09, size = 555, normalized size = 3.07

$$-\frac{-2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{9/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3\sec(c+\right.}$$

```
[Out] 119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(d*x
+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3-2/3*sin(d*x+
c)/a/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x+c)^(
5/2)/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3\sec(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(9/2)*
(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(5/2)*(a^3 + a^3*
Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{5a}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(}$$

$$= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119}{10a^3}$$

Mathematica [C] time = 2.78, size = 402, normalized size = 1.94

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(5134\cos\left(\frac{1}{2}(c-dx)\right)+4148\cos\left(\frac{1}{2}(3c+dx)\right)+4664\cos\left(\frac{1}{2}(c+3dx)\right)+2476\cos\left(\frac{1}{2}(5c+3dx)\right)+3340\cos\left(\frac{1}{2}(3c+5dx)\right)}{96d\cos^{\frac{9}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-1/96*((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2
] + 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5
*d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(
9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)
/2]^5)/(d*Cos[c + d*x]^(9/2)) + ((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x)
)) + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I
)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(
1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(5*a^3*(1 + Sec[c + d*x])^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^6\sec(dx+c)^3 + 3a^3\cos(dx+c)^6\sec(dx+c)^2 + 3a^3\cos(dx+c)^6\sec(dx+c) + a^3\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^6*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^6*sec(d*x + c) + a^3*cos(d*x
 + c)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)), x)
```

maple [A] time = 6.67, size = 453, normalized size = 2.19

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{32\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{118\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(32/15*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+11
8/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+48*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*
c)^2)^(1/2)+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)^5-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{11/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\cos(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(}{35d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

```
[Out] 12/35*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*cos(d*x+
c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*s
ec(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\cos(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(}{35d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (32*a*Sin[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (1
6*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{\text{se}}$$
$$= \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}\left(24\sqrt{c}\right.$$
$$= \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}}{7d\sqrt{a}}$$
$$= \frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A] time = 0.21, size = 80, normalized size = 0.52

$$\frac{(140\sin(c+dx)+42\sin(2(c+dx))+12\sin(3(c+dx))+5\sin(4(c+dx)))\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}{140d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(140*Sin[c + d*x] + 42*Sin[2
*(c + d*x)] + 12*Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(140*d*(1 + Cos[c
+ d*x]))
```

fricas [A] time = 0.66, size = 79, normalized size = 0.52

$$\frac{2\left(5\cos(dx+c)^3+6\cos(dx+c)^2+8\cos(dx+c)+16\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{35\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

maple [A] time = 1.15, size = 80, normalized size = 0.52

$$-\frac{2\left(5\left(\cos^4(dx+c)\right)+\cos^3(dx+c)+2\left(\cos^2(dx+c)\right)+8\cos(dx+c)-16\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}(dx+c)\right)}{35d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [B] time = 0.87, size = 293, normalized size = 1.92

$$\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)
*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{7/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x+
c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+c)*cos(d*x+c)^(
1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (8
*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}\left(4\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{}}{}$$
$$= \frac{8a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{15}\left(8\sqrt{}\right.$$
$$= \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{8a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.18, size = 61, normalized size = 0.53

$$\frac{\sqrt{\cos(c+dx)}\,(8\cos(c+dx)+3\cos(2(c+dx))+19)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (Sqrt[Cos[c + d*x]]*(19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 0.84, size = 69, normalized size = 0.60

$$\frac{2\left(3\cos(dx+c)^2+4\cos(dx+c)+8\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.24, size = 70, normalized size = 0.61

$$-\frac{2\left(3\left(\cos^3(dx+c)\right)+\cos^2(dx+c)+4\cos(dx+c)-8\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,(dx+c)\right)}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [B] time = 0.97, size = 203, normalized size = 1.77

$$\frac{\sqrt{2}\left(30\,\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\cos\left(\frac{2}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 4/3*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)
*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \\ &= \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 49, normalized size = 0.64

$$\frac{2\sqrt{\cos(c+dx)}(\cos(c+dx)+2)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
+ d*x)/2])/(3*d)
```

fricas [A] time = 1.15, size = 57, normalized size = 0.74

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos\left(dx+c\right)+2\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) + 2)*sqrt(cos(d*x
 + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 1.23, size = 58, normalized size = 0.75

$$-\frac{2\left(\cos^{2}\left(dx+c\right)+\cos\left(dx+c\right)-2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,\left(dx+c\right)\right)}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(
d*x+c)^(1/2)/sin(d*x+c)
```

maxima [A] time = 1.23, size = 113, normalized size = 1.47

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.401 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4264, 3804}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 39, normalized size = 1.08

$$\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

fricas [A] time = 1.27, size = 49, normalized size = 1.36

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(dx+c) + a}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 0.96, size = 50, normalized size = 1.39

$$-\frac{2\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/s
in(d*x+c)
```

maxima [A] time = 0.59, size = 20, normalized size = 0.56

$$\frac{2\sqrt{2}\sqrt{a}\,\sin\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\cos(c+d\,x)}\,\sqrt{a + \frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(c+dx)+1)}\,\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4264, 3801, 215}

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx\\
&= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 74, normalized size = 1.30

$$-\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

fricas [A] time = 0.73, size = 180, normalized size = 3.16

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cdots}{a\cos(dx+c\cdots}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*
x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\sec(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.03, size = 142, normalized size = 2.49

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}(dx+c)\right)\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(\cos(dx+c)\cdots}{4}\right.\right.}{2d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-arctan(1/4*(-2/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2)))*(-2/(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)
```

maxima [B] time = 1.55, size = 241, normalized size = 4.23

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3803, 3801, 215}

$$\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx}$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\right.}{d}$$

$$= \frac{\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.24, size = 90, normalized size = 0.98

$$\frac{2a\sin(c+dx)\left(\frac{1}{2}\cos(c+dx)+\frac{\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\right)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(Cos[c + d*x]/2 + ArcSin[Sqrt[1 - Sec[c + d*x]]]/(2*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(3/2)))*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*Sqrt[a*(1 + S
ec[c + d*x])])
```

fricas [A] time = 0.54, size = 325, normalized size = 3.53

$$\left[\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*((cos(d*x + c)^2 + cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 1.00, size = 178, normalized size = 1.93

$$-\frac{(-1+\cos(dx+c))\left(-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)+\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(c\cdots}{\cdots}\right.\right.}{2d\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)^{2}\sqrt{c\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(-arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+arctan(1/4*(-2/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+2*sin(d*x+c)*(-2/(
1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)
))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

maxima [B] time = 1.50, size = 662, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{4d}$$

```
[Out] 3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+1/2*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)
)^(1/2)+3/4*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3803, 3801, 215}

$$\frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (3*a*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+\ldots$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}\left(3\sqrt{\ldots}\right.$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(3\sqrt{\cos\ldots\right.}{\ldots}$$

$$= \frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec\ldots}}$$

Mathematica [A] time = 0.49, size = 100, normalized size = 0.74

$$\frac{2a\sin(c+dx)\left(\frac{1}{8}\cos(c+dx)(3\cos(c+dx)+2)+\frac{3\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\right)}{d\cos^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sin[c + d*x])/(d*Cos[c
 + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.55, size = 355, normalized size = 2.61

$$\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(d\ldots}{\ldots}\right.}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(co
s(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2),
 1/8*(2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(c
os(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.01, size = 213, normalized size = 1.57

$$\left(3\left(\cos^2(dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1-\sin(dx+c)}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/16/d*(3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c))*2^(1/2))-3*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+6*(-2/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 0.72, size = 1264, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
```

```
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{26a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{208\ldots}{105d\sqrt{\cos(c+\ldots}}$$

```
[Out] 26/35*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a^2*cos(
d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)/d/c
os(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3813, 21, 3805, 3804}

$$\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{26a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{208\ldots}{105d\sqrt{\cos(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (104*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (26*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(13a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}(5$$

$$= \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + 2a$$

$$= \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+d}{105d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.28, size = 72, normalized size = 0.45

$$\frac{a\sqrt{\cos(c+dx)}(253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx))+494)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+}}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Co
s[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

fricas [A] time = 0.57, size = 84, normalized size = 0.52

$$\frac{2\left(15\,a\cos(dx+c)^3+39\,a\cos(dx+c)^2+52\,a\cos(dx+c)+104\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{105\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

maple [A] time = 1.17, size = 83, normalized size = 0.52

$$-\frac{2\left(15\left(\cos^4\left(dx+c\right)\right)+24\left(\cos^3\left(dx+c\right)\right)+13\left(\cos^2\left(dx+c\right)\right)+52\cos\left(dx+c\right)-104\right)\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{a(1}{}}}{105d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-104
)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

maxima [B] time = 0.56, size = 303, normalized size = 1.88

$$\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+175\,a\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,d\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+d}}{5d}$$

```
[Out] 2/5*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+8/5*a^2*sin(d*x+c)
/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*cos(d*x+c)^(1/2
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4264, 3812, 3809, 3804}

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+d}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\Big(3\sqrt{\cos(c+dx}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\cos^{\frac{3}{2}}(c+dx)(a}{}$$
$$= \frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(}}{5d}$$

Mathematica [A] time = 0.22, size = 60, normalized size = 0.52

$$\frac{a\sqrt{\cos(c+dx)}(6\cos(c+dx)+\cos(2(c+dx))+13)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
[Out] (a*Sqrt[Cos[c + d*x]]*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 +
Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d)
```

fricas [A] time = 0.58, size = 72, normalized size = 0.62

$$\frac{2\left(a\cos(dx+c)^2+3\,a\cos(dx+c)+6\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{5\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.17, size = 71, normalized size = 0.61

$$-\frac{2\left(\cos^3(dx+c)+2\left(\cos^2(dx+c)\right)+3\cos(dx+c)-6\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{5d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*cos(d*x+c)^(1/2)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

maxima [B] time = 1.56, size = 210, normalized size = 1.81

$$\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] 8/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+
c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3809, 3804}

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\Big(4a\sqrt{\cos(c+d\\ &= \frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(\end{aligned}$$

Mathematica [A] time = 0.16, size = 50, normalized size = 0.63

$$\frac{2a\sqrt{\cos(c+dx)}(\cos(c+dx)+5)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(
c + d*x)/2])/(3*d)
```

fricas [A] time = 1.01, size = 61, normalized size = 0.77

$$\frac{2\left(a\cos\left(dx+c\right)+5\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 1.02, size = 61, normalized size = 0.77

$$-\frac{2\left(\cos^{2}\left(dx+c\right)+4\cos\left(dx+c\right)-5\right)\left(\sqrt{\cos}\,\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+4*cos(d*x+c)-5)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/sin(d*x+c)*a
```

maxima [A] time = 1.58, size = 38, normalized size = 0.48

$$\frac{\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.408 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^
(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3813, 21, 3801, 215}

$$\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \left(2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\ldots\right.$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx\ldots\right.$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\ldots\right.}{\ldots}$$

$$= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} + \frac{\ldots}{d\sqrt{\cos(\ldots}}$$

Mathematica [A] time = 0.14, size = 81, normalized size = 0.84

$$\frac{2a^2\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}+\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{\cos(c+dx)-1}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(Sqrt[1 - Sec[c + d*x]] + ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c
+ d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])
])
```

fricas [A] time = 0.47, size = 298, normalized size = 3.10

$$\left[\frac{4a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)\ldots}{\cos(dx+c)^3\ldots}\right.}{2(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.14, size = 172, normalized size = 1.79

$$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-}{2d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/2/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2^(1/2)*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*cos
(d*x+c)-4)/sin(d*x+c)*a
```

maxima [B] time = 1.29, size = 274, normalized size = 2.85

$$\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sqrt{2}\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A] time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3814, 21, 3801, 215}

$$\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c-}{}$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec}$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int\right.}{d}$$

$$= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2\sin(c+}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

Mathematica [A] time = 0.31, size = 92, normalized size = 0.97

$$-\frac{a^2\sin(c+dx)\left(\frac{3\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}-\sqrt{1-\sec(c+dx)}\right)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a^2*(-Sqrt[1 - Sec[c + d*x]] + (3*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[c
 + d*x]])*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]))
```

fricas [A] time = 0.75, size = 337, normalized size = 3.55

$$\left[\frac{4a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + 3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sq
rt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*a*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a*cos(d*
x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
```

```
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.08, size = 182, normalized size = 1.92

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)+3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)\right.\right.}{4d\sqrt{\cos(dx+c)}\,\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+3*arctan(1/4*
(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*
x+c)+2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c)^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

maxima [B] time = 1.79, size = 1143, normalized size = 12.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
```

```
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

```
[Out] 7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+1/2*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+
c))^(1/2)+7/4*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3814, 21, 3803, 3801, 215}

$$\frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (7*a^2*Sin[c + d*x])/(4*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}}{}$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(7a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}($$

$$(7a$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} -$$

$$= \frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

Mathematica [A] time = 0.43, size = 99, normalized size = 0.71

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\right.\right.}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2]*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin[(3*(c + d*x))/
2]))/(8*d*Cos[c + d*x]^(3/2))
```

fricas [A] time = 0.55, size = 369, normalized size = 2.64

$$\left[\frac{4\left(7a\cos(dx+c)+2a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+7\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{a}\,\mathrm{lo}}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sq
rt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^
2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), 1/8*(2*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

maple [A] time = 0.98, size = 212, normalized size = 1.51

$$-\frac{(-1+\cos(dx+c))\left(7\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-7\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}}{}\right.\right.}{8d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+14*(-2/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)/(-2/
(1+cos(d*x+c)))^(1/2)*a
```

maxima [B] time = 1.44, size = 2244, normalized size = 16.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqr
t(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3
```

```
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*
log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
```

```
d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((2*(2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{11a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^2}{12d\cos^{\frac{5}{2}}(c+}$$

```
[Out] 11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x
+c))^(1/2)+11/12*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1
1/8*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3814, 21, 3803, 3801, 215}

$$\frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(12*d*Cos[c + d*
x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c-}{}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}\left(11a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}($$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(11$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{8d\,\mathrm{cc}}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{8d\,\mathrm{cc}}$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+d.}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a}}$$

Mathematica [A] time = 0.52, size = 112, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+66\sqrt{2}\cos^{3}\right.}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[2]*ArcTanh[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*(Sin[(3*(c + d*
x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 0.57, size = 391, normalized size = 2.17

$$\left[\frac{4\left(33\,a\cos\left(dx+c\right)^{2}+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+33\left(a\cos\left(dx+c\right)^{4}\cdots\right.}{96\left(d\cos\left(dx+c\right)^{4}+d\,c\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 33*(a*cos(d*x + c)^
4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos
(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*
x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*
x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.01, size = 242, normalized size = 1.34

$$\frac{\left(-1+\cos\left(dx+c\right)\right)\left(33\sqrt{2}\,\left(\cos^{3}\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos(dx+c)+1-\sin(dx+c)\right)\sqrt{2}}{4}\right)-33\sqrt{2}\,\left(\cos^{3}\left(dx+c\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/48/d*(-1+cos(d*x+c))*(33*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^3*2^(1/2)-33*arctan(1/4*(-2/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^3*2^(1/2)-66*(-2/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2-44*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*sin(d*x+c)-16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+
c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)/(-2/(1+cos(d*x+c)))^(1/
2)*a
```

maxima [B] time = 1.40, size = 2361, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
```

```
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=201

$$\frac{38a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{146a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{584a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{1168}{315d\sqrt{\cos(c+}}$$

```
[Out] 146/105*a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/63*a^3*
cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+a*
sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4264, 3813, 4015, 3805, 3804}

$$\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{146a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
 + (584*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]
]) + (146*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*
x]]) + (38*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*
d)
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d} + \frac{1}{9}\left(2a\sqrt{\cos(c+\right.$$

$$= \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}{9d}$$

$$= \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{584a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{584a^3\sqrt{\cos(c+dx)}\,\sin(c}{315d\sqrt{a+a\sec(c+dx}}$$

Mathematica [A] time = 0.24, size = 90, normalized size = 0.45

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\left(35\cos^4(c+dx)+130\cos^3(c+dx)+219\cos^2(c+dx)+292\cos(c+dx)+584\right)}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584 + 292*Cos[c + d*x] + 219*Cos[c + d*x]^2 + 13
0*Cos[c + d*x]^3 + 35*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*
x])/(315*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.55, size = 105, normalized size = 0.52

$$\frac{2\left(35\,a^2\cos(dx+c)^4+130\,a^2\cos(dx+c)^3+219\,a^2\cos(dx+c)^2+292\,a^2\cos(dx+c)+584\,a^2\right)\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c
)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

maple [A] time = 1.11, size = 95, normalized size = 0.47

$$-\frac{2\left(35\left(\cos^5(dx+c)\right)+95\left(\cos^4(dx+c)\right)+89\left(\cos^3(dx+c)\right)+73\left(\cos^2(dx+c)\right)+292\cos(dx+c)-584\right)\left(\sqrt{}\right.}{315d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)^2+2
92*cos(d*x+c)-584)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin
(d*x+c)*a^2
```

maxima [B] time = 1.30, size = 422, normalized size = 2.10

$$\frac{\sqrt{2}\left(8190\,a^2\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+2100\,a^2\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{9}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+$$

```
[Out] 2/7*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*cos(d*x+c)^(
5/2)*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)/d+64/21*a^3*sin(d*x+c)/d/cos(d*x+c)^
(1/2)/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec
(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{4264, 3812, 3809, 3804\}$

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\Big(5\sqrt{\cos(c+dx}$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2\cos^{\frac{5}{2}}(c+dx)(}{}$$

$$= \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\,s}}{21d}$$

Mathematica [A] time = 0.26, size = 74, normalized size = 0.47

$$\frac{a^2\sqrt{\cos(c+dx)}(101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx))+208)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)}}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
[Out] (a^2*Sqrt[Cos[c + d*x]]*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*C
os[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d)
```

fricas [A] time = 0.51, size = 92, normalized size = 0.59

$$\frac{2\left(3\,a^2\cos(dx+c)^3+12\,a^2\cos(dx+c)^2+23\,a^2\cos(dx+c)+46\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c}{21\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
46*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

maple [A] time = 1.18, size = 85, normalized size = 0.54

$$-\frac{2\left(3\left(\cos^4(dx+c)\right)+9\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+23\cos(dx+c)-46\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos}{\cos(}}}{21d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46)*co
s(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

maxima [B] time = 0.92, size = 323, normalized size = 2.07

$$\frac{\sqrt{2}\left(315\,a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^2\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}}{}$$

```
[Out] 2/5*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d*
x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*cos(d*x
+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3809, 3804}

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(8a\sqrt{\cos(c+dx)}\right.$$
$$= \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a}{}$$
$$= \frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(}}{15d}$$

Mathematica [A] time = 0.24, size = 64, normalized size = 0.54

$$\frac{a^2\sqrt{\cos(c+dx)}(28\cos(c+dx)+3\cos(2(c+dx))+89)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 1.09, size = 79, normalized size = 0.66

$$\frac{2\left(3a^2\cos(dx+c)^2+14a^2\cos(dx+c)+43a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{15(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.82, size = 75, normalized size = 0.63

$$-\frac{2\left(3\left(\cos^3(dx+c)\right)+11\left(\cos^2(dx+c)\right)+29\cos(dx+c)-43\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{15d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*cos(d*x+c)^(1/2)*
(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)*a^2
```

maxima [A] time = 1.26, size = 60, normalized size = 0.50

$$\frac{\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=138

$$\frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{c}$$

```
[Out] 2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+14/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c
))^(1/2)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3813, 4015, 3801, 215}

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2a\sqrt{\cos(c+\right.$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec}}{3d}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec}}{3d}$$

$$= \frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{}{3d\sqrt{\cos}}$$

Mathematica [A] time = 0.24, size = 93, normalized size = 0.67

$$\frac{2a^3\sin(c+dx)\left((\cos(c+dx)+8)\sqrt{1-\sec(c+dx)}+3\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*((8 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*ArcSin[Sqrt[1 - Sec[c
 + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sq
rt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.70, size = 339, normalized size = 2.46

$$\left[\frac{4\left(a^2\cos(dx+c)+8a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)}{}\right.}{6(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a
*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(a^2*cos(d*x
+ c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
```

```
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

maple [A] time = 1.06, size = 185, normalized size = 1.34

$$-\frac{\left(3\sqrt{2}\, \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\, (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\, \sin (dx + c) - 3\sqrt{2}\, \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\, (\cos(\cdots}{\cdots}\right.\right.}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6/d*(3*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*2^(1/2)*arctan(1/4*(-
2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2+28*cos(d*x+c)-32)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)*a^2
```

maxima [B] time = 1.45, size = 593, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.416 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\,\mathrm{se}}}{d\sqrt{\cos(c+}}$$

```
[Out] 5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1
/2)+a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3814, 4015, 3801, 215}

$$\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\Big(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d} + \frac{}{d\sqrt{\cos(}}$$

Mathematica [A] time = 0.29, size = 90, normalized size = 0.68

$$\frac{a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+2)+5\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{\cos(c+dx)-1}\,\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c
+ d*x]]*(2 + Sec[c + d*x]))*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.59, size = 373, normalized size = 2.83

$$\left[\frac{4\left(2\,a^2\cos(dx+c)+a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+5\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 1/2*(2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
```

```
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 1.07, size = 197, normalized size = 1.49

$$-\frac{\left(5\sin(dx+c)\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sqrt{2}-5\sin(dx+c)\cos(}{4d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(5*sin(d*x+c)*cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^(1/2)-5*sin(d*x+c)
*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)*(
a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\,\mathrm{co}}$$

```
[Out] 19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+9/4*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x
+c))^(1/2)+1/2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3814, 4016, 3801, 215}

$$\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{sin}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+\ldots}$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{8}\left(19a^2\ldots\right.$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(19a^2\sqrt{\ldots}\right.}{\ldots}$$

$$= \frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9a^3\sin(c+\ldots}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+\ldots}}$$

Mathematica [A] time = 0.65, size = 95, normalized size = 0.68

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{1-\sec(c+dx)}\,(2\sec(c+dx)+11)-\frac{19\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}\right)}{4d\sqrt{\cos(c+dx)-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sec[c + d*x])]*((-19*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[
c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(11 + 2*Sec[c + d*x]))*Tan[(c + d*x)/2])
/(4*d*Sqrt[-1 + Cos[c + d*x]])
```

fricas [A] time = 0.68, size = 385, normalized size = 2.75

$$\left[\frac{4\left(11\,a^2\cos\left(dx+c\right)+2\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+19\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\ldots}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x
+ c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos
(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 +
```

```
d*cos(d*x + c)^2), 1/8*(2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)
^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(
d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 1.04, size = 214, normalized size = 1.53

$$\frac{(-1+\cos(dx+c))\left(19\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-19\left(\cos^2(dx+c)\right)\sqrt{\cdots}}{8d\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(19*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))-19*cos(d*x+c)^2*2^(1/2)*arctan(
1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))-22*(-2/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c)^(3/2)/sin(d*x+c)^2*a^2
```

maxima [B] time = 10.41, size = 2826, normalized size = 20.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^
2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 7
6*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
```

c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*x +

```
 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2
*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d
*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin}{12d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+13/12*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d
*x+c))^(1/2)+25/8*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+
1/3*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A] time = 0.34, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3814, 4016, 3803, 3801, 215}

$$\frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\cos^{\frac{5}{2}}(c+dx)}+\frac{25}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (25*a^3*Sin[c + d*x])/(8*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{8}\Big(2$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}$$

Mathematica [C] time = 5.47, size = 180, normalized size = 1.00

$$\frac{a^2(\cos(c+dx)+1)^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-75ie^{\frac{1}{2}i(c+dx)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-e^{2i(c+dx)}\right)\cos^3(c+dx)-\right.}{96d\cos^{\frac{5}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
75*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeometric2F1[1/4, 1, 5/4, -E^
((2*I)*(c + d*x))] - (25*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeo
metric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (8 + 34*Cos[c + d*x] + 75*Co
s[c + d*x]^2)*Sin[(c + d*x)/2]))/(96*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 0.71, size = 411, normalized size = 2.28

$$\left[\frac{4\left(75\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+75\left(a^2\cos\left(dx+c\right)^4\right.}{96\left(d\cos\left(dx+c\right)^4+d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d
*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(75*a^2*cos(d*x + c)^2 +
 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

maple [A] time = 1.03, size = 244, normalized size = 1.36

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)\left(75\sqrt{2}\,\left(\cos^3\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos(dx+c)+1+\sin(dx+c)\right)\sqrt{2}}{4}\right)-75\sqrt{2}\,\left(\cos^3\left(dx+c\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(75*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)^3*2^(1/2)-75*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)^3*2^(1/2)+150*(-2/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+68*(-2/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*sin(d*x+c)+16*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(5/2)/(-2/(1+cos(d*x+c)))^(
1/2)*a^2
```

maxima [B] time = 0.95, size = 3469, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)
*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4
56*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*
d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^
2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*
x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*
x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7
5*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan
```

```
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) +
a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*
sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(
6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*
cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*
sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((cos(6*d*x + 6*c)^2 + 6*(cos(
6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x
 + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d
*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(6*d*x + 6*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{163a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{163a^3}{96d\cos^{\frac{5}{2}}(c+d}$$

```
[Out] 163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d+17/24*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec
(d*x+c))^(1/2)+163/96*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1
/2)+163/64*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a^2
*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A] time = 0.40, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3814, 4016, 3803, 3801, 215}

$$\frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Cos[
c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(96*d*Cos
[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(64*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
```

```
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{4}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{48}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{}}$$

Mathematica [C] time = 5.57, size = 190, normalized size = 0.86

$$\frac{a^2(\cos(c+dx)+1)^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-489ie^{\frac{1}{2}i(c+dx)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-e^{2i(c+dx)}\right)\cos^4(c+dx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
489*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^4*Hypergeometric2F1[1/4, 1, 5/4, -E
```

```
^((2*I)*(c + d*x))] - (163*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^4*Hyperg
eometric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (48 + 184*Cos[c + d*x] + 3
26*Cos[c + d*x]^2 + 489*Cos[c + d*x]^3)*Sin[(c + d*x)/2]))/(768*d*Cos[c + d
*x]^(7/2))
```

fricas [A] time = 0.58, size = 437, normalized size = 1.99

$$\left[\frac{4\left(489\,a^{2}\cos\left(dx+c\right)^{3}+326\,a^{2}\cos\left(dx+c\right)^{2}+184\,a^{2}\cos\left(dx+c\right)+48\,a^{2}\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx\right.}{768\left(d\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*
x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((
a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d
*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(c
os(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/
384*(2*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x +
 c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*c
os(d*x + c)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

maple [A] time = 1.07, size = 276, normalized size = 1.25

$$\frac{\left(489\left(\cos^{4}\left(dx+c\right)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-489\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\mathrm{s}}{4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/768/d*(489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c))*2^(1/2))-489*cos(d*x+c)^4*2^(1/2)*arctan(1/4*(-2/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+978*sin(d*x+c)*cos(d*x
+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+652*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*co
s(d*x+c)^2+368*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c)^(7/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2
```

maxima [B] time = 1.66, size = 3860, normalized size = 17.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x
 + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 4
8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c
) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x +
 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos
(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a
^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^
2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x +
2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*
x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*
d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2
+ 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x
 + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x
+ 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*
```

```
x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*si
n(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*
cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2
+ a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4
*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^
2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*
d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(
4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*c
os(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2
*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*si
n(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*
(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*
a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d
*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c)
 + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2
)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c
) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2
)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c)
 + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(
6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sq
rt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}}{}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/5*cos(d*x+c)^(3
/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)
/(a+a*sec(d*x+c))^(1/2)-2/15*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3823, 4022, 4013, 3808, 206}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}}{5a}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+}{5d\sqrt{a+a}}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+}{5d\sqrt{a+a}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{26\,\mathrm{s}}{15d\sqrt{\cos(c+d}}$$

Mathematica [A] time = 0.31, size = 136, normalized size = 0.72

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(13\sec^2(c+dx)-\sec(c+dx)+3\right)+15\sqrt{2}\sec^{\frac{5}{2}}(c+dx)\tan^{-1}\left(\right.\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c +
 d*x] + 13*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.95, size = 324, normalized size = 1.71

$$\left[\frac{4\left(3\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{15\sqrt{2}\left(a\cos(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}+\cdots}{\cdots}\right)}{\cdots}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(4*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) +
 a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqr
t(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*cos(d*x
 + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

maple [A] time = 1.14, size = 120, normalized size = 0.63

$$\frac{\left(15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)-6\left(\cos^{3}\left(dx+c\right)\right)+8\left(\cos^{2}\left(dx+c\right)\right)-28\cos\left(dx\cdots\right.\right.}{15d\sin\left(dx+c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-6*cos(d*x+c)^3+8*cos(d*x+c)^2-28*cos(d*x+c)+26)*(a*(1
+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)/a
```

maxima [B] time = 1.54, size = 357, normalized size = 1.89

$$\frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(\cdots}{\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*sin(d*x+c)/d/c
os(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a
+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3823, 4013, 3808, 206}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(\cdots}{\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}}{3a}$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{\cos(c+\right.$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{2\sin}{3d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.22, size = 116, normalized size = 0.77

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(2(1-\sec(c+dx))^{3/2}-3\sqrt{2}\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(1 - Sec[c + d*x])^(3/2) - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2))*Sin[c + d*
x])/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.86, size = 300, normalized size = 1.99

$$\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)-1\right)\sqrt{\cos(dx+c)}\sin(dx+c) + \frac{3\sqrt{2}\left(a\cos(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\mathrm{co}}}{\sqrt{a}}}{\cos(dx+c)^2+2\,\mathrm{c}}\right.}{\sqrt{a}}}{6\left(ad\cos(dx+c)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(cos
(d*x + c))*sin(d*x + c) + 3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
```

```
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*cos(d*x + c) +
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

maple [A] time = 1.09, size = 110, normalized size = 0.73

$$-\frac{\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)+2\left(\cos^2(dx+c)\right)-4\cos(dx+c)+2\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\cdots}{3d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*(3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos(d*x+c)+2)*(a*(1+cos(d*x+c))/cos(d
*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)/a
```

maxima [B] time = 1.48, size = 282, normalized size = 1.87

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/co
s(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3812, 3808, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c\ldots}}{\sqrt{a+a\sec\ldots}}$$
$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{2a-\ldots}\right.}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{2\sin\ldots}{d\sqrt{\cos(c+dx)}\ldots}$$

Mathematica [A] time = 0.10, size = 100, normalized size = 0.88

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.72, size = 281, normalized size = 2.49

$$\left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx\ldots}{2(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

maple [A] time = 0.97, size = 98, normalized size = 0.87

$$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx+c)-2\cos(dx+c)+2\right)}{d\sin(dx+c)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*cos(
d*x+c)+2)/sin(d*x+c)/a
```

maxima [A] time = 1.27, size = 104, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx\right.\right.}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4264, 3808, 206}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\text{si}}{\sqrt{a+a\sec(c+}}\right.}{d}\\
&= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 1.70

$$-\frac{\sqrt{2}\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])]))
```

fricas [A] time = 0.62, size = 160, normalized size = 2.86

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 1.07, size = 91, normalized size = 1.62

$$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)-1\right)}{d\sin(dx+c)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*arctan(1/2*sin(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(
d*x+c)^2-1)/a
```

maxima [A] time = 1.28, size = 90, normalized size = 1.61

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,d\right.\right.}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/
2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1
/2)
```

Rubi [A] time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4264, 3821, 3801, 215, 3808, 206}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
```

```
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\right) + \frac{\left(\sqrt{\cos\right.}}{}$$

$$= \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c}{\sqrt{a+a\sec(c+dx}}\right.}{d}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.09, size = 109, normalized size = 0.81

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.47, size = 342, normalized size = 2.53

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(}{\cos(d}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
```

```
^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*s
qrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) +
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 0.98, size = 174, normalized size = 1.29

$$\frac{\left(\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)-2\arctan\left(\right.\right.}{2d\sin(dx+c)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
))*2^(1/2))-2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c))*2^(1/2))-2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/si
n(d*x+c)^2*(cos(d*x+c)^2-1)/a
```

maxima [B] time = 0.75, size = 476, normalized size = 3.53

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan(\sin(dx+c),\cos(dx+c))\right)^{2}+\sin\left(\frac{1}{2}\arctan(\sin(dx+c),\cos(dx+c))\right)^{2}+2\sin\left(\frac{1}{2}\,\mathrm{a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
```

```
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos (c+d\,x)^{3/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{\cos(c+dx)}}{}$$

```
[Out] -arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d/a^(1/2)+arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2
)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/
2)+sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4264, 3822, 4023, 3808, 206, 3801, 215}

$$\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a}$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.86

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+2\sin^{-}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*
ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c +
d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.58, size = 520, normalized size = 3.10

$$\left[\frac{\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos(dx+c)-2\right)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+}{4\left(ad\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/2*(2*sqrt(2)*(a
*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + (cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 1.06, size = 214, normalized size = 1.27

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1-\sin(dx+c)\right)\sqrt{2}}{4}\right)\sqrt{2}\,\cos\left(dx+c\right)-\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1+\right.}{4}\right.\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(arctan(1/4*(-2/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)-arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)+
4*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+2*sin(d*x+c)*
```

```
(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/sin(d
*x+c)^2*(cos(d*x+c)^2-1)/a
```

maxima [B] time = 2.80, size = 876, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-}}{\sqrt{a}\,d}$$

```
[Out] 7/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(
1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^
(1/2)+1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)-1/4*sin(d*x+
c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4264, 3822, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-}}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (7*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{s}{}}{4a}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx}}$$

$$= \frac{7\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{}$$

Mathematica [A]   time = 0.39, size = 178, normalized size = 0.84

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}-\sin\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(-ArcSin[Sqrt[1 - Sec[c + d*x]]] - 8
*ArcSin[Sqrt[Sec[c + d*x]]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])
/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) - Sq
rt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A]   time = 0.88, size = 550, normalized size = 2.61

$$\left[-\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx}{}\right.}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] [-1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(c
os(d*x + c))*sin(d*x + c) - 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/
(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*
x + c)^2), 1/8*(8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(
d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(cos(d*x + c)^3 + cos(d*x +
 c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec (dx+c)+a}\ \cos (dx+c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)

maple [A] time = 1.05, size = 247, normalized size = 1.17

$$\frac{(-1+\cos (dx+c))\left(-7\left(\cos^2 (dx+c)\right)\sqrt{2}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right)+7\arctan\left(\sqrt{-\frac{2}{1+\cos(dx}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)

[Out] 1/8/d*(-1+cos(d*x+c))*(-7*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))+7*cos(d*x+c)^2*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))+2*(-2/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+16*cos(d*x+c)^2*arctan(1/2*sin(d*x+c
)*(-2/(1+cos(d*x+c)))^(1/2))-4*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*
x+c)^(3/2)/a

maxima [B] time = 1.90, size = 1646, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*

```
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+}{2d(a\sec(c+dx)+a}$$

```
[Out] -1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+9/10*cos(d*x+c)^(3/2)*sin(d*x+
c)/a/d/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*se
c(d*x+c))^(1/2)-13/10*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2
)
```

Rubi [A] time = 0.59, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3817, 4022, 4013, 3808, 206}

$$-\frac{15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+}{2d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d
) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (4
9*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (13*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}}{2a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(}{10ad\sqrt{a+a\sec(c}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{\cos(}}{10ad\sqrt{a}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{\cos(}}{10ad\sqrt{a}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{2d(a+a\sec(c}$$

Mathematica [A] time = 0.92, size = 152, normalized size = 0.64

$$\frac{75\sqrt{2}\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{1-\sec(c+dx)}\left(-2\sin(2(c+dx))+\right.}{10d\sqrt{\cos(c+dx)-1}\,(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(4
*(9 + Cos[c + d*x]^2)*Sin[c + d*x] - 2*Sin[2*(c + d*x)] + 49*Tan[c + d*x]))
/(10*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))

fricas [A] time = 0.49, size = 400, normalized size = 1.69

$$\left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{40\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x +
c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(75*sqrt
(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))
) + 2*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)

maple [A] time = 1.14, size = 193, normalized size = 0.81

$$-\frac{\left(75\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-8\left(\cos^{5}\left(dx+c\right)\right)+24\left(\cos^{4}\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)

[Out] -1/20/d*(75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-8*cos(d*x+c)^5+24*cos(d*x+c)^4-75*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-96*cos(d*x+c)^3+54*cos(d*x+c)^2+124*cos(d*x+c)-98)*(a*(1+cos(d*x+c))/
cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^3/a^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{19\sin(c+}{6ad\sqrt{\cos(c+dx)}\sqrt{a\,s}}$$

```
[Out] -1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(3/2)+11/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-19/6*sin(d*x+c)/a/d/cos(d*x+c)
^(1/2)/(a+a*sec(d*x+c))^(1/2)+7/6*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3817, 4022, 4013, 3808, 206}

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{19\sin(c+}{6ad\sqrt{\cos(c+dx)}\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - (19
*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
```

```
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+2a\sec(c+dx}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}}}{2a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c}}{6ad\sqrt{a}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c}}{6ad\sqrt{a}}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\,\mathrm{si}}{2d(a+a\sec(c}$$

Mathematica [A] time = 0.65, size = 133, normalized size = 0.68

$$\frac{\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}\,(4\cos(c+dx)-19\sec(c+dx)-12)-33\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan\right.}{6d\sqrt{\cos(c+dx)-1}\,(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-12 + 4*Cos[c + d*x] - 19*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 33*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*
x)/2]^2*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.66, size = 380, normalized size = 1.93

$$\left[\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{24\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)*(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*cos(d*x +
 c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.11, size = 183, normalized size = 0.93

$$\frac{\left(33\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+8\left(\cos^4\left(dx+c\right)\right)-33\arctan\left(\frac{\sin(d}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*(33*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+8*cos(d*x+c)^4-33*arctan(1/2*sin(d*x+c)*
(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-40*cos(d*x+
c)^3+18*cos(d*x+c)^2+52*cos(d*x+c)-38)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)^(1/2)/sin(d*x+c)^3/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\cdots}{2d\sqrt{\cos(c+\cdots}}$$

```
[Out] -1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-7/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c)^(
1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3817, 4013, 3808, 206}

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\cdots}{2d\sqrt{\cos(c+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*Si
n[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
```

```
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\sec(c}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{\sin}{2d\sqrt{\cos(c+dx}}$$

Mathematica [A] time = 0.93, size = 138, normalized size = 0.88

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left((4\cos(c+dx)+5)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+7\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+\right.}{2d\sqrt{\cos(c+dx)-1}\,(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
 Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + (5 + 4*Cos[c + d*x])*Sqrt
[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(2*d*Sqrt[-1 + Cos[c +
d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.71, size = 360, normalized size = 2.29

$$\left[\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
```

```
s(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c
) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sq
rt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(4*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 1.06, size = 173, normalized size = 1.10

$$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(7\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)\sin(dx+c)-7}{4d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(7*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
*sin(d*x+c)-7*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-8*cos(d*x+c)^3+6*cos(d*x+c)^2+12*cos(d*x+c)-10)/si
n(d*x+c)^3/a^2
```

maxima [B] time = 0.72, size = 7176, normalized size = 45.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)
^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) -
 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
 + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*
```

```
x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2
*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(
1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/
2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(
1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*s
in(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2
*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^
```

```
2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*
c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8*
(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*
d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d
*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(3
/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*cos
(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d
*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x
 + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2
*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x +
1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
 1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin
(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*(
3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*
x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x + 1
/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c
```

```
)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2*
d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2
*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1
/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x + 1
/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3
/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))*
sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1
/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x +
 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
 + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19*
cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/
2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1/
2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*c
os(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/((4*sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/2*
c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^
2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x + 5
/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2
+ 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*
sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c
)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*sq
rt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2
*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*d*
x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 +
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*co
s(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
```

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^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1
/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*si
n(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos(3
/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*cos
(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*
c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)
+ 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2*d
*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2*d
*x + 3/2*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{\left(c + dx \right)}}}{\left(a \left(\sec{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3811, 3808, 206}

$$\frac{3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{4a}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(3\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d} - \frac{}{2d\cos}$$

Mathematica [A] time = 0.44, size = 131, normalized size = 1.12

$$-\frac{\sin(c+dx)\left(2\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}+3\sqrt{2}\,(\sec(c+dx)+1)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{4ad\sqrt{\cos(c+dx)-1}\,(\cos(c+dx)+1)\sqrt{\sec(c+dx)}\,\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/4*((2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 3*Sqrt[2]*ArcTan[(Sqrt
[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]))*Sin[c +
 d*x])/(a*d*Sqrt[-1 + Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.50, size = 340, normalized size = 2.91

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 1.12, size = 138, normalized size = 1.18

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}\,(dx+c)\right)\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)+\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos(dx+c)-\right.}{4d\sin(dx+c)^3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(3*arctan(1/2*si
n(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)-(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*
(cos(d*x+c)^2-1)/a^2
```

maxima [B] time = 0.60, size = 1031, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*si
n(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3810, 3808, 206}

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) +
Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{4a}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{}{2d\cos^{\frac{3}{2}}(}$$

Mathematica [B] time = 0.87, size = 248, normalized size = 2.12

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[
c + d*x]])/Sqrt[1 - Sec[c + d*x]]]) - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x
]]]*(1 + Cos[c + d*x]) + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]) +
Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 - Sec[c
 + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.47, size = 338, normalized size = 2.89

$$\left[\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 0.99, size = 136, normalized size = 1.16

$$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin\left(dx+c\right)-\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)+\sqrt{-\frac{2}{1+\cos(dx+c)}}\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{c}\right.}{4d\sin\left(dx+c\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-(-2/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/c
os(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(c
os(d*x+c)^2-1)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/
2)/d-5/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4264, 3816, 4023, 3808, 206, 3801, 215}

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.84, size = 248, normalized size = 1.43

$$-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\sec\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c +
```

```
d*x]) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.51, size = 550, normalized size = 3.16

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*a
rctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 1.01, size = 230, normalized size = 1.32

$$\frac{\left(2\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1+\sin(dx+c)\right)\sqrt{2}}{4}\right)\sin\left(dx+c\right)-2\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos(dx+c)+1-\sin(dx+c)\right)\sqrt{}}{4}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c))*2^(1/2))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-(-2/(1+c
os(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-2
/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

maxima [B] time = 0.69, size = 2122, normalized size = 12.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d
*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
```

```
in(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[Out] -1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-3*arcsinh(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/
2)/d+9/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d
*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+3/2*sin(d
*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4264, 3816, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/
2)*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,s}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,s}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,s}}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.97, size = 242, normalized size = 1.13

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(6\cos(c+dx)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+4\sqrt{(\cos(c+dx)-1)\,\mathrm{se}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
+ d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d
*x]]]*(1 + Cos[c + d*x]) + 18*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x])
 + 4*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] + 6*Cos[c + d*x]*Sqrt[(-1 + C
os[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*
(1 + Sec[c + d*x]))^(3/2))
```

fricas [A] time = 0.50, size = 614, normalized size = 2.87

$$\left[\frac{9\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
```

```
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2
*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -
1/4*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*co
s(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2*cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d
*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

maple [A] time = 1.01, size = 270, normalized size = 1.26

$$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\cos(dx+c)\sin(dx+c)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*arctan(1/4*(-
2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d
*x+c)*2^(1/2)-3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+9*arctan(1/2*sin(d*x+c)*(-2/(1+c
os(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-3*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)^2+(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+2*(-2/(1+cos(d*x+c)))^(1/2))/co
s(d*x+c)^(1/2)/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2
```

maxima [B] time = 1.69, size = 4934, normalized size = 23.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
```

```
2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c
)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2
*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(
2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*

```
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))/((sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*cos(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(
2*d*x + 2*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sqrt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{95\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{299\sin}{48a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(5/2)-17/16*sin(d*x+c)*
cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+163/32*arctanh(1/2*sin(d*x+c)*a
^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-299/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(
a+a*sec(d*x+c))^(1/2)+95/48*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {4264, 3817, 4020, 4022, 4013, 3808, 206}

$$\frac{95\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (
17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (
299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(95*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+3a\sec(c+d}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c-}}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{\cos(c+dx)}\,\mathrm{si}}{48a^2d\sqrt{a+a\sec}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\,\mathrm{sin}}{48a^2d\sqrt{\cos(c+dx}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\,\mathrm{sin}}{48a^2d\sqrt{\cos(c+dx}}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\,\mathrm{s}}{4d(a+a\sec(c}$$

Mathematica [A] time = 1.17, size = 144, normalized size = 0.61

$$-\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(-32\cos(c+dx)+299\sec^2(c+dx)+503\sec(c+dx)+160\right)+1956\sqrt{2}\cos^4\left(\right.\right.}{96d\sqrt{\cos(c+dx)-1}\,(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
[Out] -1/96*((1956*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d
*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(160
 - 32*Cos[c + d*x] + 503*Sec[c + d*x] + 299*Sec[c + d*x]^2))*Sin[c + d*x])/
(d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.48, size = 448, normalized size = 1.89

$$\left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\mathrm{si}}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{192\left(a^3d\cos(dx+c)^3+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*
x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(cos(d*x + c)^3 +
3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
 2*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.18, size = 244, normalized size = 1.03

$$-\frac{(-1+\cos(dx+c))^2\left(489\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)\sin(dx+c)+978\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d*(-1+cos(d*x+c))^2*(489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+978*cos(d*x+c)*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)+64*cos(d*x+c)^4+489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*
(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-384*cos(d*x+c)^3-686*cos(d*x+c)^2+408*
cos(d*x+c)+598)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*
x+c)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{}{16ad\sqrt{c}}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-13/16*sin(d*x+c)/
a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-75/32*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4264, 3817, 4020, 4013, 3808, 206}

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{}{16ad\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - (13
*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (49
*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
```

```
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\sec(c}}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{s}{4d\sqrt{\cos(c+d}}$$

Mathematica [A] time = 0.96, size = 141, normalized size = 0.72

$$\frac{\sqrt{1-\sec(c+dx)}\,(32\sin(c+dx)+\tan(c+dx)(49\sec(c+dx)+85))+150\sqrt{2}\,\sin(c+dx)\cos^4\left(\frac{1}{2}(c+dx)\right)\mathrm{se}}{16d\sqrt{\cos(c+dx)-1}\,(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (150*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
32*Sin[c + d*x] + (85 + 49*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[-1 + Cos
[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.46, size = 428, normalized size = 2.17

$$\left[\frac{75\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}+2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin\cdots}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\cdots\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d*x + c)^2 + 85*
cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.16, size = 234, normalized size = 1.19

$$-\frac{\left(\sqrt{\cos}\,\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(-1+\cos\left(dx+c\right)\right)^{2}\left(-75\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^{2}\left(d\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-150*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+64*cos(d*x+c)^3-
75*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+106*cos(d*x+c)^2-72*cos(d*x+c)-98)/sin(d*x+c)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{19\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\cos^{\frac{3}{2}}(c}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)-9/16*sin(d*x+c)/a
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4264, 3817, 4012, 3808, 206}

$$\frac{19\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - (9*S
in[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
```

```
-1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+d}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+d}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} - \frac{}{4a}$$

Mathematica [A] time = 1.32, size = 168, normalized size = 1.07

$$-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\left(9\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+13\sqrt{-((\sec(c+dx)-1)\sec(c+dx}\right.}{16d\sqrt{1-\sec(c+dx)}\,(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/16*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(9*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2) + 38*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + 13*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c +
 d*x]))^(5/2))
```

fricas [A] time = 0.53, size = 408, normalized size = 2.60

$$\left[\frac{19\sqrt{2}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{\cos(dx+c)^{2}+2\cos(dx+c)}\right.}{64\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(1
9*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(13*
cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 1.18, size = 200, normalized size = 1.27

$$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))^2\left(13\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right)+19\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{}{1+}}}{2}\right.\right.}{16d\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
^2*(13*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*arctan(1/2*sin(d*x+c)*(-2/
(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)+19*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-9*(
-2/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

maxima [B] time = 1.83, size = 3049, normalized size = 19.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
```

```
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2*
d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
+ 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
+ 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1
```

```
/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*sin
(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a^2*
cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x + 4
*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d*x
+ c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*
d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*cos
(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*
c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x
 + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*
sin(3*d*x + 3*c))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\cos^{\frac{5}{2}}}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+5/16*sin(d*x+c)/a
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^(1
/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4264, 3811, 3810, 3808, 206}

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{}{4d\cos^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (5*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
```

```
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{8a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{}{4d\cos^{\frac{5}{2}}}$$

Mathematica [A] time = 3.72, size = 224, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-8\sin(c+dx)\sec^{\frac{5}{2}}(c+dx) - \frac{5(\sec(c+dx)+1)\left(2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)-\tan(c+dx)(\sec\right.}{}\right.}{32d(a(\sec(c+dx)+1))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-8*Sec[c + d*x]^(5/2)*Sin[c + d*x]
- (5*(1 + Sec[c + d*x])*(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c
+ d*x] - (1 + Sec[c + d*x])*(2*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sq
rt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]] + 2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x]))/
Sqrt[1 - Sec[c + d*x]]))/(32*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.50, size = 408, normalized size = 2.60

$$\left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(d}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(
d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

maple [A] time = 1.12, size = 198, normalized size = 1.26

$$\frac{(-1+\cos(dx+c))^2\left(5\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\cos(dx+c)\sin(dx+c)-5\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\left(\cos^2(dx+c)\right.\right.}{16d\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(5*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
)*cos(d*x+c)*sin(d*x+c)-5*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*arctan(1
/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*(-2/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)+(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1
/2)*cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

maxima [B] time = 2.50, size = 2875, normalized size = 18.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin
(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +
 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
```

*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d *x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2 *d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/ 2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1 2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8* (4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin (3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/ 2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos (3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/ 2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3 /2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/ 2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/ 2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d* x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d *x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c os(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d* x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3 /2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co s(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3* sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*ar ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2 *d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/ 2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/ 2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2* c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3 /2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d *x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3 /2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*c os(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arc tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3 /2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2* c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin (3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2 *d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2 *c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2 *d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)

```
, cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))/(
(16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x +
 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) +
sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*co
s(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^2*s
in(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{}{4d\cos^{\frac{5}{2}}(c}$$

```
[Out] 1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/
d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/
2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3810, 3808, 206}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{}{4d\cos^{\frac{5}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 + Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (3*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{8a}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d} + \frac{}{4d\,\mathrm{c}}$$

Mathematica [B] time = 1.35, size = 341, normalized size = 2.17

$$14\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+6\sin(c+dx)\sqrt{-((\sec(c+dx)-1)\sec(c+dx))}-3\sqrt{2}\tan(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] + 14*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 6*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 6*Sqrt[2]*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 3*Sqrt[2]
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*T
an[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec[c +
 d*x])*Tan[c + d*x]) + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Se
c[c + d*x])*Tan[c + d*x]))/(32*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^
2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [A] time = 0.49, size = 408, normalized size = 2.60

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^2+2\cos(dx+c)+}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
```

```
+ c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(
d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

maple [A] time = 1.12, size = 200, normalized size = 1.27

$$-\frac{(-1+\cos(dx+c))^2\left(3\sqrt{-\frac{2}{1+\cos(dx+c)}}\left(\cos^2(dx+c)\right)-3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\cos(dx+c)\sin(dx+c)\cdots}{16d\sin(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(3*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-3*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)+4*(-2/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*sin(d*x+c)-7*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+}}\right.}{a^{5/2}d}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)-11/16*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2)*tan(d*x+c)/(a
+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*arc
tanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4264, 3816, 4019, 4023, 3808, 206, 3801, 215}

$$-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+}}\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)) - (11*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3
/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{4}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{}$$

Mathematica [A] time = 1.21, size = 328, normalized size = 1.53

$$\frac{-30\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) - 22\sin(c+dx)\sqrt{-((\sec(c+dx)-1)\sec(c+dx))} + 43\sqrt{2}\tan(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] - 30*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 22*
Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + 86*Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 43*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x
]*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec
[c + d*x])*Tan[c + d*x]) - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2
 + Sec[c + d*x])*Tan[c + d*x]))/(32*d*Cos[c + d*x]^(9/2)*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.53, size = 638, normalized size = 2.98

$$\left[\frac{43\sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*
sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(dx+c)+a)^{\frac{5}{2}} \cos(dx+c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 1.10, size = 396, normalized size = 1.85

$$\frac{(-1+\cos(dx+c))^2 \left(16 \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\, (\cos(dx+c)+1+\sin(dx+c))\sqrt{2}}{4}\right) \cos(dx+c) \sin(dx+c)\sqrt{2} - 16 \arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)+1+sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-16*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+
c)*2^(1/2)+16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c))*2^(1/2))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*sin(d*x+c)-43*arctan(1/2*sin(d*x+c)*(
-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)+11*(-2/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)^2-43*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+
c)+4*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-15*(-2/(1+cos(d*x+c)))^(1/2))*(a*
(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(1+cos(d
*x+c)))^(1/2)/a^3
```

maxima [B] time = 1.00, size = 4988, normalized size = 23.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2
)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2
*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)
*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6
*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
```

cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*a

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*(19*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 19*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
1*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*
c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2
*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*s
in(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 176*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 +
16*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*
sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8
*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)
*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d
*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*s
qrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{115\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{5\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+}{\sqrt{a\sec(c+d}}\right.}{a^{5/2}d}$$

```
[Out] -1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/
a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-5*arcsinh(a^(1/2)*tan(d*x+c)/(a
+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32*ar
ctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+35/16*sin(d*x+c)/a^2
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3816, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-5\sqrt{\cos}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^
(7/2)*(a + a*Sec[c + d*x])^(5/2)) - (15*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^
(5/2)*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{s}}$$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{115\tanh^{-}}{}$$

Mathematica [A] time = 1.54, size = 348, normalized size = 1.37

$$\frac{\sqrt{\sec(c+dx)}\left(32\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+110\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[
1 - Sec[c + d*x]]]*Sin[c + d*x] + 110*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(
3/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*
x] + 70*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 230*Sqrt[2
]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x]
- 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*S
ec[c + d*x]*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x]
+ (2 + Sec[c + d*x])*Tan[c + d*x]) + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c
+ d*x] + (2 + Sec[c + d*x])*Tan[c + d*x])))/(32*d*Sqrt[-1 + Cos[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

fricas [A] time = 0.53, size = 702, normalized size = 2.76

$$\left[\frac{115\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{\cos(dx+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(35*cos(d*x + c)^2
+ 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*
cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*co
s(d*x + c)), -1/32*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(35*cos(d
*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*
cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2)), x)
```

maple [B] time = 1.14, size = 444, normalized size = 1.75

$$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(40\left(\cos^2(dx+c)\right)\sin(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))\sqrt{2}}{4}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(40*sin(d*x+c)
*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)
)*2^(1/2))*2^(1/2)-40*sin(d*x+c)*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(1/2))*2^(1/2)+40*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^
(1/2)-40*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))*2^(
1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+115*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*
sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-35*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(
1/2)+115*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*
x+c)-20*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+39*(-2/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)+16*(-2/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-2/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^5/a^3
```

maxima [B] time = 3.57, size = 9048, normalized size = 35.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
6*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*si
n(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(
sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*
sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x +
4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c
)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x +
 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c)
 + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*
cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c)
+ 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c)
+ 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
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sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4
*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*s
qrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2
)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*
x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(
2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4
*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*
x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(
2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7
*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x
 + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(s
qrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x
+ 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x
+ 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) +
 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c
) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c
) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4
*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
```

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c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*si
n(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt
(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d
*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c
) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*s
in(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) +
7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x +
 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(c
os(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(
6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*s
in(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
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cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x +
6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 115*(2*
(7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x
+ 6*c)^2 + 14*(7*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*
c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7
*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x
 + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
4*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(
2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2
*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 16*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 16*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x
+ 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*
cos(2*d*x + 2*c) + 4*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(11/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 16*(75*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 24*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos
(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*cos(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300
*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) +
8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 32*(24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*
cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(cos(6*d*x +
 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 560*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 140*(cos(6*d*x + 6*c) +
 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2
```

```
*c), cos(2*d*x + 2*c))) + 560*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a^2
*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*a^2*co
s(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 64*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*sin(4*d*x + 4*c)^
2 + 98*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*a^2*sin(2
*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 64*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 14*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(7*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6
*c) + 14*(7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) +
8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2
)*a^2*cos(2*d*x + 2*c) + 8*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2)*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 16*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqr
t(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*
c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 8*sq
rt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*
a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*
sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a
^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=244

$$-\frac{a^3(7-4n)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\sin(e+fx)\cos(e+fx)(d\cos(e}{f(1-n)(n+1)\sqrt{}}$$

```
[Out] -a^3*(7-4*n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2
)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a^3*(1-4*n)*cos(f*x+e)*(d*cos(f
*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/
(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(5-2*n)*(d*cos(f*x+e))^n*tan(f*x+e)/f/(n^
2-3*n+2)+(d*cos(f*x+e))^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(2-n)
```

Rubi [A] time = 0.40, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3814, 3997, 3787, 3772, 2643}

$$-\frac{a^3(7-4n)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\sin(e+fx)\cos(e+fx)(d\cos(e}{f(1-n)(n+1)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*(7 - 4*n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f*x]^2])) - (a^3*(1
 - 4*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (
3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x
]^2]) + (a^3*(5 - 2*n)*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n)*(2 - n))
 + ((d*Cos[e + f*x])^n*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 - n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+a\sec(e$$

$$= \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2-n)} + \frac{\left(a(d\cos(e+\right.}{}$$

$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a\right.}{f(2}$$

$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a\right.}{f(2}$$

$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a\right.}{f(2}$$

$$= -\frac{a^3(7-4n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 1.59, size = 308, normalized size = 1.26

$$\frac{ia^3 2^{-n-3}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^n\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(\frac{8e^{3i(e+fx)}\,{}_2F_1\left(1,\frac{n-1}{2};\frac{5-n}{2};-e^{2i(e+fx)}\right)}{(n-3)\left(1+e^{2i(e+fx)}\right)^2}+\frac{12e^{2i(e+fx)}\,{}_2}{(n-}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (I*2^(-3 - n)*a^3*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))^n*Cos[e + f*x
]^(3 - n)*(d*Cos[e + f*x])^n*((8*E^((3*I)*(e + f*x))*Hypergeometric2F1[1, (
-1 + n)/2, (5 - n)/2, -E^((2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))^2*(
-3 + n)) + (12*E^((2*I)*(e + f*x))*Hypergeometric2F1[1, n/2, 2 - n/2, -E^((
2*I)*(e + f*x))])/((1 + E^((2*I)*(e + f*x)))*(-2 + n)) + (6*E^(I*(e + f*x))
*Hypergeometric2F1[1, (1 + n)/2, (3 - n)/2, -E^((2*I)*(e + f*x))])/(-1 + n)
 + ((1 + E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, (2 + n)/2, 1 - n/2, -E^(
(2*I)*(e + f*x))])/n)*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3)/f
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(d*cos(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

maple [F] time = 8.53, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^n\left(a+a\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(d\,\cos\left(e+f\,x\right)\right)}^n\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3,x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\left(d\cos\left(e+fx\right)\right)^n\,dx+\int 3\left(d\cos\left(e+fx\right)\right)^n\sec\left(e+fx\right)dx+\int 3\left(d\cos\left(e+fx\right)\right)^n\sec^2\left(e+fx\right)dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((d*cos(e + f*x))**n, x) + Integral(3*(d*cos(e + f*x))**n*sec
(e + f*x), x) + Integral(3*(d*cos(e + f*x))**n*sec(e + f*x)**2, x) + Integr
al((d*cos(e + f*x))**n*sec(e + f*x)**3, x))
```

3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{2a^2\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\sin(e+fx)\cos(e+fx)(d\cos(e+f}{f(1-n)(n+1)\sqrt{\sin}$$

```
[Out] -2*a^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(
f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-a^2*(1-2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hyp
ergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(si
n(f*x+e)^2)^(1/2)+a^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

Rubi [A] time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3772, 2643, 4046}

$$-\frac{2a^2\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\sin(e+fx)\cos(e+fx)(d\cos(e+f}{f(1-n)(n+1)\sqrt{\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 - 2*n)*Cos[e + f*
x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Co
s[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+a\sec(e+$$

$$= \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}\left(a^2+a^2\sec^2(e$$

$$= \frac{a^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)}{d}$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{a^2(}{}$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a^2(1}{}$$

Mathematica [C] time = 1.29, size = 266, normalized size = 1.49

$$\frac{ia^2 2^{-n-2}e^{-i(e+fx)}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{n-1}(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\left(4(n-1)ne^{2i(e+fx)}\,{}_2F_1\left(1,\frac{n}{2};2-\frac{n}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (I*2^(-2 - n)*a^2*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))^(-1 + n)*(d*C
os[e + f*x])^n*(1 + Cos[e + f*x])^2*(4*E^((2*I)*(e + f*x))*(-1 + n)*n*Hyper
geometric2F1[1, n/2, 2 - n/2, -E^((2*I)*(e + f*x))] + (1 + E^((2*I)*(e + f*
x)))*(-2 + n)*(4*E^(I*(e + f*x))*n*Hypergeometric2F1[1, (1 + n)/2, (3 - n)/
2, -E^((2*I)*(e + f*x))] + (1 + E^((2*I)*(e + f*x)))*(-1 + n)*Hypergeometri
c2F1[1, (2 + n)/2, 1 - n/2, -E^((2*I)*(e + f*x))]))*Sec[(e + f*x)/2]^4)/(E^
(I*(e + f*x))*f*(-2 + n)*(-1 + n)*n*Cos[e + f*x]^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(fx+e\right)+a\right)^2\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

maple [F] time = 5.46, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^n\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(d\cos\left(e+fx\right)\right)^{n}\,dx+\int 2\left(d\cos\left(e+fx\right)\right)^{n}\sec\left(e+fx\right)dx+\int \left(d\cos\left(e+fx\right)\right)^{n}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*cos(e + f*x))**n, x) + Integral(2*(d*cos(e + f*x))**n*sec
(e + f*x), x) + Integral((d*cos(e + f*x))**n*sec(e + f*x)**2, x))
```

3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos\right.}{df(n+1)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+
e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi **[A]** time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4225, 16, 2748, 2643}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos\right.}{df(n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (d\cos(e+fx))^n(a+a\cos(e+fx))\sec(e+fx)\,dx$$
$$= d\int (d\cos(e+fx))^{-1+n}(a+a\cos(e+fx))\,dx$$
$$= a\int (d\cos(e+fx))^n\,dx + (ad)\int (d\cos(e+fx))^{-1+n}\,dx$$
$$= -\frac{a(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d}{}$$

Mathematica [A] time = 0.14, size = 105, normalized size = 0.80

$$-\frac{a\sqrt{\sin^2(e+fx)}\,(d\cos(e+fx))^n\left(n\cot(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)+(n+1)\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2}\right.\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
[Out] -((a*(d*Cos[e + f*x])^n*((1 + n)*Csc[e + f*x]*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + n*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

maple [F] time = 2.72, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^n\left(a+a\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^n\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \left(d \cos{\left(e + f x \right)}\right)^{n}\, dx + \int \left(d \cos{\left(e + f x \right)}\right)^{n} \sec{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral((d*cos(e + f*x))**n, x) + Integral((d*cos(e + f*x))**n*sec(e +
f*x), x))
```

3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{(n+1)\sin(e+fx)\cos^2(e+fx)(d\cos}{af(n+2}$$

```
[Out] (d*cos(f*x+e))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*(d*cos(f*x+e))^n*
hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(sin(f*
x+e)^2)^(1/2)+(1+n)*cos(f*x+e)^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1+1/2*n],
[2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(2+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3820, 3787, 3772, 2643}

$$-\frac{\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{(n+1)\sin(e+fx)\cos^2(e+fx)(d\cos}{af(n+2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e + f*x]*
(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x
]^2]*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 + n)*Cos[e + f*x]^2*(d*
Cos[e + f*x])^n*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2
]*Sin[e + f*x])/(a*f*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{a+a\sec(e+fx)}\,dx$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(d(1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int(d\sec(e+}{a^2}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int(d\sec(e+f}{a}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1+n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{a}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+f}{af\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^n}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

maple [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\ \cos\left(e+f\,x\right)\right)^{n}}{a+\frac{a}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x) + 1), x)/a
```

3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=215

$$\frac{2(n+2)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(2n+3)\sin(e+fx)\cos(e+fx)(d\cos(e+fx)}{3a^2f\sqrt{\sin^2(e+}}$$

```
[Out] 2/3*(2+n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*s
in(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(3+2*n)*cos(f*x+e)*(d*cos(f*x+e))^
n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/(si
n(f*x+e)^2)^(1/2)-2/3*(2+n)*(d*cos(f*x+e))^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e)
)-1/3*(d*cos(f*x+e))^n*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] time = 0.41, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3817, 4020, 3787, 3772, 2643}

$$\frac{2(n+2)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(2n+3)\sin(e+fx)\cos(e+fx)(d\cos(e+fx)}{3a^2f\sqrt{\sin^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(2 + n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
+ f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3 + 2*n)*Cos[e +
 f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (2*(2 + n)*(d*Cos
[e + f*x])^n*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - ((d*Cos[e + f*x])
^n*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{(a+a\sec(e+fx))^2}\,dx$$

$$= -\frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-}}{}}{3a^2}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\cos(\right.}{}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left(2n(2+\right.}{}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left(2n(2+\right.}{}$$

$$= \frac{2(2+n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3+2n)\cos(}{}$$

Mathematica [F] time = 9.05, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^n}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(a+a\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{\left(a+\frac{a}{\cos\left(e+fx\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**
2
```

3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx}{8d}$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/
4*b*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^4(c+dx)\,dx+b\int \sec^5(c+dx)\,dx\\
&= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(3b)\int \sec^3(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int (1\right.}{}\\
&= \frac{a\tan(c+dx)}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+d}{4d}\\
&= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.47, size = 99, normalized size = 1.16

$$\frac{9\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^{3}+9\,b\cos\left(dx+c\right)^{2}+8\,a\cos\left(dx+c\right)+6\,b\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*b*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.22, size = 164, normalized size = 1.93

$$\frac{9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-\cdots\right)}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 -
 40*a*tan(1/2*d*x + 1/2*c)^5 - 9*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*tan(1/2*d*
x + 1/2*c)^3 - 9*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c) - 15*
b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.74, size = 92, normalized size = 1.08

$$\frac{2a\tan\left(dx+c\right)}{3d}+\frac{a\tan\left(dx+c\right)\left(\sec^{2}\left(dx+c\right)\right)}{3d}+\frac{b\left(\sec^{3}\left(dx+c\right)\right)\tan\left(dx+c\right)}{4d}+\frac{3b\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4*b*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 95, normalized size = 1.12

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,b\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.57, size = 152, normalized size = 1.79

$$\frac{\left(\frac{5\,b}{4}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{10\,a}{3}+\frac{3\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,b}{4}-\frac{10\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) - tan(c/2 + (d*x)/2)^7*(2*a - (5*b)/4)
- tan(c/2 + (d*x)/2)^3*((10*a)/3 - (3*b)/4) + tan(c/2 + (d*x)/2)^5*((10*a)/
3 + (3*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c
/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*x)/2
)))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx)) \sec^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/
3*b*tan(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3787, 3768, 3770, 3767}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^3(c+dx)\,dx+b\int \sec^4(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx-\frac{b\,\text{Subst}\left(\int \left(1+x^2\right)\,d\right.}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\,\text{ta}}{}\end{aligned}$$

Mathematica [A] time = 0.16, size = 60, normalized size = 0.95

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.50, size = 88, normalized size = 1.40

$$\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,b\cos\left(dx+c\right)^2+3\,a\cos\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*b)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.23, size = 122, normalized size = 1.94

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,b\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^5 + 4*
b*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c) - 6*b*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.74, size = 72, normalized size = 1.14

$$\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{2b\tan\left(dx+c\right)}{3d}+\frac{b\tan\left(dx+c\right)\left(\sec^2\left(dx+c\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x)
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*b*tan(d
*x+c)/d+1/3/d*b*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.41, size = 70, normalized size = 1.11

$$\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)b-3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d

mupad [B] time = 2.81, size = 109, normalized size = 1.73

$$\frac{(a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+(-a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))/cos(c + d*x)^3,x)

[Out] (tan(c/2 + (d*x)/2)^5*(a - 2*b) - tan(c/2 + (d*x)/2)*(a + 2*b) + (4*b*tan(c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c)),x)

[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**3, x)

3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3787, 3767, 8, 3768, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx+b\int \sec^3(c+dx)\,dx\\
&= \frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}b\int \sec(c+dx)\,dx-\frac{a\,\text{Subst}(\int 1\,dx,x,-}{d}\\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

fricas [A] time = 0.51, size = 74, normalized size = 1.57

$$\frac{b\cos(dx+c)^2\log(\sin(dx+c)+1)-b\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2a\cos(dx+c)+b)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.21, size = 107, normalized size = 2.28

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

maple [A] time = 0.74, size = 51, normalized size = 1.09

$$\frac{a\tan(dx+c)}{d}+\frac{b\sec(dx+c)\tan(dx+c)}{2d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b*ln(sec(d*x+c)+tan(d*x+
c))
```

maxima [A] time = 0.33, size = 58, normalized size = 1.23

$$-\frac{b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4a\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

mupad [B]   time = 1.48, size = 85, normalized size = 1.81

$$\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a-b\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a+b\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] (b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(2*a - b) - tan(c/2
 + (d*x)/2)*(2*a + b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 +
1))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3787, 3770, 3767, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec(c+dx)\,dx+b\int \sec^2(c+dx)\,dx\\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

fricas [B] time = 0.46, size = 60, normalized size = 2.50

$$\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.20, size = 63, normalized size = 2.62

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.56, size = 32, normalized size = 1.33

$$\frac{b\tan\left(dx+c\right)}{d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] b*tan(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.54, size = 29, normalized size = 1.21

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b*tan(d*x + c))/d
```

mupad [B] time = 0.79, size = 47, normalized size = 1.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x),x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
(d*x)/2)^2 - 1))
```

sympy [A] time = 4.27, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*tan(c + d*x))/d, Ne(d, 0
)), (x*(a + b*sec(c))*sec(c), True))
```

3.450 $\int (a+b\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*x+b*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3770}

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[c + d*x],x]
[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\,dx &= ax+b\int \sec(c+dx)\,dx\\ &= ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[c + d*x],x]
[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

fricas [B] time = 0.47, size = 36, normalized size = 2.25

$$\frac{2\,adx+b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="fricas")
[Out] 1/2*(2*a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 0.14, size = 49, normalized size = 3.06

$$ax+\frac{b\left(\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)-2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*b*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

maple [A] time = 0.04, size = 24, normalized size = 1.50

$$ax+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(d*x+c),x)
```

```
[Out] a*x+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.52, size = 23, normalized size = 1.44

$$ax+\frac{b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*log(sec(d*x + c) + tan(d*x + c))/d
```

mupad [B] time = 0.80, size = 57, normalized size = 3.56

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cos(c + d*x),x)
```

```
[Out] (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [A] time = 1.80, size = 41, normalized size = 2.56

$$ax+b\left(\begin{cases}\frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\: d\neq 0\\\frac{x\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

3.451 $\int \cos(c + dx)(a + b\sec(c + dx))\, dx$

Optimal. Leaf size=15

$$\frac{a\sin(c + dx)}{d} + bx$$

[Out] b*x+a*sin(d*x+c)/d

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3787, 2637, 8}

$$\frac{a\sin(c + dx)}{d} + bx$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]

[Out] b*x + (a*Sin[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(a + b\sec(c + dx))\, dx &= a\int \cos(c + dx)\, dx + b\int 1\, dx \\ &= bx + \frac{a\sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d} + \frac{a\cos(c)\sin(dx)}{d} + bx$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]

[Out] b*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d

fricas [A] time = 0.54, size = 17, normalized size = 1.13

$$\frac{bdx + a\sin\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] (b*d*x + a*sin(d*x + c))/d
```

giac [B] time = 0.16, size = 39, normalized size = 2.60

$$\frac{(dx+c)b+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] ((d*x + c)*b + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

maple [A] time = 0.50, size = 21, normalized size = 1.40

$$\frac{a\sin(dx+c)+(dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] 1/d*(a*sin(d*x+c)+(d*x+c)*b)
```

maxima [A] time = 0.38, size = 20, normalized size = 1.33

$$\frac{(dx+c)b+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] ((d*x + c)*b + a*sin(d*x + c))/d
```

mupad [B] time = 0.74, size = 17, normalized size = 1.13

$$\frac{a\,\sin(c+d\,x)+b\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x)),x)
[Out] (a*sin(c + d*x) + b*d*x)/d
```

sympy [A] time = 1.86, size = 15, normalized size = 1.00

$$a\left(\begin{cases}\sin(c) & \text{for}\ d=0\\ \frac{\sin(c+dx)}{d} & \text{otherwise}\end{cases}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] a*Piecewise((sin(c), Eq(d, 0)), (sin(c + d*x)/d, True)) + b*x
```

3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2635, 8, 2637}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx&=a\int \cos^2(c+dx)\,dx+b\int \cos(c+dx)\,dx\\&=\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\&=\frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 0.92

$$\frac{a(2(c+dx)+\sin(2(c+dx)))+4b\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] (4*b*Sin[c + d*x] + a*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.45, size = 29, normalized size = 0.76

$$\frac{adx + (a\cos(dx + c) + 2b)\sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*b)*sin(d*x + c))/d
```

giac [B] time = 0.16, size = 82, normalized size = 2.16

$$\frac{(dx + c)a - \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/2*((d*x + c)*a - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3
 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^2)/d
```

maple [A] time = 0.62, size = 38, normalized size = 1.00

$$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x)
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b*sin(d*x+c))
```

maxima [A] time = 0.32, size = 34, normalized size = 0.89

$$\frac{(2dx + 2c + \sin(2dx + 2c))a + 4b\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*b*sin(d*x + c))/d
```

mupad [B] time = 0.81, size = 31, normalized size = 0.82

$$\frac{a\,x}{2} + \frac{a\,\sin(2\,c + 2\,d\,x)}{4\,d} + \frac{b\,\sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x)),x)
[Out] (a*x)/2 + (a*sin(2*c + 2*d*x))/(4*d) + (b*sin(c + d*x))/d
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] 1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos^3(c+dx)\,dx+b\int \cos^2(c+dx)\,dx\\ &= \frac{b\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}b\int 1\,dx-\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(\right.}{d}\\ &= \frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 1.06

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b(c+dx)}{2d}+\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Si
n[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.48, size = 42, normalized size = 0.78

$$\frac{3\,bdx + \left(2\,a\cos\left(dx+c\right)^{2} + 3\,b\cos\left(dx+c\right) + 4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b*d*x + (2*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

giac [B] time = 0.15, size = 98, normalized size = 1.81

$$\frac{3\,(dx+c)b + \frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} + 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} + 6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*b + 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*
c)^5 + 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 1.00, size = 49, normalized size = 0.91

$$\frac{\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3} + b\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

maxima [A] time = 0.32, size = 46, normalized size = 0.85

$$-\frac{4\left(\sin\left(dx+c\right)^{3} - 3\,\sin\left(dx+c\right)\right)a - 3\,(2\,dx + 2\,c + \sin\left(2\,dx+2\,c\right))b}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*b)/d
```

mupad [B] time = 0.83, size = 55, normalized size = 1.02

$$\frac{b\,x}{2} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] (b*x)/2 + (2*a*sin(c + d*x))/(3*d) + (b*cos(c + d*x)*sin(c + d*x))/(2*d) +
(a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cos^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin
(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos^4(c+dx)\,dx+b\int \cos^3(c+dx)\,dx\\
&= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx-\frac{b\,\mathrm{Subst}\left(\int (1\right.}{}\\
&= \frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx}{4d}\\
&= \frac{3ax}{8}+\frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin}{4d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.47, size = 53, normalized size = 0.70

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,b\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*b)*sin(d*x + c))/d
```

giac [B] time = 0.17, size = 140, normalized size = 1.84

$$\frac{9\,(dx+c)a-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1
/2*c)^7 - 9*a*tan(1/2*d*x + 1/2*c)^5 - 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*ta
n(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2
*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 1.02, size = 60, normalized size = 0.79

$$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.33, size = 57, normalized size = 0.75

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a-32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*b)/d
```

mupad [B] time = 0.83, size = 75, normalized size = 0.99

$$\frac{3\,a\,x}{8}+\frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{a\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (2*b*sin(c + d*x))/(3*d) + (3*a*cos(c + d*x)*sin(c + d*x))/(8*d
) + (a*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b*cos(c + d*x)^2*sin(c + d*x))
/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\cos^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**4, x)
```

3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] 3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*sin
(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3787, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos^5(c+dx)\,dx+b\int \cos^4(c+dx)\,dx\\ &=\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3b)\int \cos^2(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int \left(1-2\right.\right.}{}\\ &=\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}-\\ &=\frac{3bx}{8}+\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c-}{4d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 89, normalized size = 0.97

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{3b(c+dx)}{8d}+\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d) + (
a*Sin[c + d*x]^5)/(5*d) + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])
/(32*d)
```

fricas [A] time = 0.54, size = 64, normalized size = 0.70

$$\frac{45\,bdx+\left(24\,a\cos\left(dx+c\right)^{4}+30\,b\cos\left(dx+c\right)^{3}+32\,a\cos\left(dx+c\right)^{2}+45\,b\cos\left(dx+c\right)+64\,a\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*b*d*x + (24*a*cos(d*x + c)^4 + 30*b*cos(d*x + c)^3 + 32*a*cos(d*x
 + c)^2 + 45*b*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 154, normalized size = 1.67

$$\frac{45\left(dx+c\right)b+\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)*b + 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x
+ 1/2*c)^9 + 160*a*tan(1/2*d*x + 1/2*c)^7 - 30*b*tan(1/2*d*x + 1/2*c)^7 + 4
64*a*tan(1/2*d*x + 1/2*c)^5 + 160*a*tan(1/2*d*x + 1/2*c)^3 + 30*b*tan(1/2*d
*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 1.03, size = 70, normalized size = 0.76

$$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.34, size = 69, normalized size = 0.75

$$\frac{32\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

mupad [B] time = 4.75, size = 113, normalized size = 1.23

$$\frac{3\,b\,x}{8}+\frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a}{3}-\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{116\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}+\left(\frac{8\,a}{3}+\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{5\,b}{4}\cdots\right.}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*b*x)/8 + (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) + tan(c/2 + (d*x)/2)^3*((8*
a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)/4) + tan(c/2 + (d*x)/2)^7*(
(8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)^2
+ 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left(c + d x \right)}\right) \cos^{5}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**5, x)
```

3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3a}{}$$

```
[Out] 3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(5*a^2+4*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d*
x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*ta
n(d*x+c)/d+1/15*(5*a^2+4*b^2)*tan(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3768, 3770, 4046, 3767}

$$\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*a^2 + 4*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*a^2 + 4*b^2)*Tan[c
 + d*x]^3)/(15*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^5(c+dx)\,dx + \int \sec^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx$$
$$= \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{b^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{2}(3ab)\int \text{se}$$
$$= \frac{3ab\sec(c+dx)\tan(c+dx)}{4d} + \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{b^2\sec^4(c+}{}$$
$$= \frac{3ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\left(5a^2+4b^2\right)\tan(c+dx)}{5d} + \frac{3ab\sec(c+dx)\,\text{ta}}{4d}$$

Mathematica [A] time = 0.58, size = 118, normalized size = 0.87

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{ab\tan(c+dx)\sec^3(c+dx)}{2d} + \frac{3ab\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (3*a*b*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x]))/(4*d) + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/
d + (b^2*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

fricas [A] time = 0.51, size = 136, normalized size = 1.01

$$\frac{45\,ab\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,ab\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\left(45\,ab\cos(dx+c)^3+8\right.}{120\,d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(5*a^2 + 4*b^2)*cos(d
*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 12*b^2
)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.23, size = 272, normalized size = 2.01

$$45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\,\text{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x +
1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*a^2*tan(1/2*d*x + 1/2*c)^7 +
 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 80*b^2*tan(1/2*d*x + 1/2*c)^7 + 200*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 232*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^2*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.91, size = 157, normalized size = 1.16

$$\frac{2a^2\tan(dx+c)}{3d}+\frac{a^2\left(\sec^2(dx+c)\right)\tan(dx+c)}{3d}+\frac{ab\left(\sec^3(dx+c)\right)\tan(dx+c)}{2d}+\frac{3ab\sec(dx+c)\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3
*tan(d*x+c)/d+3/4*a*b*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a*b*ln(sec(d*x+c)+tan(d
*x+c))+8/15*b^2*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*tan(d*x+c)/d+4/15*b^2*sec
(d*x+c)^2*tan(d*x+c)/d
```

maxima [A] time = 0.34, size = 132, normalized size = 0.98

$$\frac{40\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)b^2-15ab\left(\frac{2(3}{\sin(}\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 8*(3*tan(d*x + c)^5 + 10*
tan(d*x + c)^3 + 15*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.82, size = 221, normalized size = 1.64

$$\frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{16\,a^2}{3}+a\,b-\frac{8\,b^2}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,a^2}{3}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((20*a^2)/3
 + (116*b^2)/15) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan(c
/2 + (d*x)/2)^3*(a*b + (16*a^2)/3 + (8*b^2)/3) - tan(c/2 + (d*x)/2)^7*((16*
a^2)/3 - a*b + (8*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b^2))
/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**4, x)
```

3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\,\text{ta}}{}$$

```
[Out] 1/8*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(4*a^2+3*b^2
)*sec(d*x+c)*tan(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d*x
+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3788, 3767, 4046, 3768, 3770}

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
4*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sec^3(c+dx)\,dx - \frac{(2al}{}$$
$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+}{}$$
$$= \frac{\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\mathrm{sec}}{}$$

Mathematica [A] time = 0.28, size = 82, normalized size = 0.75

$$\frac{3\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(3\left(4a^2+3b^2\right)\sec(c+dx) + 16ab\left(\tan^2(c+dx)+3\right) + 6b^2\,\mathrm{se}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 + 3*b^2)*
Sec[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

fricas [A] time = 0.50, size = 133, normalized size = 1.21

$$\frac{3\left(4\,a^2+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right) - 3\left(4\,a^2+3\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(32\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + 3
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16
*a*b*cos(d*x + c) + 3*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 6*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

giac [B] time = 0.27, size = 258, normalized size = 2.35

$$\frac{3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 48\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 + 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + 3*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 -
 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 15*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan
(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 + 9
*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*tan(1/2*
d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)
/d
```

maple [A] time = 0.92, size = 142, normalized size = 1.29

$$\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d} + \frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d} + \frac{4ab\tan\left(dx+c\right)}{3d} + \frac{2ab\tan\left(dx+c\right)\left(\sec^2\left(dx+\right.\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*b
*tan(d*x+c)/d+2/3/d*a*b*tan(d*x+c)*sec(d*x+c)^2+1/4*b^2*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*b^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 144, normalized size = 1.31

$$\frac{32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)ab-3\,b^{2}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.66, size = 184, normalized size = 1.67

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+\frac{3\,b^2}{4}\right)}{d}+\frac{\left(a^2-4\,a\,b+\frac{5\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(-a^2+\frac{20\,a\,b}{3}+\frac{3\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-a^2\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^2 + (3*b^2)/4))/d + (tan(c/2 + (d*x)/2)^5*((2
0*a*b)/3 - a^2 + (3*b^2)/4) + tan(c/2 + (d*x)/2)*(4*a*b + a^2 + (5*b^2)/4)
+ tan(c/2 + (d*x)/2)^7*(a^2 - 4*a*b + (5*b^2)/4) - tan(c/2 + (d*x)/2)^3*((2
0*a*b)/3 + a^2 - (3*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=80

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] a*b*arctanh(sin(d*x+c))/d+1/3*(3*a^2+2*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3788, 3768, 3770, 4046, 3767, 8}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^3(c+dx)\,dx + \int \sec^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d} + (ab)\int \sec(c$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\sec^2(c+dx)\,\mathrm{ta}}{3d}$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(}{d}$$

Mathematica [A] time = 0.22, size = 71, normalized size = 0.89

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Ta
n[c + d*x])/d + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 0.48, size = 100, normalized size = 1.25

$$\frac{3\,ab\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(3\,ab\cos(dx+c)+\left(3\,a^2\right.\right.}{6\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (3*a^2 + 2*b^2)*cos(d*x + c)^2
 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.24, size = 178, normalized size = 2.22

$$\frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/
2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.92, size = 89, normalized size = 1.11

$$\frac{a^2\tan(dx+c)}{d}+\frac{ab\sec(dx+c)\tan(dx+c)}{d}+\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{2b^2\tan(dx+c)}{3d}+\frac{b^2\left(\sec^2(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c)/d+1/d*a*b*ln(sec(d*x+c)+tan(d*x+
c))+2/3*b^2*tan(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

maxima [A] time = 0.34, size = 84, normalized size = 1.05

$$\frac{2\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)b^{2}-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,a^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 3*a*b*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

mupad [B] time = 3.07, size = 141, normalized size = 1.76

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2-\frac{4\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+2\,a\,b\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
+ 2*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 + (4*b^2)/3) + tan(c/2 + (d*x)/2)*(2
*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left(c + dx \right)}\right)^{2} \sec^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3788, 3767, 8, 4046, 3770}

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{b^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(2a^2+b^2\right)\int \sec(c+dx)\,dx - \frac{(2ab)\,\mathrm{Su}}{}$$
$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan}{2d}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.76

$$\frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx)) + b\tan(c+dx)(4a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b*(4*a + b*Sec[c + d*x])*Tan[c + d*x
])/(2*d)
```

fricas [A] time = 0.49, size = 93, normalized size = 1.58

$$\frac{\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(4\,ab\cos\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 + b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + b^2)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.25, size = 129, normalized size = 2.19

$$\frac{\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 + b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan(1
/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.74, size = 78, normalized size = 1.32

$$\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{2ab\tan\left(dx+c\right)}{d} + \frac{b^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d} + \frac{b^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c)*tan
(d*x+c)/d+1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 80, normalized size = 1.36

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4a^2\log(\sec(dx+c)+\tan(dx+c))-8ab\tan}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b*tan(d
*x + c))/d
```

mupad [B] time = 1.53, size = 99, normalized size = 1.68

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2+b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a\,b-b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b^2+4\,a\,b\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^2 + b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
- b^2) - tan(c/2 + (d*x)/2)*(4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan
(c/2 + (d*x)/2)^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

3.460 $\int (a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=33

$$a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] a^2*x+2*a*b*arctanh(sin(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\,dx &= a^2x+(2ab)\int \sec(c+dx)\,dx+b^2\int \sec^2(c+dx)\,dx\\ &= a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.97

$$\frac{a^2dx+2ab\tanh^{-1}(\sin(c+dx))+b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2,x]
[Out] (a^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + b^2*Tan[c + d*x])/d
```

fricas [B] time = 0.47, size = 74, normalized size = 2.24

$$\frac{a^2dx\cos(dx+c)+ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)+b^2\sin(dx+c}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] (a^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*
x + c)*log(-sin(d*x + c) + 1) + b^2*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.19, size = 77, normalized size = 2.33

$$\frac{(dx+c)a^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] ((d*x + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

maple [A] time = 0.56, size = 49, normalized size = 1.48

$$a^2x+\frac{b^2\tan(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2,x)
[Out] a^2*x+b^2*tan(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2*c
```

maxima [A] time = 0.33, size = 40, normalized size = 1.21

$$a^2x+\frac{2\,ab\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] a^2*x + 2*a*b*log(sec(d*x + c) + tan(d*x + c))/d + b^2*tan(d*x + c)/d
```

mupad [B] time = 0.87, size = 181, normalized size = 5.48

$$\frac{2\,a^2\,\mathrm{atan}\left(\frac{64\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}+\frac{256\,a^4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}\right)}{d}-\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{128\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,a^5\,b+512\,a^3\,b^3}+\frac{512\,a^3\,b^3\tan\left(\frac{c}{2}\right.}{128\,a^5\,b+512\,a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2) + (256*a^4*b
^2*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2)))/d - (2*b^2*tan(c/2 + (d*x)/
2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a^5*b*tan(c/2 + (d*x
)/2))/(128*a^5*b + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128*a^5
*b + 512*a^3*b^3)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2, x)
```

3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=33

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

[Out] 2*a*b*x+b^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3788, 8, 4045, 3770}

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]

[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int 1\,dx+\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx\\&=2abx+\frac{a^2\sin(c+dx)}{d}+b^2\int \sec(c+dx)\,dx\\&=2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.39

$$\frac{a^2\sin(c)\cos(dx)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

fricas [A] time = 0.45, size = 52, normalized size = 1.58

$$\frac{4\,abdx + b^2\log\left(\sin\left(dx + c\right) + 1\right) - b^2\log\left(-\sin\left(dx + c\right) + 1\right) + 2\,a^2\sin\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + b^2*log(sin(d*x + c) + 1) - b^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

giac [B] time = 0.22, size = 78, normalized size = 2.36

$$\frac{2\,(dx + c)ab + b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a*b + b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

maple [A] time = 0.57, size = 49, normalized size = 1.48

$$2abx + \frac{a^2\sin\left(dx + c\right)}{d} + \frac{b^2\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{2abc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2*a*b*x+a^2*sin(d*x+c)/d+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*c
```

maxima [A] time = 0.39, size = 51, normalized size = 1.55

$$\frac{4\,(dx + c)ab + b^2\left(\log\left(\sin\left(dx + c\right) + 1\right) - \log\left(\sin\left(dx + c\right) - 1\right)\right) + 2\,a^2\sin\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b + b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.85, size = 73, normalized size = 2.21

$$\frac{a^2\,\sin\left(c + d\,x\right)}{d} + \frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cos(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

```
[Out] 1/2*(a^2+2*b^2)*x+2*a*b*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3788, 2637, 4045, 8}

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos(c+dx)\,dx+\int \cos^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx\\ &= \frac{2ab\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(a^2+2b^2\right)\int 1\,dx\\ &= \frac{1}{2}\left(a^2+2b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 0.92

$$\frac{2\left(a^2+2b^2\right)(c+dx)+a^2\sin(2(c+dx))+8ab\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])/(4*
d)
```

fricas [A] time = 0.45, size = 40, normalized size = 0.80

$$\frac{\left(a^{2}+2\,b^{2}\right)dx+\left(a^{2}\cos\left(dx+c\right)+4\,ab\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*b^2)*d*x + (a^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

giac [B] time = 0.20, size = 96, normalized size = 1.92

$$\frac{\left(a^{2}+2\,b^{2}\right)\left(dx+c\right)-\frac{2\left(a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*(d*x + c) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/
2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.56, size = 51, normalized size = 1.02

$$\frac{a^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\sin\left(dx+c\right)ab+b^{2}\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*sin(d*x+c)*a*b+b^2*(d*
x+c))
```

maxima [A] time = 0.47, size = 47, normalized size = 0.94

$$\frac{\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}+4\left(dx+c\right)b^{2}+8\,ab\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*b^2 + 8*a*b*sin(d*x
 + c))/d
```

mupad [B] time = 0.85, size = 42, normalized size = 0.84

$$\frac{a^{2}\,x}{2}+b^{2}\,x+\frac{a^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + b^2*x + (a^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \cos^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=58

$$\frac{(a^2+b^2)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+abx$$

```
[Out] a*b*x+(a^2+b^2)*sin(d*x+c)/d+a*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)
^3/d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2635, 8, 4044, 3013}

$$\frac{(a^2+b^2)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a*b*x + ((a^2 + b^2)*Sin[c + d*x])/d + (a*b*Cos[c + d*x]*Sin[c + d*x])/d -
(a^2*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos^2(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\cos(c+dx)\sin(c+dx)}{d} + (ab)\int 1\,dx + \int \cos(c+dx)\left(b^2+a^2\cos\right.$$
$$= abx + \frac{ab\cos(c+dx)\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(a^2+b^2-a^2x^2\right)\,dx, x, -\right.}{d}$$
$$= abx + \frac{\left(a^2+b^2\right)\sin(c+dx)}{d} + \frac{ab\cos(c+dx)\sin(c+dx)}{d} - \frac{a^2\sin^3(c}{3d}$$

Mathematica [A] time = 0.15, size = 59, normalized size = 1.02

$$\frac{3\left(3a^2+4b^2\right)\sin(c+dx)+a(a\sin(3(c+dx))+12b(c+dx)+6b\sin(2(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(3*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*b*(c + d*x) + 6*b*Sin[2*(c + d*x)]
+ a*Sin[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.45, size = 52, normalized size = 0.90

$$\frac{3\,abdx + \left(a^2\cos\left(dx+c\right)^2 + 3\,ab\cos\left(dx+c\right) + 2\,a^2 + 3\,b^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*d*x + (a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 2*a^2 + 3*b^2)*
sin(d*x + c))/d
```

giac [B] time = 0.21, size = 153, normalized size = 2.64

$$\frac{3\,(dx+c)ab + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 3\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a*b + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x + 1/2*c)^3 +
6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d
*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.91, size = 63, normalized size = 1.09

$$\frac{\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + 2ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + b^2\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^2*sin(d*x+c))

maxima [A] time = 0.34, size = 60, normalized size = 1.03

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab-6\,b^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 6*b^2*sin(d*x + c))/d

mupad [B] time = 0.83, size = 72, normalized size = 1.24

$$\frac{2\,a^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b^{2}\,\sin\left(c+d\,x\right)}{d}+a\,b\,x+\frac{a^{2}\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^2,x)

[Out] (2*a^2*sin(c + d*x))/(3*d) + (b^2*sin(c + d*x))/d + a*b*x + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{2}\cos^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2,x)

[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)

3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+4b^2\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c}{d}$$

```
[Out] 1/8*(3*a^2+4*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x
+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2633, 4045, 2635, 8}

$$\frac{\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+4b^2\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + 4*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((3*a^2 + 4*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(3a^2+4b^2\right)\int \cos^2(c+dx)\,dx - \frac{(2ab)\,S}{}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)}{4d}$$
$$= \frac{1}{8}\left(3a^2+4b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.17, size = 86, normalized size = 0.85

$$\frac{24\left(a^2+b^2\right)\sin(2(c+dx))+3a^2\sin(4(c+dx))+36a^2c+36a^2dx-64ab\sin^3(c+dx)+192ab\sin(c+dx)+48b}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (36*a^2*c + 48*b^2*c + 36*a^2*d*x + 48*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*
a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*a^2*Sin[4*(c + d*x
)])/(96*d)
```

fricas [A] time = 0.46, size = 77, normalized size = 0.76

$$\frac{3\left(3a^2+4b^2\right)dx+\left(6a^2\cos(dx+c)^3+16ab\cos(dx+c)^2+32ab+3\left(3a^2+4b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*d*x + (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
 + 32*a*b + 3*(3*a^2 + 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.19, size = 224, normalized size = 2.22

$$\frac{3\left(3a^2+4b^2\right)(dx+c) - \frac{2\left(15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-48ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-80ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1}{}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*(d*x + c) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a
*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*a^2*tan(1/2*d
*x + 1/2*c)^5 - 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*b^2*tan(1/2*d*x + 1/2*c)
^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*
tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x +
 1/2*c) - 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 1.05, size = 89, normalized size = 0.88

$$\frac{a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.33, size = 82, normalized size = 0.81

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^2-64\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)ab+24\,(2\,dx+2\,c}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 64*(s
in(d*x + c)^3 - 3*sin(d*x + c))*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2)/d
```

mupad [B] time = 0.87, size = 93, normalized size = 0.92

$$\frac{3\,a^2\,x}{8}+\frac{b^2\,x}{2}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c + 4*d
*x))/(32*d) + (b^2*sin(2*c + 2*d*x))/(4*d) + (3*a*b*sin(c + d*x))/(2*d) + (
a*b*sin(3*c + 3*d*x))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\cos^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**4, x)
```

3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=111

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\mathrm{c}}{4d}$$

```
[Out] 3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*co
s(d*x+c)^3*sin(d*x+c)/d-1/3*(2*a^2+b^2)*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5
/d
```

Rubi [A] time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 2635, 8, 4044, 3013, 373}

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\mathrm{c}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((2*a^2 + b^2)*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
```

```
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\,dx + \int \cos^5(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(3ab)\int \cos^2(c+dx)\,dx + \int \cos^3(c$$
$$= \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{4}(3ab)$$
$$= \frac{3abx}{4} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} -$$
$$= \frac{3abx}{4} + \frac{\left(a^2+b^2\right)\sin(c+dx)}{d} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + ab\cos$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.77

$$\frac{-80\left(2a^2+b^2\right)\sin^3(c+dx)+240\left(a^2+b^2\right)\sin(c+dx)+48a^2\sin^5(c+dx)+15ab(12(c+dx)+8\sin(2(c+d}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (240*(a^2 + b^2)*Sin[c + d*x] - 80*(2*a^2 + b^2)*Sin[c + d*x]^3 + 48*a^2*Si
n[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(240*d)
```

fricas [A] time = 0.45, size = 86, normalized size = 0.77

$$\frac{45\,abdx+\left(12\,a^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3+45\,ab\cos\left(dx+c\right)+4\left(4\,a^2+5\,b^2\right)\cos\left(dx+c\right)^2+32\,a^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(45*a*b*d*x + (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*
cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 32*a^2 + 40*b^2)*sin(d*x
+ c))/d
```

giac [B] time = 0.22, size = 247, normalized size = 2.23

$$45\,(dx+c)ab + \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+160\,b^2\,t}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*
d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 + 80*a^2*tan(1/2*d*x + 1/2*c
)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*
a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*a^2*tan(1/
```

```
2*d*x + 1/2*c)^3 + 30*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 1.32, size = 95, normalized size = 0.86

$$\frac{\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

maxima [A] time = 0.34, size = 94, normalized size = 0.85

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 80*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*b^2)/d
```

mupad [B] time = 0.89, size = 117, normalized size = 1.05

$$\frac{5a^2\sin(c+dx)}{8d}+\frac{3b^2\sin(c+dx)}{4d}+\frac{3abx}{4}+\frac{5a^2\sin(3c+3dx)}{48d}+\frac{a^2\sin(5c+5dx)}{80d}+\frac{b^2\sin(3c+3dx)}{12d}+\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (5*a^2*sin(c + d*x))/(8*d) + (3*b^2*sin(c + d*x))/(4*d) + (3*a*b*x)/4 + (5*
a^2*sin(3*c + 3*d*x))/(48*d) + (a^2*sin(5*c + 5*d*x))/(80*d) + (b^2*sin(3*c
 + 3*d*x))/(12*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x))/(
16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$\frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\left(3a^2-16b^2\right)\tan(c+dx)(a+b\sec(c+dx))^2}{60bd}-\frac{a\left(6a^2-71b^2\right)\tan(c+dx)\,\mathrm{se}}{120d}$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*arctanh(sin(d*x+c))/d-1/30*(3*a^4-52*a^2*b^2-16*b^4)*ta
n(d*x+c)/b/d-1/120*a*(6*a^2-71*b^2)*sec(d*x+c)*tan(d*x+c)/d-1/60*(3*a^2-16*
b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/20*a*(a+b*sec(d*x+c))^3*tan(d*x+c)
/b/d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d
```

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3840, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{\left(-52a^2b^2+3a^4-16b^4\right)\tan(c+dx)}{30bd}+\frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\left(3a^2-16b^2\right)\tan(c+dx)(a+b\sec(}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - ((3*a^4 - 52*a^2*b^2 - 16
*b^4)*Tan[c + d*x])/(30*b*d) - (a*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(120*d) - ((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b*d
) - (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + ((a + b*Sec[c + d*x]
)^4*Tan[c + d*x])/(5*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] && !LtQ[m, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c+dx)(4b-a\sec(c+dx))(a+}{5b}$$
$$= -\frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} +$$
$$= -\frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} - \frac{a(a+b\sec(c+dx))^3\,\text{ta}}{20bd}$$
$$= -\frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+dx)}{120d} - \frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx}{60bd}$$
$$= -\frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+dx)}{120d} - \frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx}{60bd}$$
$$= \frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+}{120d}$$
$$= \frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{30bd}$$

Mathematica [A] time = 0.90, size = 120, normalized size = 0.63

$$\frac{15a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8b\left(5\left(3a^2+2b^2\right)\tan^2(c+dx)+15\left(3a^2+b^2\right)+3b^2\tan^4(c+\right.\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (15*a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*a^2 + 9
*b^2)*Sec[c + d*x] + 90*a*b^2*Sec[c + d*x]^3 + 8*b*(15*(3*a^2 + b^2) + 5*(3
*a^2 + 2*b^2)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(120*d)
```

fricas [A] time = 0.46, size = 170, normalized size = 0.90

$$\frac{15\left(4a^3+9ab^2\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(4a^3+9ab^2\right)\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 + 9*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 + 9*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(15*a^2*b + 4*b^
3)*cos(d*x + c)^4 + 90*a*b^2*cos(d*x + c) + 15*(4*a^3 + 9*a*b^2)*cos(d*x +
c)^3 + 24*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

giac [B] time = 0.30, size = 367, normalized size = 1.94

$$15\left(4a^3+9ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4a^3+9ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
+ 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 225*a*b^2*tan(1/2*d*x + 1/2*c)^9
 - 120*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*a^
2*b*tan(1/2*d*x + 1/2*c)^7 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 160*b^3*tan(
1/2*d*x + 1/2*c)^7 - 1200*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 464*b^3*tan(1/2*d*
x + 1/2*c)^5 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 960*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*b^3*tan(1/2*d*x + 1/2*c)^3 -
60*a^3*tan(1/2*d*x + 1/2*c) - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 225*a*b^2*ta
n(1/2*d*x + 1/2*c) - 120*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

maple [A] time = 1.08, size = 206, normalized size = 1.09

$$\frac{a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a^2b\tan(dx+c)}{d}+\frac{a^2b\tan(dx+c)\left(\sec^2(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2
*b*tan(d*x+c)+1/d*a^2*b*tan(d*x+c)*sec(d*x+c)^2+3/4/d*b^2*a*tan(d*x+c)*sec(
d*x+c)^3+9/8*a*b^2*sec(d*x+c)*tan(d*x+c)/d+9/8/d*b^2*a*ln(sec(d*x+c)+tan(d*
x+c))+8/15/d*b^3*tan(d*x+c)+1/5/d*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*b^3*ta
n(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.47, size = 181, normalized size = 0.96

$$\frac{240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2b+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)b^3-45ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3 - 45*a*b^2*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.77, size = 258, normalized size = 1.37

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^3+\frac{9\,a\,b^2}{4}\right)}{d}-\frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^3-16\,a^2\,b+\frac{3\,a\,b^2}{2}-\frac{8\,b^3}{3}\right)\,\mathrm{t}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((9*a*b^2)/4 + a^3))/d - (tan(c/2 + (d*x)/2)^7*(
(3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) - tan(c/2 + (d*x)/2)^3*((3*a*b^
2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) + tan(c/2 + (d*x)/2)*((15*a*b^2)/4 + 6
*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2*b + (116*b^3)/15) - ta
n(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2*b^3))/(d*(5*tan(c/2 +
(d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^3\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=130

$$\frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d}+\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\tan(c+d}{}$$

```
[Out] 3/8*b*(4*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a*(a^2+4*b^2)*tan(d*x+c)/d+1/8*
b*(2*a^2+3*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(a+b*sec(d*x+c))^2*tan(d*x+c)
/d+1/4*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3835, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d}+\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\tan(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(a^2 + 4*b^2)*Tan[c +
d*x])/(2*d) + (b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) + ((a + b*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
```

```
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{3}{4}\int \sec(c+dx)(b+a\sec(c+dx))(a\\ &= \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d}+\frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\\ &= \frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(a+b\sec(c+dx))^2\tan(c+dx}{4d}\\ &= \frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(a+b\sec(c+dx))^2\tan(c+dx}{4d}\\ &= \frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+d}{8d}\\ &= \frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d}+\frac{b\left(2a^2+\right.}{}\end{aligned}$$

Mathematica [A] time = 0.45, size = 90, normalized size = 0.69

$$\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8a\left(a^2+b^2\tan^2(c+dx)+3b^2\right)+3b\left(4a^2+b^2\right)\sec(c+dx)+2b\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*a^2 + b^2)*
Sec[c + d*x] + 2*b^3*Sec[c + d*x]^3 + 8*a*(a^2 + 3*b^2 + b^2*Tan[c + d*x]^2
)))/(8*d)
```

fricas [A] time = 0.48, size = 140, normalized size = 1.08

$$\frac{3\left(4a^2b+b^3\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4a^2b+b^3\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(8ab^2\right.}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(4*a^2*b + b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2*b +
 b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a*b^2*cos(d*x + c) + 8*(
a^3 + 2*a*b^2)*cos(d*x + c)^3 + 2*b^3 + 3*(4*a^2*b + b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.28, size = 330, normalized size = 2.54

$$3\left(4a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-12a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b + b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a^3*tan(1/2*d*x + 1/2*c)^7 - 1
2*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*ta
n(1/2*d*x + 1/2*c)^7 - 24*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d
*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 24*a*b^2*tan(1/2*d*x + 1/2*c)
 - 5*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 1.10, size = 160, normalized size = 1.23

$$\frac{a^3\tan(dx+c)}{d}+\frac{3a^2b\sec(dx+c)\tan(dx+c)}{2d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2ab^2\tan(dx+c)}{d}+\frac{b^2a\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] a^3*tan(d*x+c)/d+3/2/d*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*ln(sec(d*x+c
)+tan(d*x+c))+2*a*b^2*tan(d*x+c)/d+1/d*b^2*a*tan(d*x+c)*sec(d*x+c)^2+1/4/d*
b^3*tan(d*x+c)*sec(d*x+c)^3+3/8*b^3*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b^3*ln(se
c(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 158, normalized size = 1.22

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^2-b^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - b^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*b*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

mupad [B] time = 4.80, size = 226, normalized size = 1.74

$$\frac{3b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(4a^2+b^2\right)}{4d}-\frac{\left(2a^3-3a^2b+6ab^2-\frac{5b^3}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(-6a^3+3a^2b-10ab^2-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (3*b*atanh(tan(c/2 + (d*x)/2))*(4*a^2 + b^2))/(4*d) - (tan(c/2 + (d*x)/2)^7
*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b^2 +
 3*a^2*b + 6*a^3 - (3*b^3)/4) - tan(c/2 + (d*x)/2)^5*(10*a*b^2 - 3*a^2*b +
6*a^3 + (3*b^3)/4) - tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3 + (5*b^3
)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*
x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^3 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{6d}+\frac{b\tan(c+dx)(a\ \cdots}{3\ \cdots}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/3*b*(4*a^2+b^2)*tan(d*x+c)/d+5/
6*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/3*b*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3830, 3997, 3787, 3770, 3767, 8}

$$\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{6d}+\frac{b\tan(c+dx)(a\ \cdots}{3\ \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*b*(4*a^2 + b^2)*Tan[c
+ d*x])/(3*d) + (5*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*(a + b*Sec[c
 + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
```

```
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\,(3\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{6}\int \mathrm{s}\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\left(2b\right.\\ &= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+}{}\\ &= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2b\left(4a^2+b^2\right)\tan(c+dx)}{3d}+\frac{5ab^2\sec}{}\end{aligned}$$

Mathematica [A] time = 0.25, size = 70, normalized size = 0.71

$$\frac{\left(6a^3+9ab^2\right)\tanh^{-1}(\sin(c+dx))+b\tan(c+dx)\left(18a^2+9ab\sec(c+dx)+2b^2\tan^2(c+dx)+6b^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((6*a^3 + 9*a*b^2)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(18*a^2 + 6*b^2 +
 9*a*b*Sec[c + d*x] + 2*b^2*Tan[c + d*x]^2))/(6*d)
```

fricas [A] time = 0.45, size = 126, normalized size = 1.27

$$\frac{3\left(2a^3+3ab^2\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2a^3+3ab^2\right)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(9a\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 +
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a*b^2*cos(d*x + c) +
 2*b^3 + 2*(9*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

giac [B] time = 0.28, size = 205, normalized size = 2.07

$$\frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(18a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9\right.}{}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 + 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b*tan(1/2*d*x + 1/2*c
)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*a^
```

```
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b*tan(1/
2*d*x + 1/2*c) + 9*a*b^2*tan(1/2*d*x + 1/2*c) + 6*b^3*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.91, size = 118, normalized size = 1.19

$$\frac{a^3 \ln (\sec (dx+c)+\tan (dx+c))}{d}+\frac{3a^2b \tan (dx+c)}{d}+\frac{3a\,b^2 \sec (dx+c) \tan (dx+c)}{2d}+\frac{3b^2a \ln (\sec (dx+c)+\text{t}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*tan(d*x+c)+3/2*a*b^2*sec(d*x+c)
*tan(d*x+c)/d+3/2/d*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b^3*tan(d*x+c)+1/
3/d*b^3*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.34, size = 106, normalized size = 1.07

$$\frac{4\left(\tan (dx+c)^3+3\,\tan (dx+c)\right)b^3-9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log (\sin (dx+c)+1)+\log (\sin (dx+c)-1)\right)+12\,a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^3 - 9*a*b^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3
*log(sec(d*x + c) + tan(d*x + c)) + 36*a^2*b*tan(d*x + c))/d
```

mupad [B] time = 3.15, size = 157, normalized size = 1.59

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^3+3\,a\,b^2\right)}{d}-\frac{\left(6\,a^2\,b-3\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-12\,a^2\,b-\frac{4\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 + 2*a^3))/d - (tan(c/2 + (d*x)/2)^5*(6*
a^2*b - 3*a*b^2 + 2*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + (4*b^3)/3) + ta
n(c/2 + (d*x)/2)*(3*a*b^2 + 6*a^2*b + 2*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (c+dx))^3 \sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

3.469 $\int (a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] a^3*x+1/2*b*(6*a^2+b^2)*arctanh(sin(d*x+c))/d+5/2*a*b^2*tan(d*x+c)/d+1/2*b^
2*(a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a*b^2*Tan[c + d*
x])/(2*d) + (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^3\,dx &= \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\sec(c+dx)+5ab^2\sec^2(\\ &= a^3x+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\left(5ab^2\right)\int \sec^2(c+dx)\,dx+\frac{1}{2}\left(b\left(6a^2+b\right.\right.\\ &= a^3x+\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{\left(5ab^2\right)}{}\\ &= a^3x+\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))\,\mathrm{ta}}{2d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 55, normalized size = 0.75

$$\frac{2a^3dx+b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+b^2\tan(c+dx)(6a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b^2*(6*a + b*Sec[c + d
*x])*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.49, size = 112, normalized size = 1.53

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2+\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(d\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + (6*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (6*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a*
b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.20, size = 145, normalized size = 1.99

$$\frac{2\left(dx+c\right)a^3+\left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,ab^2\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*b^2*tan(1/2*d*
x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c) -
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.73, size = 95, normalized size = 1.30

$$a^3x+\frac{a^3c}{d}+\frac{3a^2b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3a\,b^2\tan\left(dx+c\right)}{d}+\frac{b^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{b^3\ln\left(\sec\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*tan(d*x+c)/d+1/
2*b^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.36, size = 93, normalized size = 1.27

$$a^3x-\frac{b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}+\frac{3\,a^2b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^2*b*log(sec(d*x + c) + tan(d*x + c))/d +
 3*a*b^2*tan(d*x + c)/d
```

mupad [B] time = 0.95, size = 136, normalized size = 1.86

$$\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
 + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(
c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec{\left (c + d x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3, x)
```

3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

```
[Out] 3*a^2*b*x+3*a*b^2*arctanh(sin(d*x+c))/d+a*(a^2-b^2)*sin(d*x+c)/d+b^2*(a+b*s
ec(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3842, 4047, 8, 4045, 3770}

$$\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x]
)/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \int \cos(c+dx)\left(a\left(a^2-b^2\right)+3a^2b\sec\right.$$
$$= \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \left(3a^2b\right)\int 1\,dx + \int \cos(c+dx)\left(a\left(a^2\right.\right.$$
$$= 3a^2bx + \frac{a\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + (3a$$
$$= 3a^2bx + \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{b^2(a+b\,\mathrm{se}}{}$$

Mathematica [A] time = 0.34, size = 88, normalized size = 1.31

$$\frac{a^3\sin(c+dx)+3ab\left(ac+adx-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b*(a*c + a*d*x - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[c + d*x] + b^3*Tan[c + d*x])/
d
```

fricas [A] time = 0.49, size = 94, normalized size = 1.40

$$\frac{6\,a^2bdx\cos(dx+c)+3\,ab^2\cos(dx+c)\log(\sin(dx+c)+1)-3\,ab^2\cos(dx+c)\log(-\sin(dx+c)+1)+2\left(a^3\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^2*b*d*x*cos(d*x + c) + 3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + b^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.23, size = 131, normalized size = 1.96

$$\frac{3\,(dx+c)a^2b+3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\tan(}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^2*b + 3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan
(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 0.64, size = 68, normalized size = 1.01

$$3a^2bx + \frac{3b^2a\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^3\sin(dx+c)}{d} + \frac{b^3\tan(dx+c)}{d} + \frac{3a^2bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 3*a^2*b*x+3/d*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+a^3*sin(d*x+c)/d+1/d*b^3*tan(
d*x+c)+3/d*a^2*b*c
```

maxima [A] time = 0.37, size = 66, normalized size = 0.99

$$\frac{6\,(dx+c)a^2b+3\,ab^2\Big(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\Big)+2\,a^3\sin\left(dx+c\right)+2\,b^3\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^2*b + 3*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*a^3*sin(d*x + c) + 2*b^3*tan(d*x + c))/d
```

mupad [B] time = 0.92, size = 97, normalized size = 1.45

$$\frac{a^3\,\sin\left(c+d\,x\right)}{d}+\frac{b^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x))/d + (b^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a^2*b*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a*b^2*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 1/2*a*(a^2+6*b^2)*x+b^3*arctanh(sin(d*x+c))/d+5/2*a^2*b*sin(d*x+c)/d+1/2*a^
2*cos(d*x+c)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3841, 4047, 8, 4045, 3770}

$$\frac{1}{2}ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*x)/2 + (b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*
x])/(2*d) + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)\left(5a^2\right.$$
$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)\left(5a^2\right.$$
$$= \frac{1}{2}a\left(a^2+6b^2\right)x + \frac{5a^2b\sin(c+dx)}{2d} + \frac{a^2\cos(c+dx)(a+b\sec(c+dx}{2d}$$
$$= \frac{1}{2}a\left(a^2+6b^2\right)x + \frac{b^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^2b\sin(c+dx)}{2d} + \frac{a^2\cos}{}$$

Mathematica [A] time = 0.14, size = 105, normalized size = 1.33

$$\frac{a^3\sin(2(c+dx)) + 2a\left(a^2+6b^2\right)(c+dx) + 12a^2b\sin(c+dx) - 4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) +}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(a^2 + 6*b^2)*(c + d*x) - 4*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 4*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x]
 + a^3*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.48, size = 72, normalized size = 0.91

$$\frac{b^3\log(\sin(dx+c)+1) - b^3\log(-\sin(dx+c)+1) + \left(a^3+6\,ab^2\right)dx + \left(a^3\cos(dx+c)+6\,a^2b\right)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(sin(d*x + c) + 1) - b^3*log(-sin(d*x + c) + 1) + (a^3 + 6*a*b^
2)*d*x + (a^3*cos(d*x + c) + 6*a^2*b)*sin(d*x + c))/d
```

giac [A] time = 0.25, size = 137, normalized size = 1.73

$$\frac{2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \left(a^3+6\,ab^2\right)(dx+c) - \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^3 + 6*a*b^2)*(d*x + c) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 -
 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.50, size = 90, normalized size = 1.14

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d} + \frac{a^3x}{2} + \frac{a^3c}{2d} + \frac{3a^2b\sin(dx+c)}{d} + 3b^2ax + \frac{3a\,b^2c}{d} + \frac{b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c+3*a^2*b*sin(d*x+c)/d+
3*b^2*a*x+3/d*a*b^2*c+1/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.38, size = 76, normalized size = 0.96

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^3 + 12\,(dx + c)ab^2 + 2\,b^3\left(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)\right) + 12\,a^2b\sin}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a*b^2 + 2*b^3*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^2*b*sin(d*x + c))/d
```

mupad [B] time = 1.03, size = 123, normalized size = 1.56

$$\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(2*c + 2*d*x))/(4*d) + (3*a^2*b
*sin(c + d*x))/d + (6*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^3\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)
```

3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=100

$$\frac{a(2a^2+9b^2)\sin(c+dx)}{3d}+\frac{1}{2}bx(3a^2+2b^2)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{6d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\,\mathrm{se}}{3d}$$

```
[Out] 1/2*b*(3*a^2+2*b^2)*x+1/3*a*(2*a^2+9*b^2)*sin(d*x+c)/d+7/6*a^2*b*cos(d*x+c)
*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3841, 4047, 2637, 4045, 8}

$$\frac{a(2a^2+9b^2)\sin(c+dx)}{3d}+\frac{1}{2}bx(3a^2+2b^2)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{6d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\,\mathrm{se}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/(3*d) + (7*a^2*b
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x]
)*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{1}{3}\int \cos^2(c+dx)\left(7a^2b\right.\\ &= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{1}{3}\int \cos^2(c+dx)\left(7a^2b\right.\\ &= \frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d}+\frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^2(c+\right.}{}\\ &= \frac{1}{2}b\left(3a^2+2b^2\right)x+\frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d}+\frac{7a^2b\cos(c+dx)\sin(c+}{6d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.80

$$\frac{a^3\sin(3(c+dx))+9a\left(a^2+4b^2\right)\sin(c+dx)+9a^2b\sin(2(c+dx))+18a^2bc+18a^2bdx+12b^3c+12b^3dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (18*a^2*b*c + 12*b^3*c + 18*a^2*b*d*x + 12*b^3*d*x + 9*a*(a^2 + 4*b^2)*Sin[
c + d*x] + 9*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.46, size = 66, normalized size = 0.66

$$\frac{3\left(3a^2b+2b^3\right)dx+\left(2a^3\cos\left(dx+c\right)^2+9a^2b\cos\left(dx+c\right)+4a^3+18ab^2\right)\sin\left(dx+c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^2*b*cos(d*x + c)
 + 4*a^3 + 18*a*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.23, size = 170, normalized size = 1.70

$$\frac{3\left(3a^2b+2b^3\right)(dx+c)+\frac{2\left(6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*(d*x + c) + 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^
2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^3*tan(1/
2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/
2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.76, size = 76, normalized size = 0.76

$$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3b^2a\sin\left(dx+c\right)+b^3\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*b^2*a*sin(d*x+c)+b^3*(d*x+c))
```

maxima [A] time = 0.34, size = 73, normalized size = 0.73

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}b-12\left(dx+c\right)b^{3}-36\,ab^{2}\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2*b - 12*(d*x + c)*b^3 - 36*a*b^2*sin(d*x + c))/d
```

mupad [B] time = 0.86, size = 77, normalized size = 0.77

$$b^3\,x+\frac{3\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,a^2\,b\,x}{2}+\frac{3\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

```
[Out] b^3*x + (3*a^3*sin(c + d*x))/(4*d) + (a^3*sin(3*c + 3*d*x))/(12*d) + (3*a^2
*b*x)/2 + (3*a^2*b*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2*sin(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{3}\cos^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**3, x)
```

3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=123

$$\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2+4b^2\right)-\frac{3a^2b\sin^3(c+dx)}{4d}+\frac{a^2\sin(c+$$

```
[Out] 3/8*a*(a^2+4*b^2)*x+1/4*b*(11*a^2+4*b^2)*sin(d*x+c)/d+3/8*a*(a^2+4*b^2)*cos
(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))*sin(d*x+c)/d-3/4
*a^2*b*sin(d*x+c)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4047, 2635, 8, 4044, 3013}

$$\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2+4b^2\right)-\frac{3a^2b\sin^3(c+dx)}{4d}+\frac{a^2\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*x)/8 + (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(4*d) + (3*a*(a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])*Sin[c + d*x])/(4*d) - (3*a^2*b*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\left(9\right.$$
$$= \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\left(9\right.$$
$$= \frac{3a\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+$$
$$= \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{3a\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+$$
$$= \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\cos(c+$$

Mathematica [A] time = 0.28, size = 100, normalized size = 0.81

$$\frac{8b\left(9a^2+4b^2\right)\sin(c+dx)+a\left(8\left(a^2+3b^2\right)\sin(2(c+dx))+a^2\sin(4(c+dx))+12a^2c+12a^2dx+8ab\sin(3(c\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (8*b*(9*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*a^2*c + 48*b^2*c + 12*a^2*d*x + 4
8*b^2*d*x + 8*(a^2 + 3*b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + a^2
*Sin[4*(c + d*x)]))/(32*d)
```

fricas [A] time = 0.44, size = 84, normalized size = 0.68

$$\frac{3\left(a^3+4ab^2\right)dx+\left(2a^3\cos(dx+c)^3+8a^2b\cos(dx+c)^2+16a^2b+8b^3+3\left(a^3+4ab^2\right)\cos(dx+c)\right)\sin(d}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^2*b*cos(d*x + c)^2
 + 16*a^2*b + 8*b^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.25, size = 297, normalized size = 2.41

$$3\left(a^3+4ab^2\right)(dx+c)-\frac{2\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-8b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*(d*x + c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*tan(1/2
```

```
*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^2*b*tan(1/2*d*x + 1/2
*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 24*b^3*tan(1/2*d*x + 1/2*c)^5 + 3
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*tan(1/2*d*x +
1/2*c) - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*c) - 8*
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 1.00, size = 102, normalized size = 0.83

$$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3b^2a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*b^2*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+b^3*sin(d*x+c))
```

maxima [A] time = 0.37, size = 95, normalized size = 0.77

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^2b+24\,(2\,dx+2\,c+s}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^2 + 32*b^3*sin(d*x + c))/d
```

mupad [B] time = 3.83, size = 250, normalized size = 2.03

$$\frac{\left(-\frac{5\,a^3}{4}+6\,a^2\,b-3\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^3}{4}+10\,a^2\,b-3\,a\,b^2+6\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{3\,a^3}{4}+10\,a^2\,b+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) - tan(c/2 +
(d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) + tan(c/2 + (d*x)/2)^5*(
10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 + 6*a
^2*b + (5*a^3)/4 + 2*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2
)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atan((3*a*
tan(c/2 + (d*x)/2)*(a^2 + 4*b^2))/(4*(3*a*b^2 + (3*a^3)/4)))*(a^2 + 4*b^2))
/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=160

$$-\frac{a\left(4a^2+15b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(9a^2+4\right.$$

```
[Out] 1/8*b*(9*a^2+4*b^2)*x+1/5*a*(4*a^2+15*b^2)*sin(d*x+c)/d+1/8*b*(9*a^2+4*b^2)
*cos(d*x+c)*sin(d*x+c)/d+11/20*a^2*b*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*cos(
d*x+c)^4*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/15*a*(4*a^2+15*b^2)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4047, 2633, 4045, 2635, 8}

$$-\frac{a\left(4a^2+15b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(9a^2+4\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(9*a^2 + 4*b^2)*x)/8 + (a*(4*a^2 + 15*b^2)*Sin[c + d*x])/(5*d) + (b*(9*a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (11*a^2*b*Cos[c + d*x]^3*Sin
[c + d*x])/(20*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(5*d) - (a*(4*a^2 + 15*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\left(11a^2\right.$$

$$= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\left(11a^2\right.$$

$$= \frac{11a^2b\cos^3(c+dx)\sin(c+dx)}{20d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c}{5d}$$

$$= \frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{11}{}$$

$$= \frac{1}{8}b\left(9a^2+4b^2\right)x+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\cos(c+d}{8d}$$

Mathematica [A] time = 0.30, size = 130, normalized size = 0.81

$$\frac{50a^3\sin(3(c+dx))+6a^3\sin(5(c+dx))+120\left(3a^2b+b^3\right)\sin(2(c+dx))+60a\left(5a^2+18b^2\right)\sin(c+dx)+45a^2b}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (540*a^2*b*c + 240*b^3*c + 540*a^2*b*d*x + 240*b^3*d*x + 60*a*(5*a^2 + 18*b
^2)*Sin[c + d*x] + 120*(3*a^2*b + b^3)*Sin[2*(c + d*x)] + 50*a^3*Sin[3*(c +
 d*x)] + 120*a*b^2*Sin[3*(c + d*x)] + 45*a^2*b*Sin[4*(c + d*x)] + 6*a^3*Sin
[5*(c + d*x)])/(480*d)
```

fricas [A] time = 0.49, size = 110, normalized size = 0.69

$$\frac{15\left(9a^2b+4b^3\right)dx+\left(24a^3\cos(dx+c)^4+90a^2b\cos(dx+c)^3+64a^3+240ab^2+8\left(4a^3+15ab^2\right)\cos(dx+c\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^2*b*cos(d*x
 + c)^3 + 64*a^3 + 240*a*b^2 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2 + 15*(9*
a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.23, size = 332, normalized size = 2.08

$$15\left(9a^2b+4b^3\right)(dx+c)+\frac{2\left(120a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-225a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+360ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-60b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+160a^3\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*(d*x + c) + 2*(120*a^3*tan(1/2*d*x + 1/2*c)^9 -
 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*b
^3*tan(1/2*d*x + 1/2*c)^9 + 160*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*a^2*b*tan(1
/2*d*x + 1/2*c)^7 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*b^3*tan(1/2*d*x
+ 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*x + 1/2*
c)^5 + 160*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 9
60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*
tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*tan(1/2*d
*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/
d
```

maple [A] time = 1.28, size = 123, normalized size = 0.77

$$\frac{\frac{a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b^2a\left(2+\cos^2(dx+c)\right)\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+b^2*a*(2+cos(d*x+c)^2
)*sin(d*x+c)+b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.46, size = 119, normalized size = 0.74

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3+45\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 + 45
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 480*(sin(d
*x + c)^3 - 3*sin(d*x + c))*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^
3)/d
```

mupad [B] time = 3.93, size = 287, normalized size = 1.79

$$\frac{\left(2a^3-\frac{15a^2b}{4}+6ab^2-b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{8a^3}{3}-\frac{3a^2b}{2}+16ab^2-2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{116a^3}{15}+20ab^2\right)\tan}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3*a^2*b)/2 + (8*a^3)/3 + 2*b^3) + tan(c/
2 + (d*x)/2)^9*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) + tan(c/2 + (d*x)/2)^
7*(16*a*b^2 - (3*a^2*b)/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a*b^
2 + (15*a^2*b)/4 + 2*a^3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116*a^3
)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (b*atan
((b*tan(c/2 + (d*x)/2)*(9*a^2 + 4*b^2))/(4*((9*a^2*b)/4 + b^3)))*(9*a^2 + 4
*b^2))/(4*d)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=185

$$-\frac{b\left(5a^2+b^2\right)\sin^3(c+dx)}{3d}+\frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d}+\frac{a\left(5a^2+18b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\left(5a^2+18b\right.}{}$$

```
[Out] 1/16*a*(5*a^2+18*b^2)*x+1/6*b*(17*a^2+6*b^2)*sin(d*x+c)/d+1/16*a*(5*a^2+18*
b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*a*(5*a^2+18*b^2)*cos(d*x+c)^3*sin(d*x+c)/
d+1/6*a^2*cos(d*x+c)^5*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/3*b*(5*a^2+b^2)*sin(
d*x+c)^3/d+13/30*a^2*b*sin(d*x+c)^5/d
```

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4047, 2635, 8, 4044, 3013, 373}

$$-\frac{b\left(5a^2+b^2\right)\sin^3(c+dx)}{3d}+\frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d}+\frac{a\left(5a^2+18b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\left(5a^2+18b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 18*b^2)*x)/16 + (b*(17*a^2 + 6*b^2)*Sin[c + d*x])/(6*d) + (a*(5
*a^2 + 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(5*a^2 + 18*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*
Sin[c + d*x])/(6*d) - (b*(5*a^2 + b^2)*Sin[c + d*x]^3)/(3*d) + (13*a^2*b*Si
n[c + d*x]^5)/(30*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5(c+dx)\left(13a^2\right.\\
&= \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5(c+dx)\left(13a^2\right.\\
&= \frac{a\left(5a^2+18b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2\cos^5(c+dx)(a+b\sec(c+\right.}{6d}\\
&= \frac{a\left(5a^2+18b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\left(5a^2+18b^2\right)\cos^3(c+dx)\sin}{24d}\\
&= \frac{1}{16}a\left(5a^2+18b^2\right)x+\frac{a\left(5a^2+18b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\left(5a^2+\right.}{}\\
&= \frac{1}{16}a\left(5a^2+18b^2\right)x+\frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d}+\frac{a\left(5a^2+18b^2\right)\cos(c}{16d}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 159, normalized size = 0.86

$$\frac{45\left(5a^3+16ab^2\right)\sin(2(c+dx))+45a^3\sin(4(c+dx))+5a^3\sin(6(c+dx))+300a^3c+300a^3dx+360b\left(5a^2+2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (300*a^3*c + 1080*a*b^2*c + 300*a^3*d*x + 1080*a*b^2*d*x + 360*b*(5*a^2 + 2
*b^2)*Sin[c + d*x] + 45*(5*a^3 + 16*a*b^2)*Sin[2*(c + d*x)] + 300*a^2*b*Sin
[3*(c + d*x)] + 80*b^3*Sin[3*(c + d*x)] + 45*a^3*Sin[4*(c + d*x)] + 90*a*b^
2*Sin[4*(c + d*x)] + 36*a^2*b*Sin[5*(c + d*x)] + 5*a^3*Sin[6*(c + d*x)])/(9
60*d)
```

fricas [A] time = 0.49, size = 132, normalized size = 0.71

$$\frac{15\left(5a^3+18ab^2\right)dx+\left(40a^3\cos(dx+c)^5+144a^2b\cos(dx+c)^4+10\left(5a^3+18ab^2\right)\cos(dx+c)^3+384a^2b+\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^2*b*cos(d
*x + c)^4 + 10*(5*a^3 + 18*a*b^2)*cos(d*x + c)^3 + 384*a^2*b + 160*b^3 + 16
```

```
*(12*a^2*b + 5*b^3)*cos(d*x + c)^2 + 15*(5*a^3 + 18*a*b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

giac [B] time = 0.27, size = 431, normalized size = 2.33

$$15\left(5a^3+18ab^2\right)(dx+c)-\frac{2\left(165a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-720a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+450ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-240b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-25a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*(d*x + c) - 2*(165*a^3*tan(1/2*d*x + 1/2*c)^11
 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*a*b^2*tan(1/2*d*x + 1/2*c)^11 -
240*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*a^2*
b*tan(1/2*d*x + 1/2*c)^9 + 630*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*b^3*tan(1
/2*d*x + 1/2*c)^9 + 450*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*a^2*b*tan(1/2*d*x
 + 1/2*c)^7 + 180*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*b^3*tan(1/2*d*x + 1/2
*c)^7 - 450*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*a^2*b*tan(1/2*d*x + 1/2*c)^5
- 180*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a
^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*a*b^2*t
an(1/2*d*x + 1/2*c)^3 - 880*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*a^3*tan(1/2*d*
x + 1/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) - 450*a*b^2*tan(1/2*d*x + 1/2*c
) - 240*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 1.49, size = 145, normalized size = 0.78

$$\frac{a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+3b^2a\left(\frac{\left(\cos^3(dx+c\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^2*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*b
^2*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^3*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.38, size = 145, normalized size = 0.78

$$-\frac{5\left(4\sin\left(2dx+2c\right)^3-60dx-60c-9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a^3-192\left(3\sin\left(dx+c\right)^5-10s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
*x + c))*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3)/d
```

mupad [B] time = 3.30, size = 350, normalized size = 1.89

$$\frac{\left(-\frac{11a^3}{8}+6a^2b-\frac{15ab^2}{4}+2b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{5a^3}{24}+14a^2b-\frac{21ab^2}{4}+\frac{22b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(-\frac{15a^3}{4}+\frac{156}{5}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((21*a*b^2)/4 + 14*a^2*b - (5*a^3)/24 + (22*b^3)/3) -
 tan(c/2 + (d*x)/2)^11*((15*a*b^2)/4 - 6*a^2*b + (11*a^3)/8 - 2*b^3) + tan(
c/2 + (d*x)/2)^9*(14*a^2*b - (21*a*b^2)/4 + (5*a^3)/24 + (22*b^3)/3) + tan(
c/2 + (d*x)/2)^5*((3*a*b^2)/2 + (156*a^2*b)/5 + (15*a^3)/4 + 12*b^3) - tan(
c/2 + (d*x)/2)^7*((3*a*b^2)/2 - (156*a^2*b)/5 + (15*a^3)/4 - 12*b^3) + tan(
c/2 + (d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + (11*a^3)/8 + 2*b^3))/(d*(6*tan(c/2
 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) +
(a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 18*b^2))/(8*((9*a*b^2)/4 + (5*a^3)/8
)))*(5*a^2 + 18*b^2))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=244

$$-\frac{(4a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^3}{120bd}-\frac{a(4a^2-53b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{120bd}-\frac{a(4a^4-121a^2$$

```
[Out] 1/16*(8*a^4+36*a^2*b^2+5*b^4)*arctanh(sin(d*x+c))/d-1/60*a*(4*a^4-121*a^2*b
^2-128*b^4)*tan(d*x+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*sec(d*x+c)*tan(
d*x+c)/d-1/120*a*(4*a^2-53*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/120*(4*
a^2-25*b^2)*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d-1/30*a*(a+b*sec(d*x+c))^4*tan
(d*x+c)/b/d+1/6*(a+b*sec(d*x+c))^5*tan(d*x+c)/b/d
```

Rubi [A] time = 0.45, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3840, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{a(-121a^2b^2+4a^4-128b^4)\tan(c+dx)}{60bd}+\frac{(36a^2b^2+8a^4+5b^4)\tanh^{-1}(\sin(c+dx))}{16d}-\frac{(4a^2-25b^2)\tan(c+d}{120}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) - (a*(4*a^4 - 1
21*a^2*b^2 - 128*b^4)*Tan[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b
^4)*Sec[c + d*x]*Tan[c + d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3
*Tan[c + d*x])/(120*b*d) - (a*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d)
 + ((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+dx)(5b-a\sec(c+dx))(a+$$

$$= -\frac{a(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} +$$

$$= -\frac{\left(4a^2-25b^2\right)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} - \frac{a(a+b\sec(c+dx))^4\,\mathrm{ta}}{30bd}$$

$$= -\frac{a\left(4a^2-53b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{120bd} - \frac{\left(4a^2-25b^2\right)(a+b\,\mathrm{se}}{12}$$

$$= -\frac{\left(8a^4-178a^2b^2-75b^4\right)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a\left(4a^2-53b^2\right)(a+b}{}$$

$$= -\frac{\left(8a^4-178a^2b^2-75b^4\right)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a\left(4a^2-53b^2\right)(a+b}{}$$

$$= \frac{\left(8a^4+36a^2b^2+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{\left(8a^4-178a^2b^2-75b^4\right)\sec}{240d}$$

$$= \frac{\left(8a^4+36a^2b^2+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{a\left(4a^4-121a^2b^2-128b^4\right)\mathrm{t}}{60bd}$$

Mathematica [A] time = 0.96, size = 154, normalized size = 0.63

$$\frac{15\left(8a^4+36a^2b^2+5b^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(64ab\left(5\left(a^2+2b^2\right)\tan^2(c+dx)+15\left(a^2+b^2\right)+3b^2\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(
8*a^4 + 36*a^2*b^2 + 5*b^4)*Sec[c + d*x] + 10*b^2*(36*a^2 + 5*b^2)*Sec[c +
d*x]^3 + 40*b^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(a^2 + 2*b^2)*T
an[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(240*d)
```

fricas [A] time = 0.51, size = 217, normalized size = 0.89

$$\frac{15\left(8\,a^4+36\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^4+36\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
 - 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
2*(128*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^5 + 192*a*b^3*cos(d*x + c) + 15*(8*
a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 + 40*b^4 + 64*(5*a^3*b + 4*a*b^3)*
cos(d*x + c)^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

giac [B] time = 0.31, size = 592, normalized size = 2.43

$$15\left(8a^4+36a^2b^2+5b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(8a^4+36a^2b^2+5b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12
0*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2
*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^4*
tan(1/2*d*x + 1/2*c)^11 - 360*a^4*tan(1/2*d*x + 1/2*c)^9 + 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^9 + 25*b^4*tan(1/2*d*x + 1/2*c)^9 + 240*a^4*tan(1/2*d*x + 1/
2*c)^7 - 5760*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 4992*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^4*tan(1/2*d*x + 1/2*c)^7 +
 240*a^4*tan(1/2*d*x + 1/2*c)^5 + 5760*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4992*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 450*b^4
*tan(1/2*d*x + 1/2*c)^5 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 - 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^3 + 25*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/
2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) +
960*a*b^3*tan(1/2*d*x + 1/2*c) + 165*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 1.24, size = 302, normalized size = 1.24

$$\frac{a^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{8a^3b\tan(dx+c)}{3d}+\frac{4a^3b\tan(dx+c)\left(\sec^2(dx\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a
^3*b*tan(d*x+c)+4/3/d*a^3*b*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^2*b^2*tan(d*x+c
)*sec(d*x+c)^3+9/4/d*a^2*b^2*sec(d*x+c)*tan(d*x+c)+9/4/d*a^2*b^2*ln(sec(d*x
+c)+tan(d*x+c))+32/15*a*b^3*tan(d*x+c)/d+4/5/d*a*b^3*tan(d*x+c)*sec(d*x+c)^
4+16/15/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/6/d*b^4*tan(d*x+c)*sec(d*x+c)^5+5
/24/d*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*b^4*sec(d*x+c)*tan(d*x+c)+5/16/d*b
^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.38, size = 275, normalized size = 1.13

$$640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3b+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)ab^3-5b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b + 128*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3 - 5*b^4*(2*(15*sin(d*x + c)^5
- 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)))/d
```

mupad [B] time = 4.88, size = 370, normalized size = 1.52

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^4+\frac{9\,a^2\,b^2}{2}+\frac{5\,b^4}{8}\right)}{d}+\frac{\left(a^4-8\,a^3\,b+\frac{15\,a^2\,b^2}{2}-8\,a\,b^3+\frac{11\,b^4}{8}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-3\,a^4+\frac{88\,a^3\,b}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^4 + (5*b^4)/8 + (9*a^2*b^2)/2))/d + (tan(c/2
+ (d*x)/2)^9*((56*a*b^3)/3 + (88*a^3*b)/3 - 3*a^4 + (5*b^4)/24 - (21*a^2*b^
2)/2) - tan(c/2 + (d*x)/2)^3*((56*a*b^3)/3 + (88*a^3*b)/3 + 3*a^4 - (5*b^4)
/24 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((208*a*b^3)/5 + 48*a^3*b + 2*
a^4 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 - 48*a^3*b - (2
08*a*b^3)/5 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3
*b + a^4 + (11*b^4)/8 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*(a^4 - 8*a^
3*b - 8*a*b^3 + (11*b^4)/8 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x)/2)^4 -
 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8
 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec\left(c + dx\right)\right)^{4} \sec^{3}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**3, x)
```

3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=179

$$\frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(3a^2+4b^2\right)\tan(c+dx)(a+b\sec(c+dx))^2}{15d}+\frac{ab\left(6a^2+29b^2\right)\tan(c+dx)}{30d}$$

```
[Out] 1/2*a*b*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/15*(3*a^4+28*a^2*b^2+4*b^4)*t
an(d*x+c)/d+1/30*a*b*(6*a^2+29*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*(3*a^2+4*b
^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/5*a*(a+b*sec(d*x+c))^3*tan(d*x+c)/d+1
/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/d
```

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3835, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2\left(28a^2b^2+3a^4+4b^4\right)\tan(c+dx)}{15d}+\frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(3a^2+4b^2\right)\tan(c+dx)(a+b\sec(c}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a^4 + 28*a^2*b^2
+ 4*b^4)*Tan[c + d*x])/(15*d) + (a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c +
d*x])/(30*d) + ((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(15*d)
 + (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + ((a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{4}{5}\int \sec(c+dx)(b+a\sec(c+dx))(a\\
&= \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d}+\frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}\\
&= \frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{15d}+\frac{a(a+b\sec(c+dx))^3\tan(}{5d}\\
&= \frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c+dx)}{30d}+\frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx}{15d}\\
&= \frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c+dx)}{30d}+\frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx}{15d}\\
&= \frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c}{30d}\\
&= \frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2\left(3a^4+28a^2b^2+4b^4\right)\tan(c+dx}{15d}\end{aligned}$$

Mathematica [A] time = 0.79, size = 125, normalized size = 0.70

$$\frac{15ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(20b^2\left(3a^2+b^2\right)\tan^2(c+dx)+15ab\left(4a^2+3b^2\right)\sec(c+dx)\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*a
^2*b^2 + b^4) + 15*a*b*(4*a^2 + 3*b^2)*Sec[c + d*x] + 30*a*b^3*Sec[c + d*x]
^3 + 20*b^2*(3*a^2 + b^2)*Tan[c + d*x]^2 + 6*b^4*Tan[c + d*x]^4))/(30*d)
```

fricas [A] time = 0.49, size = 182, normalized size = 1.02

$$\frac{15\left(4\,a^3b+3\,ab^3\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,a^3b+3\,ab^3\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3*b + 3*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a*b^3*cos(d*x
```

+ c) + 2*(15*a^4 + 60*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 6*b^4 + 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^3 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)

giac [B] time = 0.29, size = 461, normalized size = 2.58

$$15\left(4a^3b+3ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4a^3b+3ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(30a^4\tan\left(\frac{1}{2}dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(4*a^3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
1/2*c)^9 - 60*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 - 12
0*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^2*b
^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 40*b^4*tan(1/
2*d*x + 1/2*c)^7 + 180*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 116*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*a^4*tan(1/2*d*x + 1/2*c)
^3 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
- 30*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*
tan(1/2*d*x + 1/2*c) + 60*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 1.34, size = 225, normalized size = 1.26

$$\frac{a^4\tan(dx+c)}{d}+\frac{2a^3b\sec(dx+c)\tan(dx+c)}{d}+\frac{2a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{4a^2b^2\tan(dx+c)}{d}+\frac{2a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*tan(d*x+c)/d+2/d*a^3*b*sec(d*x+c)*tan(d*x+c)+2/d*a^3*b*ln(sec(d*x+c)+ta
n(d*x+c))+4*a^2*b^2*tan(d*x+c)/d+2/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+1/d*a*
b^3*tan(d*x+c)*sec(d*x+c)^3+3/2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+3/2/d*a*b^3*l
n(sec(d*x+c)+tan(d*x+c))+8/15/d*b^4*tan(d*x+c)+1/5/d*b^4*tan(d*x+c)*sec(d*x
+c)^4+4/15/d*b^4*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.39, size = 195, normalized size = 1.09

$$120\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2b^2+4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)b^4-15ab^3\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 + 4*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^4 - 15*a*b^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x +
 c))/d
```

mupad [B] time = 4.97, size = 304, normalized size = 1.70

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a^3\,b+3\,a\,b^3\right)}{d}-\frac{\left(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-8\,a^4+8\,a^3\,b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a*b^3 + 4*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*(
12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3*b
 - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3*b
+ 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3*b -
 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a^3*b
+ 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d
*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*
x)/2)^10 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^4\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}+\frac{\left(8a^4+24a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+}{8d}$$

[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b*(19*a^2+16*b^2)*tan(d*x+c)/d+1/24*b^2*(26*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+7/12*a*b*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b*(a+b*sec(d*x+c))^3*tan(d*x+c)/d

Rubi [A] time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3830, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d}+\frac{\left(24a^2b^2+8a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)\sec(c+}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]

[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b*(19*a^2 + 16*b^2)*Tan[c + d*x])/(6*d) + (b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + (7*a*b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (b*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
```

```
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\big(\\ &= \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\\ &= \frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{7ab(a+b\sec(c+dx))^2\tan(c+\\ &= \frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d}+\frac{7ab(a+b\sec(c+dx))^2\tan(c+\\ &= \frac{\left(8a^4+24a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\,\mathrm{t}\\ &= \frac{\left(8a^4+24a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.54, size = 101, normalized size = 0.69

$$\frac{b\tan(c+dx)\left(32a\left(3\left(a^2+b^2\right)+b^2\tan^2(c+dx)\right)+9b\left(8a^2+b^2\right)\sec(c+dx)+6b^3\sec^3(c+dx)\right)+3\left(8a^4+24a^2\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(9*b
*(8*a^2 + b^2)*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^3 + 32*a*(3*(a^2 + b^2) +
b^2*Tan[c + d*x]^2)))/(24*d)
```

fricas [A] time = 0.51, size = 163, normalized size = 1.12

$$\frac{3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx\right.\right.}{48\,d\,\mathrm{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(
32*a*b^3*cos(d*x + c) + 6*b^4 + 32*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3 + 9*(
8*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.29, size = 360, normalized size = 2.47

$$3\left(8a^4+24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8a^4+24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*a^
3*b*tan(1/2*d*x + 1/2*c)^7 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*a*b^3*t
an(1/2*d*x + 1/2*c)^7 - 15*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a*b^3*tan(1/2*d*x +
 1/2*c)^5 - 9*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^3
 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 9
*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 72*a^2*b^2*ta
n(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 1.12, size = 188, normalized size = 1.29

$$\frac{a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{4a^3b\tan\left(dx+c\right)}{d}+\frac{3a^2b^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{d}+\frac{3a^2b^2\ln\left(\sec\left(dx+c\right)+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*tan(d*x+c)+3/d*a^2*b^2*sec(d*x+
c)*tan(d*x+c)+3/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/3*a*b^3*tan(d*x+c)/d+
4/3/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d
*b^4*sec(d*x+c)*tan(d*x+c)+3/8/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 180, normalized size = 1.23

$$64\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)ab^3-3b^4\left(\frac{2\left(3\sin\left(dx+c\right)^3-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\sin\left(dx+c\right)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 3*b^4*(2*(3*sin(d*x + c)
^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log
(sec(d*x + c) + tan(d*x + c)) + 192*a^3*b*tan(d*x + c))/d
```

mupad [B] time = 4.92, size = 245, normalized size = 1.68

$$\frac{\left(-8a^3b+6a^2b^2-8ab^3+\frac{5b^4}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(24a^3b-6a^2b^2+\frac{40ab^3}{3}+\frac{3b^4}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-24a^3b\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*b^4)/4 + 6*a^2*b^2) - tan(c/2 +
 (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2
)^3*((40*a*b^3)/3 + 24*a^3*b - (3*b^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/2)^
5*((40*a*b^3)/3 + 24*a^3*b + (3*b^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d*x)/
2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)
^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 + 6*a^2*b^2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^4 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

3.479 $\int (a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=107

$$a^4x+\frac{b^2\left(17a^2+2b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\tan(c+dx)\sec(c+dx)}{3d}+\frac{b^2\tan(c}{}$$

```
[Out] a^4*x+2*a*b*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tan(d*
x+c)/d+4/3*a*b^3*sec(d*x+c)*tan(d*x+c)/d+1/3*b^2*(a+b*sec(d*x+c))^2*tan(d*x
+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$\frac{b^2\left(17a^2+2b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+a^4x+\frac{4ab^3\tan(c+dx)\sec(c+dx)}{3d}+\frac{b^2\tan(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tan[c + d*x])/(3*d) + (4*a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (b^2*(
a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4\,dx = \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}\int (a+b\sec(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\text{sec}\right.$$
$$= \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{6}\int \left(6a^4+12a\right.$$
$$= a^4x + \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \left(2ab\left(2a^2\right.\right.$$
$$= a^4x + \frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d} + \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d} + \frac{b^2(a+b}{}$$
$$= a^4x + \frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d} + \frac{b^2\left(17a^2+2b^2\right)\tan(c+dx)}{3d} + \frac{4ab^3\sec(}{}$$

Mathematica [A] time = 0.30, size = 77, normalized size = 0.72

$$\frac{3a^4dx+6ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+3b^2\tan(c+dx)\left(6a^2+2ab\sec(c+dx)+b^2\right)+b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + b^4*Tan[c + d*x]^3)/(3*d)
```

fricas [A] time = 0.53, size = 138, normalized size = 1.29

$$\frac{3\,a^4dx\cos(dx+c)^3+3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\log(-\text{sin}}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 (6*a*b^3*cos(d*x + c) + b^4 + 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.20, size = 221, normalized size = 2.07

$$3(dx+c)a^4+6\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+1\right|\right)-6\left(2\,a^3b+ab^3\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(18\,a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b^2
*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*
x + 1/2*c)^5 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^4*tan(1/2*d*x + 1/2*
c)^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b
^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.98, size = 135, normalized size = 1.26

$$a^4x+\frac{a^4c}{d}+\frac{4a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^2b^2\tan(dx+c)}{d}+\frac{2a\,b^3\sec(dx+c)\tan(dx+c)}{d}+\frac{2a\,b^3\ln(\text{sec}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*tan(d*x+c)/d+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+2/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b
^4*tan(d*x+c)+1/3/d*b^4*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.39, size = 121, normalized size = 1.13

$$a^4x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)b^4}{3\,d} - \frac{ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right)}{d} + \frac{4\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4/d - a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d +
4*a^3*b*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^2*b^2*tan(d*x + c)/d
```

mupad [B] time = 1.03, size = 185, normalized size = 1.73

$$\frac{2\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{b^4\,\sin\left(c+d\,x\right)}{3\,d\cos\left(c+d\,x\right)^3} + \frac{4\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4,x)
```

```
[Out] (2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^4*sin(c + d*x)
)/(3*d*cos(c + d*x)) + (b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (4*a*b^3*a
tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*b^3*sin(c + d*x))/(d*cos(c + d*x)^2)
+ (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(c + dx\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4, x)
```

3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=104

$$4a^3bx+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d}+\frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2\sin(c+dx)(a+b\sec$$

```
[Out] 4*a^3*b*x+1/2*b^2*(12*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a^2*(2*a^2-b^2)*si
n(d*x+c)/d+1/2*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+3*a*b^3*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3842, 4076, 4047, 8, 4045, 3770}

$$\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d}+\frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+4a^3bx+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2\sin(c+dx)(a+b\sec$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^3*b*x + (b^2*(12*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*a^2
- b^2)*Sin[c + d*x])/(2*d) + (b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d
) + (3*a*b^3*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx\\ &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{1}{2}\int \cos(c+\\ &= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{3ab^3\tan(c+dx)}{d}+\frac{1}{2}\int \cos(c+\\ &= 4a^3bx+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d}\\ &= 4a^3bx+\frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx}{2d}\end{aligned}$$

Mathematica [B] time = 0.54, size = 280, normalized size = 2.69

$$\frac{\sec^2(c+dx)\left(\left(a^4+2b^4\right)\sin(c+dx)+a^4\sin(3(c+dx))+8a^3bc+8a^3bdx-12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(8*a^3*b*c + 8*a^3*b*d*x - 12*a^2*b^2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] - b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2
*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^4*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + b*Cos[2*(c + d*x)]*(8*a^3*(c + d*x) - b*(12*a^2 + b^2)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(12*a^2 + b^2)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + (a^4 + 2*b^4)*Sin[c + d*x] + 8*a*b^3*Sin[2*(c
+ d*x)] + a^4*Sin[3*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.49, size = 130, normalized size = 1.25

$$\frac{16\,a^3bdx\cos(dx+c)^2+\left(12\,a^2b^2+b^4\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(12\,a^2b^2+b^4\right)\cos(dx+c)^2\log(-}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^3*b*d*x*cos(d*x + c)^2 + (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(2*a^4*cos(d*x + c)^2 + 8*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

giac [A] time = 0.26, size = 179, normalized size = 1.72

$$\frac{8\,(dx+c)a^3b+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\left(12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^3*b + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 + 1) + (12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*a^2*b^2
 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a*b^3*tan(1/2*d*x + 1/2*c
)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*tan(1/2*d*x + 1/2*c) - b^4*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.95, size = 114, normalized size = 1.10

$$\frac{a^4\sin(dx+c)}{d}+4a^3bx+\frac{4a^3bc}{d}+\frac{6a^2b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{4ab^3\tan(dx+c)}{d}+\frac{b^4\sec(dx+c)\tan(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^3*b*x+4/d*a^3*b*c+6/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+4*a*b^3*tan(d*x+c)/d+1/2/d*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*b^4*ln(sec(d*x
+c)+tan(d*x+c))
```

maxima [A] time = 0.36, size = 115, normalized size = 1.11

$$\frac{16\,(dx+c)a^3b-b^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,a^2b^2\left(\log(\sin(dx+c)+1)-\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^3*b - b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a*b^3*tan(d*x + c))/d
```

mupad [B] time = 1.04, size = 152, normalized size = 1.46

$$\frac{a^4\,\sin(c+d\,x)}{d}+\frac{b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin(c+d\,x)}{2\,d\,\cos(c+d\,x)^2}+\frac{12\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (a^4*sin(c + d*x))/d + (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (b^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (4*a*b^3*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))^4\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*cos(c + d*x), x)
```

3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=108

$$\frac{3a^3b\sin(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2+12b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

```
[Out] 1/2*a^2*(a^2+12*b^2)*x+4*a*b^3*arctanh(sin(d*x+c))/d+3*a^3*b*sin(d*x+c)/d+1
/2*a^2*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/2*b^2*(a^2-2*b^2)*tan(d
*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4076, 4047, 8, 4045, 3770}

$$-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2+12b^2\right)+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a^2*(a^2 + 12*b^2)*x)/2 + (4*a*b^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*b*Sin
[c + d*x])/d + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d)
 - (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c+dx)(a+b\,\mathrm{s}$$
$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx}{2d}$$
$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx}{2d}$$
$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx)}{2d}$$
$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x+\frac{4ab^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\,\mathrm{c}}{}$$

Mathematica [A] time = 0.68, size = 119, normalized size = 1.10

$$\frac{a^4\sin(2(c+dx))+16a^3b\sin(c+dx)+2a\left(a\left(a^2+12b^2\right)(c+dx)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*a*(a*(a^2 + 12*b^2)*(c + d*x) - 8*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 8*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a^3*b*Sin[c +
 d*x] + a^4*Sin[2*(c + d*x)] + 4*b^4*Tan[c + d*x])/(4*d)
```

fricas [A] time = 0.49, size = 116, normalized size = 1.07

$$\frac{4\,ab^3\cos(dx+c)\log(\sin(dx+c)+1)-4\,ab^3\cos(dx+c)\log(-\sin(dx+c)+1)+\left(a^4+12\,a^2b^2\right)dx\cos(dx+c}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a*b^3*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (a^4 + 12*a^2*b^2)*d*x*cos(d*x + c) + (a^4*cos(d*x + c
)^2 + 8*a^3*b*cos(d*x + c) + 2*b^4)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.27, size = 170, normalized size = 1.57

$$\frac{8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(a^4+12\,a^2b^2\right)(dx+c)-}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
 + (a^4 + 12*a^2*b^2)*(d*x + c) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b*t
an(1/2*d*x + 1/2*c)^3 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^3*b*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.76, size = 109, normalized size = 1.01

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4x}{2}+\frac{a^4c}{2d}+\frac{4a^3b\sin(dx+c)}{d}+6a^2b^2x+\frac{6a^2b^2c}{d}+\frac{4a\,b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+1/2*a^4*x+1/2/d*a^4*c+4*a^3*b*sin(d*x+c)/d+
6*a^2*b^2*x+6/d*a^2*b^2*c+4/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^4*tan(d
*x+c)
```

maxima [A] time = 0.34, size = 90, normalized size = 0.83

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^2b^2+8\,ab^3\big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\big)+16\,a}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^2*b^2 + 8*a*b^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^3*b*sin(d*x + c) + 4
*b^4*tan(d*x + c))/d
```

mupad [B] time = 1.03, size = 150, normalized size = 1.39

$$\frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{12\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{4\,a^3\,b\,\sin\left(c+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^4*sin(c + d*x))/(d
*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (4*a^3*b*sin(c + d*x))/d + (8*a*
b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^4\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**2, x)
```

3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=115

$$\frac{4a^3b\sin(c+dx)\cos(c+dx)}{3d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+2abx\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec}{3d}$$

```
[Out] 2*a*b*(a^2+2*b^2)*x+b^4*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*sin(d*
x+c)/d+4/3*a^3*b*cos(d*x+c)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+
c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4074, 4047, 8, 4045, 3770}

$$\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+2abx\left(a^2+2b^2\right)+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 2*a*b*(a^2 + 2*b^2)*x + (b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Sin[c + d*x])/(3*d) + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (a^2*C
os[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int \cos^2(c+dx)(a\\ &= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin\\ &= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin\\ &= 2ab\left(a^2+2b^2\right)x+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\sin\\ &= 2ab\left(a^2+2b^2\right)x+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+\end{aligned}$$

Mathematica [A] time = 0.16, size = 128, normalized size = 1.11

$$\frac{a^4\sin(3(c+dx))+12a^3b\sin(2(c+dx))+24ab\left(a^2+2b^2\right)(c+dx)+9a^2\left(a^2+8b^2\right)\sin(c+dx)-12b^4\log\left(\cos\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (24*a*b*(a^2 + 2*b^2)*(c + d*x) - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*b^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a^2*(a^2 + 8*b
^2)*Sin[c + d*x] + 12*a^3*b*Sin[2*(c + d*x)] + a^4*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.48, size = 98, normalized size = 0.85

$$\frac{3\,b^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(a^3b+2\,ab^3\right)dx+2\left(a^4\cos\left(dx+c\right)^2+6\,a^3b\cos\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*log(sin(d*x + c) + 1) - 3*b^4*log(-sin(d*x + c) + 1) + 12*(a^3*b
 + 2*a*b^3)*d*x + 2*(a^4*cos(d*x + c)^2 + 6*a^3*b*cos(d*x + c) + 2*a^4 + 18
*a^2*b^2)*sin(d*x + c))/d
```

giac [A] time = 0.24, size = 212, normalized size = 1.84

$$3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\left(a^3b+2\,ab^3\right)(dx+c)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 6*(a^3*b + 2*a*b^3)*(d*x + c) + 2*(3*a^4*tan(1/2*d*x + 1/2*
c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 +
 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^4*t
an(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.84, size = 131, normalized size = 1.14

$$\frac{\sin(dx+c)\left(\cos^2(dx+c)\right)a^4}{3d}+\frac{2a^4\sin(dx+c)}{3d}+\frac{2a^3b\cos(dx+c)\sin(dx+c)}{d}+2a^3bx+\frac{2a^3bc}{d}+\frac{6a^2b^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3*a^4*sin(d*x+c)/d+2*a^3*b*cos(d*x+c)*s
in(d*x+c)/d+2*a^3*b*x+2/d*a^3*b*c+6/d*a^2*b^2*sin(d*x+c)+4*a*b^3*x+4/d*a*b^
3*c+1/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.34, size = 102, normalized size = 0.89

$$-\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-6(2dx+2c+\sin(2dx+2c))a^3b-24(dx+c)ab^3-3b^4\left(\log(\sin(dx+c\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^3*b - 24*(d*x + c)*a*b^3 - 3*b^4*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) - 36*a^2*b^2*sin(d*x + c))/d
```

mupad [B] time = 1.11, size = 158, normalized size = 1.37

$$\frac{3a^4\sin(c+dx)}{4d}+\frac{2b^4\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{a^4\sin(3c+3dx)}{12d}+\frac{a^3b\sin(2c+2dx)}{d}+\frac{6a^2b^2\sin(c+dx)}{d}+\frac{8ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*a^4*sin(c + d*x))/(4*d) + (2*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (a^4*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(2*c + 2*d*x))/d + (6*
a^2*b^2*sin(c + d*x))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (4*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=145

$$\frac{5a^3b\sin(c+dx)\cos^2(c+dx)}{6d}+\frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d}+\frac{a^2\left(3a^2+22b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2\sin(c$$

```
[Out] 1/8*(3*a^4+24*a^2*b^2+8*b^4)*x+4/3*a*b*(2*a^2+3*b^2)*sin(d*x+c)/d+1/8*a^2*(
3*a^2+22*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/6*a^3*b*cos(d*x+c)^2*sin(d*x+c)/d+1
/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.31, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4074, 4047, 2637, 4045, 8}

$$\frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d}+\frac{a^2\left(3a^2+22b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(24a^2b^2+3a^4+8b^4\right)+\frac{5a^3b\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*x)/8 + (4*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/(
3*d) + (a^2*(3*a^2 + 22*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^3*b*Co
s[c + d*x]^2*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^3(c+dx)(a+$$
$$= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin($$
$$= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin($$
$$= \frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d} + \frac{a^2\left(3a^2+22b^2\right)\cos(c+dx)\sin(c+dx)}{8d} +$$
$$= \frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x + \frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d} + \frac{a^2\left(3a^2+22b\right.}{}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.72

$$\frac{3a^4\sin(4(c+dx)) + 32a^3b\sin(3(c+dx)) + 24a^2\left(a^2+6b^2\right)\sin(2(c+dx)) + 96ab\left(3a^2+4b^2\right)\sin(c+dx) + 12\left(3\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (12*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(c + d*x) + 96*a*b*(3*a^2 + 4*b^2)*Sin[c +
 d*x] + 24*a^2*(a^2 + 6*b^2)*Sin[2*(c + d*x)] + 32*a^3*b*Sin[3*(c + d*x)] +
 3*a^4*Sin[4*(c + d*x)])/(96*d)
```

fricas [A] time = 0.47, size = 96, normalized size = 0.66

$$\frac{3\left(3a^4+24a^2b^2+8b^4\right)dx + \left(6a^4\cos(dx+c)^3 + 32a^3b\cos(dx+c)^2 + 64a^3b + 96ab^3 + 9\left(a^4+8a^2b^2\right)\cos(dx\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^3*b
*cos(d*x + c)^2 + 64*a^3*b + 96*a*b^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c))*s
in(d*x + c))/d
```

giac [B] time = 0.24, size = 318, normalized size = 2.19

$$3\left(3a^4+24a^2b^2+8b^4\right)(dx+c) - \frac{2\left(15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 96a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 96ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 9a^4\text{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(15*a^4*tan(1/2*d*x + 1/
2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^
7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*a^
3*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*a*b^3*
tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a*b^3*tan(1/2*d*x +
 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c) - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 7
2*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.95, size = 116, normalized size = 0.80

$$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^3b\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+4*a*b^3*sin(d*x+c)+b^4*(d*x+c))
```

maxima [A] time = 0.46, size = 109, normalized size = 0.75

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^4-128\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3b+144\,(2\,dx+}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 128*(
sin(d*x + c)^3 - 3*sin(d*x + c))*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2*c
))*a^2*b^2 + 96*(d*x + c)*b^4 + 384*a*b^3*sin(d*x + c))/d
```

mupad [B] time = 0.88, size = 123, normalized size = 0.85

$$\frac{3\,a^4\,x}{8}+b^4\,x+3\,a^2\,b^2\,x+\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*a^4*x)/8 + b^4*x + 3*a^2*b^2*x + (a^4*sin(2*c + 2*d*x))/(4*d) + (a^4*sin
(4*c + 4*d*x))/(32*d) + (a^3*b*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2*c
 + 2*d*x))/(2*d) + (4*a*b^3*sin(c + d*x))/d + (3*a^3*b*sin(c + d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=173

$$\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{5d}-\frac{a^2\left(4a^2+27b^2\right)\sin^3(c+dx)}{15d}+\frac{ab\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}abx\left(3a^2\right.$$

```
[Out] 1/2*a*b*(3*a^2+4*b^2)*x+1/5*(4*a^4+29*a^2*b^2+5*b^4)*sin(d*x+c)/d+1/2*a*b*(
3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+3/5*a^3*b*cos(d*x+c)^3*sin(d*x+c)/d+1/
5*a^2*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a^2*(4*a^2+27*b^2)*
sin(d*x+c)^3/d
```

Rubi [A] time = 0.35, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{a^2\left(4a^2+27b^2\right)\sin^3(c+dx)}{15d}+\frac{\left(29a^2b^2+4a^4+5b^4\right)\sin(c+dx)}{5d}+\frac{ab\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(3*a^2 + 4*b^2)*x)/2 + ((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(5*
d) + (a*b*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*b*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(5*d) - (a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)(a$$

$$=\frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin}{5d}$$

$$=\frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin}{5d}$$

$$=\frac{ab\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\cos^3(c+dx)\sin(c+dx}{5d}$$

$$=\frac{1}{2}ab\left(3a^2+4b^2\right)x+\frac{ab\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3b\cos}{}$$

$$=\frac{1}{2}ab\left(3a^2+4b^2\right)x+\frac{\left(4a^4+29a^2b^2+5b^4\right)\sin(c+dx)}{5d}+\frac{ab\left(3a^2+4b\right.}{}$$

Mathematica [A] time = 0.52, size = 133, normalized size = 0.77

$$\frac{30\left(5a^4+36a^2b^2+8b^4\right)\sin(c+dx)+a\left(5\left(5a^3+24ab^2\right)\sin(3(c+dx))+3a^3\sin(5(c+dx))+240b\left(a^2+b^2\right)\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (30*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sin[c + d*x] + a*(360*a^2*b*c + 480*b^3*c
+ 360*a^2*b*d*x + 480*b^3*d*x + 240*b*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(5*a
^3 + 24*a*b^2)*Sin[3*(c + d*x)] + 30*a^2*b*Sin[4*(c + d*x)] + 3*a^3*Sin[5*(
c + d*x)]))/(240*d)
```

fricas [A] time = 0.48, size = 121, normalized size = 0.70

$$\frac{15\left(3a^3b+4ab^3\right)dx+\left(6a^4\cos(dx+c)^4+30a^3b\cos(dx+c)^3+16a^4+120a^2b^2+30b^4+4\left(2a^4+15a^2b^2\right.\right.}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^3*b*cos(d*x
 + c)^3 + 16*a^4 + 120*a^2*b^2 + 30*b^4 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x +
c)^2 + 15*(3*a^3*b + 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.24, size = 425, normalized size = 2.46

$$15\left(3a^3b+4ab^3\right)(dx+c)+\frac{2\left(30a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-75a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+180a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-60ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+30b^4\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*(d*x + c) + 2*(30*a^4*tan(1/2*d*x + 1/2*c)^9 -
 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*
a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 + 40*a^4*tan(1
/2*d*x + 1/2*c)^7 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*b^4*tan(1/2*d*x + 1/
2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^3
*b*tan(1/2*d*x + 1/2*c)^3 + 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^3*
tan(1/2*d*x + 1/2*c)^3 + 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*tan(1/2*d*
x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^5)/d
```

maple [A] time = 1.51, size = 138, normalized size = 0.80

$$\frac{\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4a^3b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos^2(dx+c)\right)\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^3*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*b^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4*a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^4*sin(
d*x+c))
```

maxima [A] time = 0.43, size = 133, normalized size = 0.77

$$\frac{8\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*
x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^3 + 120*b^4*sin(d*x + c))/d
```

mupad [B] time = 3.82, size = 330, normalized size = 1.91

$$\frac{\left(2a^4-5a^3b+12a^2b^2-4ab^3+2b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{8a^4}{3}-2a^3b+32a^2b^2-8ab^3+8b^4\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*x
)/2)^9*(2*a^4 - 5*a^3*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2
)^3*(8*a*b^3 + 2*a^3*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)/
2)^7*((8*a^4)/3 - 2*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)
/2)*(4*a*b^3 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)
/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*x)/2)*(
3*a^2 + 4*b^2))/(4*a*b^3 + 3*a^3*b))*(3*a^2 + 4*b^2))/d
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=213

$$\frac{7a^3b\sin(c+dx)\cos^4(c+dx)}{15d}-\frac{4ab\left(4a^2+5b^2\right)\sin^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{a^2\left(5a^2+32b^2\right)\sin(c}{24d}$$

```
[Out] 1/16*(5*a^4+36*a^2*b^2+8*b^4)*x+4/5*a*b*(4*a^2+5*b^2)*sin(d*x+c)/d+1/16*(5*
a^4+36*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(5*a^2+32*b^2)*cos(d
*x+c)^3*sin(d*x+c)/d+7/15*a^3*b*cos(d*x+c)^4*sin(d*x+c)/d+1/6*a^2*cos(d*x+c
)^5*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-4/15*a*b*(4*a^2+5*b^2)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.38, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4074, 4047, 2633, 4045, 2635, 8}

$$-\frac{4ab\left(4a^2+5b^2\right)\sin^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{a^2\left(5a^2+32b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{\left(36a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*x)/16 + (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x])/
(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^2*(5*a^2 + 32*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (7*a^3*b*Cos[c +
 d*x]^4*Sin[c + d*x])/(15*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*S
in[c + d*x])/(6*d) - (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d}+\frac{1}{6}\int \cos^5(c+dx)(a \\
&= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d}+\frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2 \mathrm{s}}{6d} \\
&= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d}+\frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2 \mathrm{s}}{6d} \\
&= \frac{a^2\left(5a^2+32b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{7a^3b\cos^4(c+dx)\sin(c+}{15d} \\
&= \frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{\left(5a^4+36a^2b^2+8b^4\right)\cos(c+dx)\sin(c}{16d} \\
&= \frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x+\frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{\left(5a^4+36\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.46, size = 156, normalized size = 0.73

$$
\frac{5a^4\sin(6(c+dx))+48a^3b\sin(5(c+dx))+45a^2\left(a^2+4b^2\right)\sin(4(c+dx))+480ab\left(5a^2+6b^2\right)\sin(c+dx)+8}{96}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (60*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(c + d*x) + 480*a*b*(5*a^2 + 6*b^2)*Sin[c
+ d*x] + 15*(15*a^4 + 96*a^2*b^2 + 16*b^4)*Sin[2*(c + d*x)] + 80*a*b*(5*a^2
 + 4*b^2)*Sin[3*(c + d*x)] + 45*a^2*(a^2 + 4*b^2)*Sin[4*(c + d*x)] + 48*a^3
*b*Sin[5*(c + d*x)] + 5*a^4*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.54, size = 150, normalized size = 0.70

$$
\frac{15\left(5a^4+36a^2b^2+8b^4\right)dx+\left(40a^4\cos(dx+c)^5+192a^3b\cos(dx+c)^4+512a^3b+640ab^3+10\left(5a^4+36\right.\right.}{240}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*d*x + (40*a^4*cos(d*x + c)^5 + 192*a
^3*b*cos(d*x + c)^4 + 512*a^3*b + 640*a*b^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d
*x + c)^3 + 64*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^2 + 15*(5*a^4 + 36*a^2*b^2
+ 8*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.25, size = 550, normalized size = 2.58

$$15\left(5a^4+36a^2b^2+8b^4\right)(dx+c)-\frac{2\left(165a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-960a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+900a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-960ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(165*a^4*tan(1/2*d*x +
 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x +
1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/2*c
)^11 - 25*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^9 +
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 3
60*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*a^3*b
*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a*b^3*t
an(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*a^4*tan(1/2*d*
x + 1/2*c)^5 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*b^4*tan(1/2*d*x + 1/2*
c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 3
60*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*a^4*tan(1/2*d*x + 1/2*c) - 960*a^3*b*ta
n(1/2*d*x + 1/2*c) - 900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*a*b^3*tan(1/2*d
*x + 1/2*c) - 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)
/d
```

maple [A] time = 1.60, size = 174, normalized size = 0.82

$$\frac{a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^3b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+6a^2b^2\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a
^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*b
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

maxima [A] time = 0.37, size = 170, normalized size = 0.80

$$-\frac{5\left(4\sin\left(2dx+2c\right)^3-60dx-60c-9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a^4-256\left(3\sin\left(dx+c\right)^5-10\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
```

```
*x + c))*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 240*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*b^4)/d
```

mupad [**B**] time = 1.09, size = 214, normalized size = 1.00

$$\frac{5a^4x}{16}+\frac{b^4x}{2}+\frac{9a^2b^2x}{4}+\frac{15a^4\sin(2c+2dx)}{64d}+\frac{3a^4\sin(4c+4dx)}{64d}+\frac{a^4\sin(6c+6dx)}{192d}+\frac{b^4\sin(2c+2dx)}{4d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (5*a^4*x)/16 + (b^4*x)/2 + (9*a^2*b^2*x)/4 + (15*a^4*sin(2*c + 2*d*x))/(64*
d) + (3*a^4*sin(4*c + 4*d*x))/(64*d) + (a^4*sin(6*c + 6*d*x))/(192*d) + (b^
4*sin(2*c + 2*d*x))/(4*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (5*a^3*b*sin(3
*c + 3*d*x))/(12*d) + (a^3*b*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(2*c
+ 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (3*a*b^3*sin(c + d*
x))/d + (5*a^3*b*sin(c + d*x))/(2*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.486 $\int (a+b\sec(c+dx))^5\,dx$

Optimal. Leaf size=158

$$a^5x+\frac{ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{6d}+\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}+\frac{b\left(40a^4+40a^2b^2+3b^4\right)\tanh^{-1}(\mathrm{si}$$

```
[Out] a^5*x+1/8*b*(40*a^4+40*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b^2*(53*a
^2+20*b^2)*tan(d*x+c)/d+1/24*b^3*(58*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+11/
12*a*b^2*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b^2*(a+b*sec(d*x+c))^3*tan(d*x
+c)/d
```

Rubi [A] time = 0.24, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3782, 4056, 4048, 3770, 3767, 8}

$$\frac{ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{6d}+\frac{b\left(40a^2b^2+40a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] a^5*x + (b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/(6*d) + (b^3*(58*a^2 + 9*b^2)*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + (11*a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
12*d) + (b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^5\,dx = \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^3+3b\left(4a^2+b^2\right)\right.$$
$$= \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{12}\int$$
$$= \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} +$$
$$= a^5x + \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+}{12d}$$
$$= a^5x + \frac{b\left(40a^4+40a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)}{24d}$$
$$= a^5x + \frac{b\left(40a^4+40a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{ab^2\left(53a^2+20b^2\right)\tan(c+d}{6d}$$

Mathematica [A] time = 0.62, size = 114, normalized size = 0.72

$$\frac{24a^5dx+3b^2\tan(c+dx)\left(b\left(40a^2+3b^2\right)\sec(c+dx)+40a\left(2a^2+b^2\right)+2b^3\sec^3(c+dx)\right)+3b\left(40a^4+40a^2b\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] (24*a^5*d*x + 3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + 3*b
^2*(40*a*(2*a^2 + b^2) + b*(40*a^2 + 3*b^2)*Sec[c + d*x] + 2*b^3*Sec[c + d*
x]^3)*Tan[c + d*x] + 40*a*b^4*Tan[c + d*x]^3)/(24*d)
```

fricas [A] time = 0.49, size = 183, normalized size = 1.16

$$\frac{48\,a^5dx\cos(dx+c)^4+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(40\,a^4b+40\,a^2b^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^5*d*x*cos(d*x + c)^4 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(40*a*b^4*cos(d*x + c) + 6*b^5 + 80*(3*a^3
*b^2 + a*b^4)*cos(d*x + c)^3 + 3*(40*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.22, size = 380, normalized size = 2.41

$$\frac{24\,(dx+c)a^5+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\log\left(\left|\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a^5 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(240*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a^2*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*b^5*tan(1/2*d*x +
1/2*c)^7 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*a^2*b^3*tan(1/2*d*x + 1
/2*c)^5 - 200*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*b^5*tan(1/2*d*x + 1/2*c)^5 +
 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
200*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*b^5*tan(1/2*d*x + 1/2*c)^3 - 240*a^3*b
^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(
1/2*d*x + 1/2*c) - 15*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

maple [A] time = 1.10, size = 205, normalized size = 1.30

$$a^5x+\frac{a^5c}{d}+\frac{5b\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{10b^2a^3\tan(dx+c)}{d}+\frac{5a^2b^3\sec(dx+c)\tan(dx+c)}{d}+\frac{5a^2b^3\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^5,x)
```

```
[Out] a^5*x+1/d*a^5*c+5/d*b*a^4*ln(sec(d*x+c)+tan(d*x+c))+10/d*b^2*a^3*tan(d*x+c)
+5/d*a^2*b^3*sec(d*x+c)*tan(d*x+c)+5/d*a^2*b^3*ln(sec(d*x+c)+tan(d*x+c))+10
/3/d*a*b^4*tan(d*x+c)+5/3/d*a*b^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^5*tan(d*x
+c)*sec(d*x+c)^3+3/8/d*b^5*sec(d*x+c)*tan(d*x+c)+3/8/d*b^5*ln(sec(d*x+c)+ta
n(d*x+c))
```

maxima [A] time = 0.36, size = 198, normalized size = 1.25

$$a^5x+\frac{5\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^4}{3d}-\frac{b^5\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4/d - 1/16*b^5*(2*(3*sin(
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d - 5/2*a^2*b^3*(2*sin(d*x +
 c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
 + 5*a^4*b*log(sec(d*x + c) + tan(d*x + c))/d + 10*a^3*b^2*tan(d*x + c)/d
```

mupad [B] time = 1.36, size = 274, normalized size = 1.73

$$\frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,b^5\,\sin(c+d\,x)}{8\,d\cos(c+d\,x)^2}+\frac{b^5\,\sin(c+d\,x)}{4\,d\cos(c+d\,x)^4}+\frac{10\,a\,b^4\,\sin(c+d\,x)}{3\,d\,\cos(c+d\,x)}+\frac{5\,a\,b^4\,\sin(c+d\,x)}{3\,d\cos(c+d\,x)^3}+\frac{10\,a^3\,b^2\,\mathrm{si}}{d\,\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^5,x)
```

```
[Out] (2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^5*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/(4*d) + (3*b^5*sin(c + d*x))/(8*d*co
s(c + d*x)^2) + (b^5*sin(c + d*x))/(4*d*cos(c + d*x)^4) - (a^2*b^3*atan((si
n(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d - (a^4*b*atan((sin(c/2 + (d
```

```
*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d + (10*a*b^4*sin(c + d*x))/(3*d*cos(c
+ d*x)) + (5*a*b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (10*a^3*b^2*sin(c +
 d*x))/(d*cos(c + d*x)) + (5*a^2*b^3*sin(c + d*x))/(d*cos(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**5, x)
```

3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d}-\frac{a\tan(c+dx)\sec(c+}{2b^2d}$$

```
[Out] -1/2*a*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d+2*a^4*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(3*a^2+2*b^2)
*tan(d*x+c)/b^3/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*sec(d*x+c)^2*tan(d*
x+c)/b/d
```

Rubi [A] time = 0.48, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3851, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d}-\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)\sec(c+}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^4*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3
*a^2 + 2*b^2)*Tan[c + d*x])/(3*b^3*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^
2*d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
```

```
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2a+2b\sec(c+dx)-3a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3b}$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec(c+dx)\left(-3a^2+ab\sec(c+dx)+\dots\right.}{a+b\sec(c+\dots}}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \dots$$

$$= \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \dots$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+\dots}{2b^2d}$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+\dots}{2b^2d}$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} + \frac{\left(3a^2+2b^2\right)\tan\dots}{3b^3d}$$

Mathematica [A] time = 2.70, size = 258, normalized size = 1.64

$$\frac{1}{2}\sec^3(c+dx)\left(4b\sin(c+dx)\left(\left(3a^2+2b^2\right)\cos(2(c+dx))+3a^2-3ab\cos(c+dx)+4b^2\right)+9a\left(2a^2+b^2\right)\cos(c+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-24*a^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Sec[c + d*x]^3*(9*a*(2*a^2 + b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*(2*
a^2 + b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*b*(3*a^2 + 4*b^2 - 3*a*b*Cos[c +
 d*x] + (3*a^2 + 2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*b^4*d)
```

fricas [A] time = 0.66, size = 557, normalized size = 3.55

$$\left[\frac{6\sqrt{a^2-b^2}\,a^4\cos\left(dx+c\right)^3\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\left(2\,a^5-a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(a^2 - b^2)*a^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(2*a
^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 - a^3
*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5
+ 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*
x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*sqrt(-a^
2 + b^2)*a^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c)))*cos(d*x + c)^3 - 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log
(sin(d*x + c) + 1) + 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*
x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x
+ c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d
*cos(d*x + c)^3)]
```

giac [B] time = 0.27, size = 286, normalized size = 1.82

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}\,b^4}-\frac{3\left(2\,a^3+ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}+\frac{3\left(2\,a^3+ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^4/(sqrt(-a^
2 + b^2)*b^4) - 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 +
3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(6*a^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)
^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*t
```

```
an(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

maple [B] time = 0.34, size = 400, normalized size = 2.55

$$\frac{2a^4 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^4\sqrt{(a-b)\,(a+b)}}-\frac{1}{3db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a}{2d\,b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{2db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^4/b^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))-1/3/d/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1
)^2*a-1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-1
/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a-1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*
ln(tan(1/2*d*x+1/2*c)-1)+1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/b/(tan(
1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*a+1/2/d/b/(tan(1/2*d
*x+1/2*c)+1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b^2/(tan(1/2*d*x+1/
2*c)+1)*a-1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)+1)
-1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.45, size = 1021, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

```
[Out] -((9*a^4*cos(c + d*x)*atanh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*s
in(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 +
(d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)
/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(
c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*c
os(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) +
2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2
*a^3*b*(a^2 - b^2)))))/2 - (3*b^3*sin(c + d*x)*(a^2 - b^2)^(1/2))/2 - (b^3*
sin(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/2 + (3*a^4*cos(3*c + 3*d*x)*atanh((8*a^
6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/2 +
 (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x
)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c
/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^
4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/
2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a
^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2)))))/2 + (3*a^3
```

```
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*(a^2 - b^2)^
(1/2))/2 - (3*a^2*b*sin(c + d*x)*(a^2 - b^2)^(1/2))/4 + (9*a^3*cos(c + d*x)
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 - b^2)^(1/2))/2 + (3*a*b
^2*sin(2*c + 2*d*x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*b*sin(3*c + 3*d*x)*(a^2 -
 b^2)^(1/2))/4 + (9*a*b^2*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*(a^2 - b^2)^(1/2))/4 + (3*a*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*cos(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/4)/(3*b^4*d*((3*cos(c + d*x)
)/4 + cos(3*c + 3*d*x)/4)*(a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^3/d-2*a^3*arctanh((a-b)^(1/2)*tan(1/2
*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)-a*tan(d*x+c)/b^2/d+1
/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.28, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3851, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a^3*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Tan
[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
```

```
, 3]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(a+b\sec(c+dx)-2a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b^2}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^3} + \frac{\left(2a^2+b^2\right)\int \sec(c+$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3\int \frac{}{1+\frac{a\cos}{}}}{b^4}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(2a^3\right)\mathrm{Sub}}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} - \frac{a\tan(c+dx)}{b^2d} + \sec($$

Mathematica [A] time = 1.14, size = 238, normalized size = 2.00

$$\frac{-4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{8a^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] - 4*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b^2*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*T
an[c + d*x])/(4*b^3*d)
```

fricas [B] time = 0.67, size = 485, normalized size = 4.08

$$\left[\frac{2\sqrt{a^2-b^2}\,a^3\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(2\,a^4-a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a^2 - b^2)*a^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*a^4
- a^2*b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^4 - a^2*b^2 -
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^2*b^2 - b^4 - 2*(a^3*b -
a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4
*(4*sqrt(-a^2 + b^2)*a^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^4 - a^2*b^2 - b^4)*cos(d*x +
c)^2*log(sin(d*x + c) + 1) + (2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^2*log(-si
n(d*x + c) + 1) - 2*(a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

giac [A] time = 0.26, size = 211, normalized size = 1.77

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}\,b^3}-\frac{\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^3}+\frac{\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3/(sqrt(-a^
2 + b^2)*b^3) - (2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a
^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2
*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

maple [B] time = 0.42, size = 262, normalized size = 2.20

$$-\frac{2a^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^3\sqrt{(a-b)\,(a+b)}}+\frac{1}{2db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{d\,b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{2db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)
*a+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/
d/b*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2+1/d/b^2/(tan(
1/2*d*x+1/2*c)+1)*a+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)+1/d/b^3*ln(tan(1/2*d*x+1
/2*c)+1)*a^2+1/2/d/b*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.65, size = 1002, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] sin(c + d*x)/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a^2*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (atan
h(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*b*d*(cos(2*c
+ 2*d*x)/2 + 1/2)) - (a*sin(2*c + 2*d*x))/(2*b^2*d*(cos(2*c + 2*d*x)/2 + 1/
2)) - (a^3*atan(((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d
*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) -
2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 -
b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)
/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2
+ (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b
 - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*
(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (
a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(b^3*d*(
cos(2*c + 2*d*x)/2 + 1/2)) - (a^3*cos(2*c + 2*d*x)*atan(((8*a^6*sin(c/2 + (
d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^
2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b
^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)
*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2
 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4
*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a
^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^
2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+2*a^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)+tan(d*x+c)/b/d
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3790, 3789, 3770, 3831, 2659, 208}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Tan[c + d*x]/(b*d
)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan(c+dx)}{bd} - \frac{a\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$
$$= \frac{\tan(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\tan(c+dx)}{bd} + \frac{a^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^3}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\tan(c+dx)}{bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+d$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{\tan(c+dx)}{bd}$$

Mathematica [A] time = 0.39, size = 115, normalized size = 1.35

$$\frac{-\frac{2a^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-2*a^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] + a*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + b*Tan[c + d*x])/(b^2*d)
```

fricas [B] time = 0.55, size = 392, normalized size = 4.61

$$\left[\frac{\sqrt{a^2-b^2}\,a^2\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right) - \left(a^3-ab^2\right)\cos(}{2\left(a^2b^2-b^4\right)d\,\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^3 - a*b^
2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin
(d*x + c) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x +
 c)), 1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^3 - a*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

giac [A] time = 0.30, size = 152, normalized size = 1.79

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}\,b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}+\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 + b
^2)*b^2) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
b))/d
```

maple [A] time = 0.35, size = 134, normalized size = 1.58

$$\frac{2a^2\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^2\sqrt{(a-b)\,(a+b)}}-\frac{1}{db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}-\frac{1}{db\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/b^2*a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))-1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)-
1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.21, size = 119, normalized size = 1.40

$$\frac{\tan\left(c+d\,x\right)}{b\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}-\frac{a^2\,\mathrm{atan}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c + d*x)/(b*d) - (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^
2*d) - (a^2*atan((a*sin(c/2 + (d*x)/2)*1i - b*sin(c/2 + (d*x)/2)*1i)/(cos(c
/2 + (d*x)/2)*(a^2 - b^2)^(1/2)))*2i)/(b^2*d*(a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] arctanh(sin(d*x+c))/b/d-2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1
/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3789, 3770, 3831, 2659, 208}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \sec(c+dx)\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2 d}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.09, size = 102, normalized size = 1.50

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]])/(b*d)
```

fricas [A] time = 0.57, size = 290, normalized size = 4.26

$$\left[\frac{\sqrt{a^2-b^2}\,a\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+\right.}{2\left(a^2b-b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^2 - b^2)*log(sin(d*x +
c) + 1) - (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(2*sq
rt(-a^2 + b^2)*a*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)
*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(
d*x + c) + 1))/((a^2*b - b^3)*d)]
```

giac [B] time = 0.24, size = 120, normalized size = 1.76

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}\,b} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/(sqrt(-a^2 + b^
2)*b) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b)/d
```

maple [A] time = 0.35, size = 88, normalized size = 1.29

$$-\frac{2a\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{db\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d/b*a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.06, size = 186, normalized size = 2.74

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,a\,\mathrm{atanh}\left(\frac{2\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,a^3\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-b^2\right)}\right)}{b\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((2*a^2*
sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*a^4*sin(c/2 + (d*x)/2) + b^2*sin(c/2 + (
d*x)/2)*(a^2 - b^2) + 2*a^3*b*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - b^2))))/(b*d*(
a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] 2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3831, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*
d)
```

fricas [A] time = 0.46, size = 185, normalized size = 3.78

$$\left[\frac{\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,\sqrt{a^2-b^2}\,d},\ \frac{\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c}{\left(a^2-b^2\right)\sin(dx+c}\right.}{\left(a^2-b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2))/(sqrt(a^2 - b^2)*d), sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^2 - b^
2)*d)]
```

giac [A] time = 0.21, size = 77, normalized size = 1.57

$$-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*d)
```

maple [A] time = 0.38, size = 44, normalized size = 0.90

$$\frac{2\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.89, size = 40, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] (2*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(1/2
)*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c + dx)}{a + b \sec (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] x/a-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3783, 2659, 208}

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sec(c+dx)}\,dx &= \frac{x}{a}-\frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a}\\
&= \frac{x}{a}-\frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\
&= \frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.02

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{c}{d}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1),x]
```

```
[Out] (c/d + x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

fricas [A] time = 0.53, size = 230, normalized size = 3.90

$$\left[\frac{2\left(a^2-b^2\right)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left(a^3-ab^2\right)d}, \frac{\left(a^2-b^2\right)dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^3 -
 a*b^2)*d), ((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*
(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

giac [B] time = 0.19, size = 218, normalized size = 3.69

$$\frac{\frac{\left(\sqrt{-a^2+b^2}\,(a-2\,b)|-a+b|-\sqrt{-a^2+b^2}\,|a||-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{\left(a^2-2\,ab+b^2\right)a^2+\left(a^2b-2\,ab^2+b^3\right)|a|}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b-\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)(a-2\,b+|}{a^2-b|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((sqrt(-a^2 + b^2)*(a - 2*b)*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a +
 b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-
(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 + (a^
2*b - 2*a*b^2 + b^3)*abs(a)) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(t
an(1/2*d*x + 1/2*c)/sqrt(-(b - sqrt((a + b)*(a - b) + b^2))/(a - b))))*(a -
 2*b + abs(a))/(a^2 - b*abs(a)))/d
```

maple [A] time = 0.43, size = 67, normalized size = 1.14

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)\,(a+b)}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d/a*b/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))+2/d/a*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.09, size = 186, normalized size = 3.15

$$\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,a\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,\left(a\,b-a^2\right)}\right)}{a\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((2*b^4*s
in(c/2 + (d*x)/2) + a^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 2*b^2*sin(c/2 + (d
*x)/2)*(a^2 - b^2) - 2*a*b^3*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/2)*
(a^2 - b^2))/(a*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - a^2))))/(a*d*(a
^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \sec\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sec(c + d*x)), x)
```

3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}$$

```
[Out] -b*x/a^2+sin(d*x+c)/a/d+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^
(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3853, 12, 3783, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sin(c+dx)}{ad} - \frac{\int \frac{b}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{\sin(c+dx)}{ad} - \frac{b\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}$$
$$= -\frac{bx}{a^2} + \frac{\sin(c+dx)}{ad} + \frac{b\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}$$
$$= -\frac{bx}{a^2} + \frac{\sin(c+dx)}{ad} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d}$$
$$= -\frac{bx}{a^2} + \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d} + \frac{\sin(c+dx)}{ad}$$

Mathematica [A] time = 0.15, size = 72, normalized size = 0.95

$$\frac{-\frac{2b^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\sin(c+dx) - b(c+dx)}{a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-(b*(c + d*x)) - (2*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + a*Sin[c + d*x])/(a^2*d)
```

fricas [A] time = 0.49, size = 277, normalized size = 3.64

$$\left[\frac{\sqrt{a^2-b^2}\,b^2\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right) - 2\left(a^2b-b^3\right)dx + 2\left(a^3-\right.}{2\left(a^4-a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^2*b - b^3)*d*x + 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), (sqrt(-a^2 + b^2)*b^2*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
2*b - b^3)*d*x + (a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

giac [A] time = 0.22, size = 126, normalized size = 1.66

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}\,a^2} - \frac{(dx+c)b}{a^2} + \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 + b
^2)*a^2) - (d*x + c)*b/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 + 1)*a))/d
```

maple [A] time = 0.64, size = 102, normalized size = 1.34

$$\frac{2b^2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\sqrt{(a-b)\,(a+b)}}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))+2/d/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/a^2*b*arct
an(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.31, size = 395, normalized size = 5.20

$$\frac{a^3\sin\left(c+d\,x\right)}{d\,\left(a^4-a^2\,b^2\right)}+\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}-\frac{a\,b^2\,\sin\left(c+d\,x\right)}{d\,\left(a^4-a^2\,b^2\right)}-\frac{2\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}+\frac{b^2\,\mathrm{atan}\left(\frac{-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1\mathrm{i}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] (a^3*sin(c + d*x))/(d*(a^4 - a^2*b^2)) + (2*b^3*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (a*b^2*sin(c + d*x))/(d*(a^4 - a^2*
b^2)) + (b^2*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(c/
2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2
)*2i - a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c/2 +
(d*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*
1i)/(a^6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2*cos(c/
2 + (d*x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2)) - (2*a^2*b*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{b\sin(c+dx)}{a^2d}+\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-b*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d-2*b^
3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+
b)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (b*Sin[c + d*x])/(a^2*d) + (
Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int \frac{\cos(c+dx)\left(-2b+a\sec(c+dx)+b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2a}\\
&= -\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\int \frac{-a^2-2b^2-ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2a^2}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{b^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{b^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^3}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx\right.}{a^3d}\\
&= \frac{\left(a^2+2b^2\right)x}{2a^3}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{b\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 97, normalized size = 0.88

$$
\frac{2\left(a^2+2b^2\right)(c+dx)+\frac{8b^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a^2\sin(2(c+dx))-4ab\sin(c+dx)}{4a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + (8*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])
/(4*a^3*d)
```

fricas [A] time = 0.50, size = 334, normalized size = 3.04

$$
\left[\frac{\sqrt{a^2-b^2}\,b^3\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+\left(a^4+a^2b^2-2b^4\right)dx-\left(2a^3\right.}{2\left(a^5-a^3b^2\right)d}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^4 + a^2*b^2 - 2*b^4)*
d*x - (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^
5 - a^3*b^2)*d), -1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4 + a^2*b^2 - 2*b^4)*d*x
+ (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
a^3*b^2)*d)]
```

giac [A] time = 0.20, size = 178, normalized size = 1.62

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}\,a^3}-\frac{(a^2+2b^2)(dx+c)}{a^3}+\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/(sqrt(-a^
2 + b^2)*a^3) - (a^2 + 2*b^2)*(d*x + c)/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

maple [B] time = 0.64, size = 222, normalized size = 2.02

$$-\frac{2b^3\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^3\sqrt{(a-b)(a+b)}}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+1/d/a/(1+tan(1/2*d*x+1/2*c
)^2)^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/
2*c)*b+1/d/a*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*
b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.81, size = 592, normalized size = 5.38

$$\frac{a\left(\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{\sin(2\,c+2\,d\,x)}{4}\right)}{d\,\left(a^2-b^2\right)}-\frac{b\,\sin\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}+\frac{b^2\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a\,d\,\left(a^2-b^2\right)}+\frac{b^3\,\sin\left(c+d\,x\right)}{a^2\,d\,\left(a^2-b^2\right)}-\frac{2\,b^4\,\mathrm{at}}{a^3\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*(atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4))/(d*(
a^2 - b^2)) - (b*sin(c + d*x))/(d*(a^2 - b^2)) + (b^2*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a*d*(a^2 - b^2)) - (b^3
*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^9*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*a^2*b^7
*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 -
 b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*a^6*b^3*si
n(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^
2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)
/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^6 - a^7 + 4*b^7 - 2*a^2*b^5 +
a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(a^2 - b^2) + 2*a*b^4*(a^2 - b^
2))))*2i)/(a^3*d*(a^2 - b^2)^(1/2)) + (b^3*sin(c + d*x))/(a^2*d*(a^2 - b^2)
) - (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{b\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{bx\left(a^2+2b^2\right)}{2a^4}+\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{3\ldots}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sin(d*x+c)/a^3/d-1/2*b*cos(d*x+c
)*sin(d*x+c)/a^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/a/d+2*b^4*arctanh((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(a^2+2b^2\right)}{2a^4}-\frac{b\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*a^2 + 3*b^2)*Sin[c +
d*x])/(3*a^3*d) - (b*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2
*Sin[c + d*x])/(3*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{\cos^2(c+dx)\left(-3b+2a\sec(c+dx)+2b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3a}$$

$$= -\frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos(c+dx)\left(-2\left(2a^2+3b^2\right)-ab\sec(c+d}{a+b\sec(c+dx)}}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int -3b}{}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\,\mathrm{s}}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\,\mathrm{s}}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\,\mathrm{s}}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - b\cos(c+$$

Mathematica [A] time = 0.33, size = 122, normalized size = 0.82

$$\frac{a^3\sin(3(c+dx)) - 6b\left(a^2+2b^2\right)(c+dx) + 3a\left(3a^2+4b^2\right)\sin(c+dx) - \frac{24b^4\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 3a^2b\sin(2(c+}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*(c + d*x) - (24*b^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sin[c + d*x] - 3*a
^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^4*d)
```

fricas [A] time = 0.57, size = 401, normalized size = 2.71

$$\left[\frac{3\sqrt{a^2-b^2}\,b^4\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right) - 3\left(a^4b+a^2b^3-2b^5\right)dx +}{6\left(a^6-a^4b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - b^2)*b^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(a^4*b + a^2*b^3 - 2
*b^5)*d*x + (4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2
 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d), 1/6
*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) - 3*(a^4*b + a^2*b^3 - 2*b^5)*d*x + (4*a^5 + 2*a^3*
b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

giac [A] time = 0.22, size = 249, normalized size = 1.68

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}\,a^4}-\frac{3\left(a^2b+2b^3\right)(dx+c)}{a^4}+\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^4/(sqrt(-a^
2 + b^2)*a^4) - 3*(a^2*b + 2*b^3)*(d*x + c)/a^4 + 2*(6*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^
2*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*
x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

maple [B] time = 0.72, size = 367, normalized size = 2.48

$$\frac{2b^4\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^4\sqrt{(a-b)\,(a+b)}}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a^4*b^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))+2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+1/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*b+2/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^3*tan(1/2*d*x+1/2*c)^5*b^2+4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2
*d*x+1/2*c)^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*b^2+2
/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^3*tan(1/2*d*x+1/2*c)*b^2-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/
2*d*x+1/2*c)*b-1/d/a^2*b*arctan(tan(1/2*d*x+1/2*c))-2/d/a^4*arctan(tan(1/2*
d*x+1/2*c))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.06, size = 654, normalized size = 4.42

$$\frac{\frac{b^2\sin(c+dx)}{4}-\frac{b^2\sin(3c+3dx)}{12}}{a\,d\,\left(a^2-b^2\right)}-\frac{b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{b\,\sin(2\,c+2\,d\,x)}{4}}{d\,\left(a^2-b^2\right)}+\frac{a\,\left(\frac{3\,\sin(c+d\,x)}{4}+\frac{\sin(3\,c+3\,d\,x)}{12}\right)}{d\,\left(a^2-b^2\right)}-\frac{b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-}{a^2\,d\,\left(a^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] ((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12)/(a*d*(a^2 - b^2)) - (b*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b*sin(2*c + 2*d*x))/4)/(d*(a^
2 - b^2)) + (a*((3*sin(c + d*x))/4 + sin(3*c + 3*d*x)/12))/(d*(a^2 - b^2))
- (b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^3*sin(2*c + 2*d*x))
/4)/(a^2*d*(a^2 - b^2)) + (b^4*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(3/2) - a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) - 8*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4
*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a
^2 - b^2)^(1/2) + 2*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^
2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^
2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^
6 - a^7 + 4*b^7 - 2*a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(
a^2 - b^2) + 2*a*b^4*(a^2 - b^2))))*2i)/(a^4*d*(a^2 - b^2)^(1/2)) - (b^4*si
n(c + d*x))/(a^3*d*(a^2 - b^2)) + (2*b^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/(a^4*d*(a^2 - b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}-\frac{b\sin(c+dx)\cos^2(c+dx)}{3a^2d}-\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx}{8a^3d}$$

```
[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sin(d*x+c)/a^4/d+1/8*
(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/d-1/3*b*cos(d*x+c)^2*sin(d*x+c)/a^2
/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(4a^2b^2+3a}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*a^2 + 3*
b^2)*Sin[c + d*x])/(3*a^4*d) + ((3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) - (b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \frac{\cos^3(c+dx)\left(-4b+3a\sec(c+dx)+3b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{4a}$$

$$= -\frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^2(c+dx)\left(-3\left(3a^2+4b^2\right)-ab\sec(c\cdots\right.}{a+b\sec(c+dx)}}{12a^2}$$

$$= \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c\cdots}{4ad}$$

$$= -\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(\cdots}{3a^2d}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+d\cdots}{8a^3d}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+d\cdots}{8a^3d}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+d\cdots}{8a^3d}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}\,d} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \cdots$$

Mathematica [A] time = 0.59, size = 153, normalized size = 0.79

$$\frac{3a^4\sin(4(c+dx)) - 8a^3b\sin(3(c+dx)) - 24ab\left(3a^2+4b^2\right)\sin(c+dx) + 24a^2\left(a^2+b^2\right)\sin(2(c+dx)) + \frac{192b^5\tan\cdots}{\cdots}}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(c + d*x) + (192*b^5*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Si
```

```
n[c + d*x] + 24*a^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)]
 + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

fricas [A] time = 0.51, size = 482, normalized size = 2.50

$$\left[\frac{12\sqrt{a^2-b^2}\,b^5\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+3\left(3\,a^6+a^4b^2+4\,a^2b^4\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(12*sqrt(a^2 - b^2)*b^5*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*(3*a^6 + a^4*b^2 +
 4*a^2*b^4 - 8*b^6)*d*x - (16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b
^2)*cos(d*x + c)^3 + 8*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^
2 - 4*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), -1/24*(24*s
qrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*d*x + (16*a^5*
b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b^2)*cos(d*x + c)^3 + 8*(a^5*b - a^
3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^2 - 4*a^2*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^7 - a^5*b^2)*d)]
```

giac [B] time = 0.24, size = 393, normalized size = 2.04

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}\,a^5}-\frac{3\left(3\,a^4+4\,a^2b^2+8\,b^4\right)(dx+c)}{a^5}+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,a^2b\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/(sqrt(-
a^2 + b^2)*a^5) - 3*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 + 2*(15*a^3*t
an(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*
d*x + 1/2*c)^7 + 24*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*tan(1/2*d*x + 1/2*c)
^5 + 40*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72
*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b*tan(1
/2*d*x + 1/2*c)^3 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12
*a*b^2*tan(1/2*d*x + 1/2*c) + 24*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*a^4))/d
```

maple [B] time = 0.66, size = 672, normalized size = 3.48

$$-\frac{2b^5\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^5\sqrt{(a-b)\,(a+b)}}-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*b^5/a^5/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))-5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b-1/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^2-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^7*b^3+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
5-10/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-6/d/a^4/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^3-1/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^2-3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^3+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2-1
0/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-6/d/a^4/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+5/4/d/a/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)*b^2-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-2/d/a^4/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^3+3/4/d/a*arctan(tan(1/2*d*x+1/
2*c))+1/d/a^3*arctan(tan(1/2*d*x+1/2*c))*b^2+2/d/a^5*arctan(tan(1/2*d*x+1/2
*c))*b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.42, size = 2678, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (b^5*atan(((b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^1
0*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5
*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5
*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b
^3 + 4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12
*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/
2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((tan(c/2
 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256
*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b
^3 + 51*a^9*b^2))/(2*a^8) - (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a
^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan(c/2 + (d*x
)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(
a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2))/(
(96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*b^11 - 104*a^4*b^10 + 88*a^5*b
^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9*a^9*b^5)/a^12 - (b^5*(a^2 - b
^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11
- 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136
*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5*((12*a^16 - 12*a^15*b +
 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 -
(b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*
a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2)))/(a
^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 2
7*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 25
6*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) -
```

```
 (b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a
^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128
*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2
)^(1/2))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^7
- a^5*b^2)) - (atan((((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 +
 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 - (tan(c/2 + (d*x)/2)*(a^4*3i
+ b^4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13
))*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(256*a*b^1
0 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7
 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a
^8))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1i)/(8*a^5) - (((((12*a^16 - 12*a^15*b
+ 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 +
 (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*
b^3 - 256*a^11*b^2))/(16*a^13))*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) - (
tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^
9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 8
1*a^8*b^3 + 51*a^9*b^2))/(2*a^8))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1i)/(8*a^5
))/((((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*
a^13*b^3 + 4*a^14*b^2)/a^12 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*8i + a^2*b^
2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(a^4*3i + b^4*
8i + a^2*b^2*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9
*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 -
216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))*(a^4*3i + b^
4*8i + a^2*b^2*4i))/(8*a^5) - (96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*
b^11 - 104*a^4*b^10 + 88*a^5*b^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9
*a^9*b^5)/a^12 + (((((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*
a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^
4*8i + a^2*b^2*4i)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(16*a^13))*(
a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5) - (tan(c/2 + (d*x)/2)*(256*a*b^10 -
27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 2
56*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8))
*(a^4*3i + b^4*8i + a^2*b^2*4i))/(8*a^5)))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1
i)/(4*a^5*d) - ((tan(c/2 + (d*x)/2)^7*(4*a*b^2 + 8*a^2*b + 5*a^3 + 8*b^3))/
(4*a^4) - (tan(c/2 + (d*x)/2)*(4*a*b^2 - 8*a^2*b + 5*a^3 - 8*b^3))/(4*a^4)
+ (tan(c/2 + (d*x)/2)^3*(40*a^2*b - 12*a*b^2 + 9*a^3 + 72*b^3))/(12*a^4) +
(tan(c/2 + (d*x)/2)^5*(12*a*b^2 + 40*a^2*b - 9*a^3 + 72*b^3))/(12*a^4))/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
 tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2b^2d\left(a^2-b^2\right)}+\frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{a\left(3a^2-2\right.}{b^3d}$$

```
[Out] 1/2*(6*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(3*a^2-4*b^2)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b)^(3/2)/d-a*(3*a
^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d+1/2*(3*a^2-b^2)*sec(d*x+c)*tan(d*x+c)/
b^2/(a^2-b^2)/d-a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.61, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan}{bd\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(3*a^2 - 4*b^2)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
b)^(3/2)*d) - (a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^
2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-ab\sec(c+dx)-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)(-a}{}}{}$$

$$= -\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)}{b\left(a^2-b^2\right)d(a+}$$

$$= -\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)}{b\left(a^2-b^2\right)d(a+}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx}{2b^2\left(a^2-b^2\right.}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx}{2b^2\left(a^2-b^2\right.}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} - \frac{a\left(3a^2\right.}{}$$

Mathematica [A] time = 6.13, size = 357, normalized size = 1.61

$$\frac{a^4\sin(c+dx)}{b^3d(b-a)(a+b)(a\cos(c+dx)+b)} + \frac{\left(-6a^2-b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2b^4d} + \frac{\left(6a^2+b^2\right)\log\left(\sin\left(\frac{1}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^3*(-3*a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((-6*a^2 - b^2)*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/(2*b^4*d) + ((6*a^2 + b^2)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(2*b^4*d) + 1/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^2) - (2*a*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])) - 1/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (2*a*Sin[(c + d
*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^4*Sin[c + d*x])/
(b^3*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x]))
```

fricas [B] time = 1.25, size = 909, normalized size = 4.09

$$\left[\frac{2\left(\left(3\,a^6-4\,a^4b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^5b-4\,a^3b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2}}{a^2\cos(dx+c)^2+2\,ab}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a^6 - 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x
 + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*
cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 +
a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x +
c)^2)*log(-sin(d*x + c) + 1) + 2*(a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*a^6*b -
5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 +
(a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2), -1/4*(4*((3*a^6 - 4*a^4*b^2)
*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
(6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4
*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a
^2*b^5 + b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^4*b^3 - 2*a^2*b
^5 + b^7 - 2*(3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*
b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2)]
```

giac [A] time = 0.31, size = 299, normalized size = 1.35

$$\frac{4 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(a^2b^3-b^5\right)\left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-a-b\right)} - \frac{4\left(3a^5-4a^3b^2\right)\left(\pi\left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-b\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2))
+ (6*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - (6*a^2 + b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(
1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

maple [A] time = 0.39, size = 405, normalized size = 1.82

$$\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^3\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{6a^5\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^4\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}+\frac{8a^3\,\mathrm{arc}}{d\,b^2\,(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b-a-b)-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1
/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+
b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/2/d/b^2/(
tan(1/2*d*x+1/2*c)-1)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/b^2/(tan(1/2
*d*x+1/2*c)-1)-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2*ln(tan(1/2*d*
x+1/2*c)-1)-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)+
```

```
1)*a+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a^2+
1/2/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 8.01, size = 3685, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^5*(3*a*b^3 - 3*a^3*b + 6*a^4 + b^4 - 5*a^2*b^2))/((a
*b^3 - b^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(b^4 - 6*a^4 + 3*a^2*b^2))/(
b*(a*b^2 - b^3)*(a + b)) + (tan(c/2 + (d*x)/2)*(3*a^3*b - 3*a*b^3 + 6*a^4 +
 b^4 - 5*a^2*b^2))/(b^3*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^2*
(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(c/2 + (d*x)/2)^4*(3*a - b)))
 - (atan((((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b
^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4
+ 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - ((6*a^2 +
 b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 +
6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2
+ (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10
 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))/(2*b^4
))*1i)/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b
- 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a
^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((
6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*
b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*t
an(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a
^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))))
/(2*b^4))*1i)/(2*b^4))/((16*(108*a^11 - 54*a^10*b + 4*a^3*b^8 - 4*a^4*b^7 +
 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^3 - 216*a^9*b^2))/(a*b^11 +
 b^12 - a^2*b^10 - a^3*b^9) - ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^1
0 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a
^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 -
a^3*b^6) - ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b
^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^
3*b^9) - (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^
3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7
- a^3*b^6))))/(2*b^4)))/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72
*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 -
26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^
7 - a^3*b^6) + ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a
^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10
- a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 1
6*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*
b^7 - a^3*b^6))))/(2*b^4)))/(2*b^4)))*(6*a^2 + b^2)*1i)/(b^4*d) - (a^3*atan
(((a^3*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(
72*a^10 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6
```

```
- 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*
b^7 - a^3*b^6) + (a^3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11
+ 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^
9) + (8*a^3*tan(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*
(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8)
)/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^
4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^3*(3*a^2
 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*
a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5
+ 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6
) - (a^3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10
 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^3*t
an(c/2 + (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8
*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b
^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 -
 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^
4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(108*a^11 - 54*a^10*
b + 4*a^3*b^8 - 4*a^4*b^7 + 41*a^5*b^6 - 9*a^6*b^5 + 63*a^7*b^4 + 81*a^8*b^
3 - 216*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (a^3*(3*a^2 - 4*b^
2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b -
 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^
6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^
3*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^
6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*a^3*tan(c/2
+ (d*x)/2)*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^
12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^
2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)
*((a + b)^3*(a - b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b
^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) - (a^3*(3*a^2 - 4*b^2)*((a + b)^3*(a
 - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b - 2*a*b^9 + b^10
+ 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a^6*b^4 + 120*a^7*
b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^3*((8*(2*b^15 +
6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b
^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^3*tan(c/2 + (d*x)/2)*(3*a^
2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11
 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a -
 b)^3)^(1/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8
+ 3*a^4*b^6 - a^6*b^4)))*(3*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d
*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{(2a^2-b^2)\tan(c+dx)}{b^2d(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{2a\tanh^{-1}(\mathrm{si}}{b^3c}$$

```
[Out] -2*a*arctanh(sin(d*x+c))/b^3/d+2*a^2*(2*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*tan(d
*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+
c))
```

Rubi [A] time = 0.35, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(2a^2-b^2)\tan(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{2a\tanh^{-1}(\mathrm{si}}{b^3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d)
 + ((2*a^2 - b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
```

```
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(a^2-ab\sec(c+dx)-\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(a^2b+2a\left(a^2-b^2\right)\right.}{a+b\sec(c+dx)}}{b^2\left(a^2-b^2\right.}$$

$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{(2a)\int\sec(c+dx)\,dx}{b^3}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+d}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+d}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{\left(2a^2-b^2\right.}{b^2\left(a\right.}$$

Mathematica [A] time = 1.55, size = 162, normalized size = 0.99

$$\frac{\frac{a^3b\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - \frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\right.}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(3/2) + 2*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*a*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*Sin[c + d*x])/((a - b)*(a
 + b)*(b + a*Cos[c + d*x])) + b*Tan[c + d*x])/(b^3*d)
```

fricas [B] time = 0.84, size = 760, normalized size = 4.63

$$\left[\frac{\left(\left(2\,a^{5}-3\,a^{3}b^{2}\right)\cos\left(dx+c\right)^{2}+\left(2\,a^{4}b-3\,a^{2}b^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{a^{2}-b^{2}}\ldots}{a^{2}\cos(dx+c)^{2}+2\,ab\cos\ldots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((a^6 - 2*a^4*b^2 + a^2*b^4
)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x +
c) + 1) + 2*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^
3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b^2 - 2*a^2*b^4 +
b^6 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 -
 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*
x + c)), (((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d
*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a
^2 - b^2)*sin(d*x + c))) - ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a
^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^6 - 2*a
^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)
)*log(-sin(d*x + c) + 1) + (a^4*b^2 - 2*a^2*b^4 + b^6 + (2*a^5*b - 3*a^3*b^
3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos
(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

giac [B] time = 0.30, size = 331, normalized size = 2.02

$$\frac{2\left(\frac{\left(2\,a^{4}-3\,a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{2}b^{3}-b^{5}\right)\sqrt{-a^{2}+b^{2}}}-\frac{2\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\ldots\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\ldots\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*
b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x +
 1/2*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2
*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2
 - b^4)) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^3)/d
```

maple [A] time = 0.45, size = 275, normalized size = 1.68

$$-\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,b^{2}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{4a^{4}\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^{3}\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{6a^{2}\,\mathrm{arcta}\ldots}{db\,(a-b)\,(a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b-a-b)+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b
))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*a/b^3*ln
(tan(1/2*d*x+1/2*c)-1)-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)-1/d/b^2/(tan(1/2*d*x+
1/2*c)+1)-2/d*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.81, size = 3159, normalized size = 19.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a - b
)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a -
 b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2*a*tan(c/2 + (d*x)/2)^2))
 + (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3
*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b
^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 -
2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x)/2)*(2
*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3
*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3)*2i)/b^3 + (a*((32*tan(c/2 + (d*x
)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16
*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (2*a*((32*(2*a*b^11 - 3*a^2*
b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7
 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)
)))/b^3)*2i)/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5*b^3 - 20*a^6*b
^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (2*a*((32*tan(c/2 + (d*x)/2)*(8*a^
8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))
/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a
^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6
) - (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8
 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3))/
b^3 + (2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5
 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)
+ (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^
6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b
^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*
b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3))/b^3))*4i)/(b^3*d) + (a^2*atan(((a^2
*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b
^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*
a^2 - 3*b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7
```

```
- 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2
)*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^
3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b
^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/
(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)
^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*((32*tan(c/2 +
 (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3
 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2)*((
32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(
a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^
2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b
^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^
2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(
b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4
+ 6*a^5*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a^2*((32*ta
n(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*
a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*a^2 - 3*
b^2)*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*
b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2
 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9
 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3
*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2
))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (a^2*((32*tan(c/2 + (d*x)/2)*(
8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b
^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2)*((32*(2*a*b^1
1 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9
 - a^2*b^7 - a^3*b^6) - (32*a^2*tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*((a + b)
^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*
b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a
^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b
^5 - a^6*b^3))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^
7 + 3*a^4*b^5 - a^6*b^3)))*(2*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/
(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=117

$$-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-2*a*(a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*tan(d*x+c)/b/(a^2-b^2)
/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 3998, 3770, 3831, 2659, 208}

$$-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
```

]

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab-\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \sec(c+dx)\,dx}{b^2} - \frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,d}{b^2\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}}{b^3\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \frac{}{1\text{-}}\right.}{b^3\left(\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{a^2\tan(c+}{b\left(a^2-b^2\right)d(a+b}$$

Mathematica [A] time = 0.39, size = 146, normalized size = 1.25

$$\frac{\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^2b\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a
*Cos[c + d*x])))/(b^2*d)
```

fricas [B] time = 0.81, size = 596, normalized size = 5.09

$$\left[\frac{\left(a^3b-2\,ab^3+\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*lo
g((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b
*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + b^2)) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)
*cos(d*x + c))*log(sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*
a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^4*b - a^2*b^3)
*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2
*a^2*b^5 + b^7)*d), -1/2*(2*(a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*
cos(d*x + c))*log(sin(d*x + c) + 1) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a
^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b - a^2*b^3)*
sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*
a^2*b^5 + b^7)*d)]
```

giac [A] time = 0.28, size = 203, normalized size = 1.74

$$\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{2\left(a^3-2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{-a^2+b^2}}+\frac{\log}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 - a - b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + log(abs(
tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

maple [B] time = 0.38, size = 225, normalized size = 1.92

$$\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,b^2\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}+\frac{4a\,\mathrm{arctan}}{d\,(a-b)\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b-a-b)-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2
)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^2*ln(tan(1/2*
d*x+1/2*c)-1)+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.73, size = 2848, normalized size = 24.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] - (atan((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b
^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a
^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 -
a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 +
3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/
b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6
- a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*
b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3
*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a
^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2
)/((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^
2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7
+ 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2
)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b
^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2 - (64*(
2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^
3*b^3) + (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^
6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^
3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a
^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3
*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2))
*2i)/(b^2*d) - (a*atan(((a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*t
an(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3
- 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (a*(a^2 - 2*b^2)*((32*(2*
a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*
b^3) + (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(
2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b
^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a
 + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*1i)/(b^8
 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3
)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^
4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (a*(a^2 - 2
*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 -
a^2*b^4 - a^3*b^3) - (32*a*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a -
 b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a
^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 -
a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*
b^2))*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))/((64*(2*a*b^4 - a^4*b +
a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (a*(a^2 -
 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*
b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*
b^3 - a^3*b^2) + (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6
 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*a*tan(c/2 + (d*x)/2)*(
a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 +
 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^
8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(b^8 -
3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)
+ (a*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a
^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 +
 b^5 - a^2*b^3 - a^3*b^2) - (a*(a^2 - 2*b^2)*((32*(2*a*b^8 - b^9 + a^2*b^7
- 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*a*tan(c/2 +
 (d*x)/2)*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a*b^9 - 2*a^2*b^8 -
```

```
4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a
^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/
2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4
- a^6*b^2)))*(a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(b^8 - 3*a^2*
b^6 + 3*a^4*b^4 - a^6*b^2)) - (2*a^2*tan(c/2 + (d*x)/2))/(d*(a + b)*(a*b -
b^2)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d+a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3836, 12, 3831, 2659, 208}

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{b\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{b\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2-b^2}$$

$$= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}$$

$$= -\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.20, size = 83, normalized size = 0.98

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/
2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

fricas [A] time = 0.49, size = 329, normalized size = 3.87

$$\left[-\frac{\left(ab\cos(dx+c)+b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-2\left(a\right.}{2\left(\left(a^5-2a^3b^2+ab^4\right)d\cos(dx+c)+\left(a^4b-2a^2b^3+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b*cos(d*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), -((a*b*cos(d*x + c) + b^2)*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
 - 2*a^2*b^3 + b^5)*d)]
```

giac [A] time = 0.23, size = 150, normalized size = 1.76

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/((a^2 - b^2)*sq
rt(-a^2 + b^2)) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan
(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

maple [A] time = 0.40, size = 118, normalized size = 1.39

$$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b-a-b)-2*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2
*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.12, size = 92, normalized size = 1.08

$$\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - tan(c/2 + (d*x)/2)^2*(
a - b))) - (2*b*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d
*(a + b)^(3/2)*(a - b)^(3/2))
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] 2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(
3/2)/d-b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3833, 12, 3831, 2659, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right.\right.}{b\left(a^2-b^2\right)d}$$

$$= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.23, size = 83, normalized size = 0.97

$$\frac{\frac{b\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - \frac{2a\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3
/2) + (b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

fricas [A] time = 0.52, size = 332, normalized size = 3.86

$$\left[-\frac{\left(a^2\cos(dx+c)+ab\right)\sqrt{a^2-b^2}\,\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+2\,(a}{2\left(\left(a^5-2a^3b^2+ab^4\right)d\cos(dx+c)+\left(a^4b-2a^2b^3+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*
(a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), ((a^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^2
*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
- 2*a^2*b^3 + b^5)*d)]
```

giac [A] time = 0.24, size = 150, normalized size = 1.74

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}} - \frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/((a^2 - b^2)*sqrt
(-a^2 + b^2)) - b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

maple [A] time = 0.34, size = 118, normalized size = 1.37

$$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b-a-b)+2*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.04, size = 92, normalized size = 1.07

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (2*a*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(3
/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - t
an(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2-2*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tan[c + d*x])/(a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\right.}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

Mathematica [A] time = 0.48, size = 138, normalized size = 1.27

$$\frac{\frac{b\left(\left(a^2-b^2\right)(c+dx)+ab\sin(c+dx)\right)+a\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{a\cos(c+dx)+b} - \frac{2b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{a^2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2),x]
```

```
[Out] ((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)*(
c + d*x) + a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)*d)
```

fricas [B] time = 0.54, size = 484, normalized size = 4.44

$$\left[\frac{2\left(a^5-2a^3b^2+ab^4\right)dx\cos(dx+c)+2\left(a^4b-2a^2b^3+b^5\right)dx+\left(2a^2b^2-b^4+\left(2a^3b-ab^3\right)\cos(dx+c)\right)\sqrt{a^2-}}{2\left(\left(a^7-2a^5b^2+a^3b^4\right)d\cos(dx+c)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 +
 b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 - b
^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b
^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 - 2*a^
5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((a^5 -
 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x - (2*a
```

```
^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^3*b^2 -
 a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b
- 2*a^4*b^3 + a^2*b^5)*d)]
```

giac [A] time = 0.20, size = 179, normalized size = 1.64

$$-\frac{\frac{2 b^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(a^3-ab^2\right)\left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-a-b\right)}+\frac{2\left(2 a^2 b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-b \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-a^2 b^2\right)\sqrt{-a^2+b^2}}-\frac{d}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)) + 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*x +
c)/a^2)/d
```

maple [B] time = 0.49, size = 204, normalized size = 1.87

$$-\frac{2b^2 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{4b \operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,(a-b)\,(a+b)\sqrt{(a-b)\,(a+b)}}+\frac{2b^3 \operatorname{arcta}}{d\,a^2\,(a-b)\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+
1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/
2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^2*arctan(tan
(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.67, size = 2886, normalized size = 26.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a
^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*
```

b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))/a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))/a^2)/((64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2)))/(a^2*d) + (b*atan(((b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (b*(2*a^2 - b^2)*((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)) - (2*b^2*tan(c/2 + (d*x)/2))/(d*(a + b)*(a*b - a^2)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-2), x)
```

3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$-\frac{2bx}{a^3}+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{2b^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*b*x/a^3+2*b^2*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d+(a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/
d+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.33, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = $\{3847, 4104, 3919, 3831, 2659, 208\}$

$$\frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{2b^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2bx}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*x)/a^3 + (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 - 2*b^2)*Sin[c
+ d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Se
c[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2+2b^2+ab\sec(c+dx)-b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-2b\left(a^2-b^2\right)+ab^2\sec(c+d\ldots}{a+b\sec(c+dx)}}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(b^2\left(3a^2-2b\ldots\right.\right.}{a^3\ldots}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(b\left(3a^2-2b^2\ldots\right.\right.}{a^3\left(\ldots\right.}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2b\left(3a^2-2b\ldots\right.\right.}{\ldots}$$

$$= -\frac{2bx}{a^3} + \frac{2b^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{\ldots}{a\left(\ldots\right.}$$

Mathematica [A] time = 0.76, size = 172, normalized size = 1.18

$$\frac{\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)+\left(a^2-b^2\right)\left(a^2\sin(2(c+dx))-4b^2(c+dx)\right)-4ab\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{a\cos(c+dx)+b} + \frac{4b^2\left(2b^2-3a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2a^3d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + (-4*a*b*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 2*a*b*(a
^2 - 2*b^2)*Sin[c + d*x] + (a^2 - b^2)*(-4*b^2*(c + d*x) + a^2*Sin[2*(c + d
*x)]))/(b + a*Cos[c + d*x]))/(2*a^3*(a - b)*(a + b)*d)
```

fricas [A] time = 0.57, size = 565, normalized size = 3.87

$$\left[-\frac{4\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)dx\cos\left(dx+c\right)+4\left(a^{4}b^{2}-2\,a^{2}b^{4}+b^{6}\right)dx-\left(3\,a^{2}b^{3}-2\,b^{5}+\left(3\,a^{3}b^{2}-2\,ab^{4}\right)\cos\left(dx+c\right)\right.}{2\left(\left(a^{8}-2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^4*b^2 - 2*a^2*
b^4 + b^6)*d*x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*s
qrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*s
qrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + (a
^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a
^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(a^5*b - 2*a
^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - (3*a
^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^5
*b - 3*a^3*b^3 + 2*a*b^5 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d)]
```

giac [B] time = 0.39, size = 837, normalized size = 5.73

$$-\frac{\frac{\left(2\,a^{7}b-5\,a^{6}b^{2}-4\,a^{5}b^{3}+9\,a^{4}b^{4}+2\,a^{3}b^{5}-4\,a^{2}b^{6}-2\,a^{2}b\left|-a^{5}+a^{3}b^{2}\right|-ab^{2}\left|-a^{5}+a^{3}b^{2}\right|+2\,b^{3}\left|-a^{5}+a^{3}b^{2}\right|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(\cdots}{\sqrt{-\frac{a^{4}b-a^{2}b^{3}+\sqrt{\left(a^{5}+a^{4}b-a^{3}b^{2}-a^{2}\cdots}}{a^{5}-a\cdots}}}\right.\right.}{a^{4}b\left|-a^{5}+a^{3}b^{2}\right|-a^{2}b^{3}\left|-a^{5}+a^{3}b^{2}\right|+\left(a^{5}-a^{3}b^{2}\right)^{2}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*a^2*b^6 - 2*
a^2*b*abs(-a^5 + a^3*b^2) - a*b^2*abs(-a^5 + a^3*b^2) + 2*b^3*abs(-a^5 + a^
3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqr
t(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4*b -
 a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a^2*b^
3))))/(a^4*b*abs(-a^5 + a^3*b^2) - a^2*b^3*abs(-a^5 + a^3*b^2) + (a^5 - a^3
*b^2)^2) + ((2*a^2*b + a*b^2 - 2*b^3)*sqrt(-a^2 + b^2)*abs(-a^5 + a^3*b^2)*
abs(-a + b) + (2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*
a^2*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) +
arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 - sqrt((a^5 + a^4*b - a^
3*b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/
(a^5 - a^4*b - a^3*b^2 + a^2*b^3))))/((a^5 - a^3*b^2)^2*(a^2 - 2*a*b + b^2)
 - (a^6*b - 2*a^5*b^2 + 2*a^3*b^4 - a^2*b^5)*abs(-a^5 + a^3*b^2)) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x +
 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - a^2*b
*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

maple [A] time = 0.56, size = 242, normalized size = 1.66

$$\frac{2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{6b^2\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{4b^4\,\mathrm{arct}}{d\,a^3\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b-a-b)+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^2*tan(1
/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^3*b*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 7.01, size = 3169, normalized size = 21.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - a^3 - 2*b^3))/(a^2*(a + b)*(a - b
)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - a^3 + 2*b^3))/(a^2*(a + b)*(a -
 b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*(a - b) + 2*b*tan(c/2 + (d*x)/2)^2))
 - (4*b*atan(((2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 +
16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a
^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 -
 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)/2)*(2*
a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(
a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3 + (2*b*((32*tan(c/2 +
 (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^
3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(2*a^11*b - 2*a
^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*
b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 +
4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*
b^2)))*2i)/a^3))/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6*a^3*b^5 + 12*a
^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(8*
b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2
))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*
b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2
) - (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4
*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^
3)*2i)/a^3 + (b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*
a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*
```

```
b^2) - (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*
a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^1
1*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3
*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3)*2i)/a^3)))/(a^3*d) - (b^2*ata
n(((b^2*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5
+ 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (
b^2*(3*a^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^
9*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 +
(d*x)/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6
+ 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3
- a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^
(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*
(a - b)^3)^(1/2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*((32*ta
n(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8
*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*
b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*
b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2
 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4
*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9
 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a
^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2
)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/((64*(8*b^8 - 4*a*b^7 - 20*a
^2*b^6 + 6*a^3*b^5 + 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*
((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*
b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b^2*(3*a
^2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 -
3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)
*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*
b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^
2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(
a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^
3)^(1/2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b^2*((32*tan(c/2 + (d*
x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 +
4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)*((32*(
2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*
b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2)*(
(a + b)^3*(a - b)^3)^(1/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 -
4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6
+ 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a^3*b^6 + 3*
a^5*b^4 - 3*a^7*b^2))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^9 - a
^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/
2)*2i)/(d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2a^2d(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{x(a^2+6b^2)}{2a^4}-\frac{2b^3(4a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{a^4d(a-b)^{3/2}(a+}$$

```
[Out] 1/2*(a^2+6*b^2)*x/a^4-2*b^3*(4*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d-b*(2*a^2-3*b^2)*sin(d*x+c)
/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*
cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.59, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3847, 4104, 3919, 3831, 2659, 208}

$$-\frac{b(2a^2-3b^2)\sin(c+dx)}{a^3d(a^2-b^2)}+\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2a^2d(a^2-b^2)}-\frac{2b^3(4a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 + 6*b^2)*x)/(2*a^4) - (2*b^3*(4*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(2*a^
2 - 3*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a
^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-a^2+3b^2+ab\sec(c+dx)-2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \int \frac{\cos(c+dx)\left(-2\right.}{}$$

$$= -\frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c+dx)}{a\left(a^2-b^2\right)d(a+}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b}{a(a}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b}{a(a}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b}{a(a}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{2b^3\left(4a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.77, size = 144, normalized size = 0.69

$$\frac{2\left(a^2+6b^2\right)(c+dx) - \frac{8b^3\left(3b^2-4a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + a^2\sin(2(c+dx)) + \frac{4ab^4\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - 8ab\sin(c+d}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + 6*b^2)*(c + d*x) - (8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - 8*a*b*Sin[c + d*x] + (4
*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*Sin[2*(c
+ d*x)])/(4*a^4*d)
```

fricas [A] time = 0.56, size = 660, normalized size = 3.17

$$\left[\frac{\left(a^7 + 4\,a^5b^2 - 11\,a^3b^4 + 6\,ab^6\right)dx\cos\left(dx + c\right) + \left(a^6b + 4\,a^4b^3 - 11\,a^2b^5 + 6\,b^7\right)dx + \left(4\,a^2b^4 - 3\,b^6 + \left(4\,a^3b^3 - \right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b +
4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*d*x + (4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a
*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (4*a^5*b^2 - 1
0*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b
 - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((a^7 + 4*a^5*
b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b + 4*a^4*b^3 - 11*a^2*
b^5 + 6*b^7)*d*x - 2*(4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - (4*a^5*b^2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 +
 a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^
3 + a^4*b^5)*d)]
```

giac [A] time = 0.21, size = 264, normalized size = 1.27

$$-\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{4\left(4\,a^2b^3-3\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}\cdot}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2))
 - (a^2 + 6*b^2)*(d*x + c)/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*
d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

maple [A] time = 0.73, size = 362, normalized size = 1.74

$$-\frac{2b^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{8b^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\left(a-b\right)\left(a+b\right)\sqrt{\left(a-b\right)\left(a+b\right)}}+\frac{6b^5\,\mathrm{a}}{d\,a^4\left(a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b-a-b)-8/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)*(a
+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)^3*b+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b+1/d/a^2*arc
tan(tan(1/2*d*x+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 8.32, size = 3738, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a
^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 +
11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*
a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6
*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a
^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b
^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1
i + b^2*6i)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^
9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*
b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2*1
i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^
4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*ta
n(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a
^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)
)))/(2*a^4))*(a^2*1i + b^2*6i)*1i)/(2*a^4))/((16*(108*b^11 - 54*a*b^10 - 21
6*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 +
4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (((8*tan(c/2 + (d*x)/2)*
(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b
^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6
*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 +
 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^
9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a
^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b +
 a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i))/(2*a^4) + (((8*tan
(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^
3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a
^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b
^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^
11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)
*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2
```

```
))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i))/(2
*a^4)))*(a^2*1i + b^2*6i)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)*(3*a*b^3 - 3*a
^3*b + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)
/2)^5*(3*a^3*b - 3*a*b^3 + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)
) - (2*tan(c/2 + (d*x)/2)^3*(a^4 - 6*b^4 + 3*a^2*b^2))/(a*(a^2*b - a^3)*(a
+ b)))/(d*(a + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a
 - 3*b) - tan(c/2 + (d*x)/2)^6*(a - b))) + (b^3*atan(((b^3*(4*a^2 - 3*b^2)*
((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b
^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6
*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^3*(
(8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b
^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^3*tan(c/2 + (
d*x)/2)*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 +
 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b
^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((
a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a
^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a
 - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10
- 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7
*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^3*((8*(2*a^15 -
12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b
^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^3*tan(c/2 + (d*x)/2)*(4*a^
2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 +
16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)
*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((a + b)^3*(a -
 b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6
 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*b^11 - 54*a*b^10 - 216*a^2*b^9 + 81*a^
3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11
*b + a^12 - a^9*b^3 - a^10*b^2) - (b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3
)^(1/2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a
^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 +
11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^3*((8*(2*a^15 - 12*a^8*
b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a
^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b
^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10
*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10
- a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^
(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b
^4 - 3*a^8*b^2) + (b^3*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(
c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3
*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^
8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6
+ 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a
^9*b^3 - a^10*b^2) + (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*((a + b)^3*(
a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b
^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^
6*b^4 - 3*a^8*b^2)))*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^10 - a
^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))
)*(4*a^2 - 3*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^
6*b^4 - 3*a^8*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3a^2d\,(a^2-b^2)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{bx\,(a^2+4b^2)}{a^5}+\frac{2b^4\,(5a^2-4b^2)\tanh^{-1}\Bigg(\frac{\sqrt{a-}}{}$$ $$\frac{}{a^5d(a-b)^{3/2}(a+b}$$

```
[Out] -b*(a^2+4*b^2)*x/a^5+2*b^4*(5*a^2-4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/3*(2*a^4+7*a^2*b^2-12*b^4
)*sin(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b
^2)/d+1/3*(a^2-4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*cos(d*x+c
)^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.84, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3847, 4104, 3919, 3831, 2659, 208}

$$\frac{(7a^2b^2+2a^4-12b^4)\sin(c+dx)}{3a^4d\,(a^2-b^2)}+\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3a^2d\,(a^2-b^2)}-\frac{b\,(a^2-2b^2)\sin(c+dx)\cos(c+dx)}{a^3d\,(a^2-b^2)}+\frac{2b^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(a^2 + 4*b^2)*x)/a^5) + (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4
 + 7*a^2*b^2 - 12*b^4)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - (b*(a^2 - 2*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 4*b^2)*Cos[c +
d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]^2*Sin[c + d*
x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^3(c+dx)\left(-a^2+4b^2+ab\sec(c+dx)-3b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-4b^2\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\cos^2(c+ \cdots}{\cdots}}{\cdots}$$

$$= -\frac{b\left(a^2-2b^2\right)\cos(c+dx)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b \cdots}{a(\cdots}$$

$$= \frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\cos(c+dx)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2- \cdots\right.}{\cdots}$$

$$= -\frac{b\left(a^2+4b^2\right)x}{a^5} + \frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\cos(c+dx)\sin \cdots}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(a^2+4b^2\right)x}{a^5} + \frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\cos(c+dx)\sin \cdots}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(a^2+4b^2\right)x}{a^5} + \frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\cos(c+dx)\sin \cdots}{a^3\left(a^2-b^2\right)d}$$

$$= -\frac{b\left(a^2+4b^2\right)x}{a^5} + \frac{2b^4\left(5a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(2a^4+7a^2b^2-12 \cdots\right.}{3a^4\left(a^2- \cdots\right.}$$

Mathematica [C] time = 1.07, size = 176, normalized size = 0.67

$$\frac{a^3\sin(3(c+dx)) + 9a\left(a^2+4b^2\right)\sin(c+dx) + \frac{24b^4\left(4b^2-5a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - 6a^2b\sin(2(c+dx)) + \frac{12 \cdots}{(b-a)(a- \cdots}}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*b*((-I)*a + 2*b)*(I*a + 2*b)*(c + d*x) + (24*b^4*(-5*a^2 + 4*b^2)*ArcT
anh[((-a + b)*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 9*a*(
a^2 + 4*b^2)*Sin[c + d*x] + (12*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b +
a*Cos[c + d*x])) - 6*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^5
*d)
```

fricas [A] time = 0.56, size = 757, normalized size = 2.90

$$\left[-\frac{6\left(a^{7}b+2\,a^{5}b^{3}-7\,a^{3}b^{5}+4\,ab^{7}\right)dx\cos\left(dx+c\right)+6\left(a^{6}b^{2}+2\,a^{4}b^{4}-7\,a^{2}b^{6}+4\,b^{8}\right)dx-3\left(5\,a^{2}b^{5}-4\,b^{7}+\left(5\,a^{3}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 6*(a^
6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*
b^4 - 4*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*
a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6
*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^
2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1/3*(3*(a^
7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2 + 2*a^
4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*b^4 - 4*a*b^
6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a
*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 +
a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x
 + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b
- 2*a^7*b^3 + a^5*b^5)*d)]
```

giac [A] time = 0.24, size = 335, normalized size = 1.28

$$\frac{\frac{6\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{6}-a^{4}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)}+\frac{6\left(5\,a^{2}b^{4}-4\,b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{7}-a^{5}b^{2}\right)\sqrt{-a^{2}+b^{2}}}-\frac{3\left(a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2))
- 3*(a^2*b + 4*b^3)*(d*x + c)/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b
*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^3*a^4))/d
```

maple [B] time = 0.68, size = 508, normalized size = 1.95

$$\frac{2b^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^4\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{10b^4\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^3\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{8b^6\,\mathrm{arc}}{d\,a^5\,(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b-a-b)+10/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*b^6/a^5/(a-b)/(a+b)/((a-b)*(a
+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/d/a^3/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^5*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x
+1/2*c)^5*b^2+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+12/
d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*b^2+2/d/a^2/(1+tan(1/
2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*b^2
-2/d/a^3*b*arctan(tan(1/2*d*x+1/2*c))-8/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^
3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.08, size = 3839, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/(
a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 + 3
6*b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*
x)/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4
*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a^2*b^
3 + a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(a -
b) + tan(c/2 + (d*x)/2)^2*(2*a + 4*b) - tan(c/2 + (d*x)/2)^6*(2*a - 4*b) +
6*b*tan(c/2 + (d*x)/2)^4)) - (2*b*atan(((b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*
x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b
^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b +
a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*
a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15
 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2
*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a
^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5 + (b*(a^2 + 4*b^2)*((32*t
an(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b
```

```
^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2)
)/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a
^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a
^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(
2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*
32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5)/((64*(64*b^1
4 - 32*a*b^13 - 112*a^2*b^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a
^6*b^8 - 5*a^7*b^7 + 5*a^8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b
*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 +
 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 -
2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^
2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^1
4*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*
x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^
13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^
5)*1i)/a^5 + (b*(a^2 + 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11
 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5
 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) -
 (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a
^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (
b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*
a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^
9*b^2)))*1i)/a^5)*1i)/a^5))*(a^2 + 4*b^2))/(a^5*d) - (b^4*atan(((b^4*(5*a^2
 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32
*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12
*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^
9*b^2) + (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*
b^5 - 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b
^4*tan(c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b
 - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*
b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5
*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*
a^9*b^2))*1i)/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2) + (b^4*(5*a^2 - 4*b^
2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11
 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^
5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2)
- (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5
*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b^4*tan(
c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^
10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^1
1 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 -
4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2
))*1i)/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))/((64*(64*b^14 - 32*a*b^13
 - 112*a^2*b^12 + 48*a^3*b^11 + 12*a^4*b^10 - 6*a^5*b^9 + 31*a^6*b^8 - 5*a^7
*b^7 + 5*a^8*b^6))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b^4*(5*a^2 - 4*
b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^
11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*
b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2
) + (b^4*((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 -
 5*a^14*b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (32*b^4*ta
n(c/2 + (d*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*
a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a
^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2
 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b
^2)))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2) + (b^4*(5*a^2 - 4*b^2)*((a +
 b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^
2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^
8*b^4 - 2*a^9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b^4*(
(32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^
```

```
4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (32*b^4*tan(c/2 + (d
*x)/2)*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^15*b - 2*a^10*b^6 +
 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2))/((a^10*b + a^11 - a^8*
b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*(
(a + b)^3*(a - b)^3)^(1/2))/(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))/(a^1
1 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*(5*a^2 - 4*b^2)*((a + b)^3*(a - b)^3
)^(1/2)*2i)/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=230

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctanh((a-b)^
(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(3*
a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d-1/2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^
2-b^2)/d/(a+b*sec(d*x+c))^2+3/2*a^3*(a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.71, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}+\frac{3a^2\left(-5a^2b^2+2a^4+4b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
 b)^(5/2)*d) + ((3*a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*
Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (
3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x
]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-2ab\sec(c+dx)-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)}{\ldots}}{\ldots}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\ldots}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\ldots}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4 d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+d\ldots}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4 d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+d\ldots}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4 d} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} + (3a\ldots$$

Mathematica [A] time = 4.85, size = 205, normalized size = 0.89

$$\frac{-\frac{6a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{a^3b\sin(c+dx)\left(a\left(4a^2-7b^2\right)\cos(c+dx)+5a^2b-8b^3\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + 6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\ldots\right.\right.}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*(5*a^2*b - 8*
b^3 + a*(4*a^2 - 7*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b
 + a*Cos[c + d*x])^2) + 2*b*Tan[c + d*x])/(2*b^4*d)
```

fricas [B] time = 1.45, size = 1354, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a^
5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos
(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((a^9 - 3*a^7*b^2 + 3
```

```
*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2
*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x +
c))*log(sin(d*x + c) + 1) + 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(
d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (
a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) +
1) + 2*(2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 +
 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*
b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^
8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^1
1)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x +
 c)), 1/2*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b -
 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6
)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) - 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6
)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)
^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(sin(d*x +
c) + 1) + 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^
8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^
4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^6*b^3 -
6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*
b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d
*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(
d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2
+ (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c))]
```

giac [A] time = 0.40, size = 383, normalized size = 1.67

$$-\frac{\frac{3\left(2a^6-5a^4b^2+4a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{-a^2+b^2}}+\frac{4a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^4b^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) + (4*a^6*tan(1/
2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*tan(1/2*d*x + 1/2*c) -
 5*a^5*b*tan(1/2*d*x + 1/2*c) + 7*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*a^3*b^3*
tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*a*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/b^4 - 3*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*tan(1/2*d*x + 1
/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

maple [B] time = 0.39, size = 735, normalized size = 3.20

$$-\frac{4a^5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^3\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2(a-b)\left(a^2+2ab+b^2\right)}+\frac{a^4}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(
```

```
1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+8/d*a^
3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*a^4/b^2/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4
)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))
^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^3/(tan(1
/2*d*x+1/2*c)-1)+3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/b^3/(tan(1/2*d*x+1/
2*c)+1)-3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.29, size = 5332, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12*
a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^4*
b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a^2*
b^3)) - (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2))/(
b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b
 + 3*a^2 - b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - ta
n(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (a*atan(((a*((8*tan(c/2 + (d*x
)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5
*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2
))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b
^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^1
4 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^
10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11
 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 - 8*a^2*b^16 -
32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^
8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*
b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*3i)/b^4 + (a*((8*
tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4
*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3
- 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a
^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^1
5 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*
a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^1
2 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^
7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2
*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*3i)
```

/b^4)/((48*(36*a^12 - 18*a^11*b + 72*a^4*b^8 + 72*a^5*b^7 - 234*a^6*b^6 - 1
26*a^7*b^5 + 288*a^8*b^4 + 81*a^9*b^3 - 162*a^10*b^2))/(a*b^15 + b^16 - 3*a
^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (3*a
*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 3
6*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9
*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9
+ 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^
3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10
 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^
4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b
^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 -
32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 -
3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4
))/b^4 + (3*a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 7
2*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*
b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10
 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a*((24*(4*a*b^17 - 8*a^2
*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11
+ 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^
3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (24*a*tan(c/2 + (d
x)/2)(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 4
8*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b
^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^
7*b^6))))/b^4))/b^4))*6i)/(b^4*d) + (a^2*atan(((a^2*((a + b)^5*(a - b)^5)^(
1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9
 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288
*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*
b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2*((24*(4*a*b^17 - 8*a^2*b^16 -
 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^
8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13
+ 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (12*a^2*tan(c/2 + (d*x)/2
)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2
*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11
 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10
 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b
^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*((a + b)^5*(a - b)^5)^(1
/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a
^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/(2*(b^14 -
 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (a^2*((a
+ b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^
2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5
+ 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 -
3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a^2*((24*(4*a*
b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 -
 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2
*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (12*a^
2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2
)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6
*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2
*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3
*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*((a +
 b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 +
 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*
b^2)*3i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^1
0*b^4)))/((48*(36*a^12 - 18*a^11*b + 72*a^4*b^8 + 72*a^5*b^7 - 234*a^6*b^6
- 126*a^7*b^5 + 288*a^8*b^4 + 81*a^9*b^3 - 162*a^10*b^2))/(a*b^15 + b^16 -
3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (
3*a^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11
*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 43

```
2*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a
^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2*
((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 32*
a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^
16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9
) - (12*a^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 -
 5*a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^1
3 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^
14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12
+ b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^
6)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*
a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + 4*b^
4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^
6 - a^10*b^4)) + (3*a^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*
(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7
- 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a
*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 -
a^7*b^6) + (3*a^2*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 + 26*a^4*b^14 +
 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*
b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 -
a^6*b^10 - a^7*b^9) + (12*a^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2
)*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4
*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 -
 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 -
a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8
- a^6*b^7 - a^7*b^6)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b^4 - 5*a^2*b
^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^
4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*
a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 4*b
^4 - 5*a^2*b^2)*3i)/(d*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^
8*b^6 - a^10*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=188

$$-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] arctanh(sin(d*x+c))/b^3/d-a*(2*a^4-5*a^2*b^2+6*b^4)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*sec(d*x+
c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*tan(d*
x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.41, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^3*d) - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d
) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) - (a^2*(2*a^2 - 5*b^2)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[
c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(a^2-2ab\sec(c+dx)-2\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \int \frac{\sec(c+d}{}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \sec(c+}{b^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3 d} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\sec(}{2b\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.36, size = 194, normalized size = 1.03

$$\frac{-\frac{a^2b\sin(c+dx)\left(a\left(2a^2-5b^2\right)\cos(c+dx)+3b\left(a^2-2b^2\right)\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{2a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(3*b*(a^2 - 2*b^2) +
 a*(2*a^2 - 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*
Cos[c + d*x])^2))/(2*b^3*d)
```

fricas [B] time = 1.45, size = 1153, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*co
s(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
 b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + b^2)) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (
a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(a^6*b^2 - 3
*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*
x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-sin
(d*x + c) + 1) - 2*(3*a^6*b^2 - 9*a^4*b^4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^
7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10
)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*a^
5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*cos(d*x + c)^
2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))
*log(sin(d*x + c) + 1) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*
a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^
3*b^5 - a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (3*a^6*b^2 - 9*a^4*b^
4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b
^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7
+ 3*a^2*b^9 - b^11)*d)]
```

giac [A] time = 0.37, size = 347, normalized size = 1.85

$$\frac{\left(2\,a^{5}-5\,a^{3}b^{2}+6\,ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\sqrt{-a^{2}+b^{2}}}+\frac{2\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,a^{3}b^{2}\tan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2)) + (2*a^5*tan(1/2*d*x
 + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a^5*tan(1/2*d*x + 1/2*c) - 3*a^
4*b*tan(1/2*d*x + 1/2*c) + 5*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1
```

```
/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

maple [B] time = 0.43, size = 685, normalized size = 3.64

$$\frac{2a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2(a-b)\left(a^2+2ab+b^2\right)}-\frac{a^3\left(\tan^3\right.}{db\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^2/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3-2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*a^3/b/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*a^3/
b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/
((a-b)*(a+b))^(1/2))-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan
h(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^3*ln(tan(1/2*d*x+1/2*
c)-1)+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.50, size = 5078, normalized size = 27.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] - (atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
+ 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 +
b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)
 - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12
 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10
*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6
- a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*
b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b
^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a
^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - ((((8*(12*a*b^14 - 4*b^1
```

5 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)/((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 + (((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 - (16*(12*a*b^8 - 2*a^8*b + 4*a^9 + 24*a^2*b^7 - 34*a^3*b^6 - 26*a^4*b^5 + 36*a^5*b^4 + 13*a^6*b^3 - 18*a^7*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*2i)/(b^3*d) - ((tan(c/2 + (d*x)/2)^3*(a^3*b - 2*a^4 + 6*a^2*b^2))/((a*b^2 - b^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(a^3*b + 2*a^4 - 6*a^2*b^2))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*a*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6

```
*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4
*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4
+ 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a
^8*b^5 - a^10*b^3)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2
)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^
3)))/((16*(12*a*b^8 - 2*a^8*b + 4*a^9 + 24*a^2*b^7 - 34*a^3*b^6 - 26*a^4*b^
5 + 36*a^5*b^4 + 13*a^6*b^3 - 18*a^7*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*
a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((8*tan(c/2 + (d
*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*
a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^1
1 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - (a
*((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b
^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a
^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*a*ta
n(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8
*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^1
0 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11
+ 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^
9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*((a + b)^5*(a
- b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b
^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^
4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5
*a^8*b^5 - a^10*b^3)) - (a*((8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b
^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^
4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^
4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + (a*((8*(12*a*b^14 - 4*b^15 + 8*a^2
*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2
*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9
 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^5*(a -
 b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^
13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*
a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^
8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^
5*b^6 - a^6*b^5 - a^7*b^4)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5
*a^2*b^2))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^
10*b^3)))*((a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2))/(2*(b^13
 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))))*((a + b)
^5*(a - b)^5)^(1/2)*(2*a^4 + 6*b^4 - 5*a^2*b^2)*1i)/(d*(b^13 - 5*a^2*b^11 +
 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

```
[Out] (a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*a*(a
^2-4*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.23, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 4003, 12, 3831, 2659, 208}

$$\frac{\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^2) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
```

```
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(-2ab-\left(a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{b\left(a^2+2b^2\right.}{a+b\,\mathrm{se}}}{2b\left(a^2\right.}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)}{2(}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)}{2b}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)}{}$$

$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\,(a}{2b\left(a^2-\right.}$$

Mathematica [A] time = 0.44, size = 113, normalized size = 0.76

$$\frac{\frac{a\sin(c+dx)\left(a^2-3ab\cos(c+dx)-4b^2\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} - \frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(5/2) + (a*(a^2 - 4*b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a -
 b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2))/(2*d)
```

fricas [**A**] time = 0.52, size = 594, normalized size = 3.99

$$\left[\frac{\left(a^2b^2+2\,b^4+\left(a^4+2\,a^2b^2\right)\cos\left(dx+c\right)^2+2\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos}{a^2\,c}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^5 - 5*a^3*
b^2 + 4*a*b^4 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
, 1/2*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^5 - 5*a^3*b^2 + 4*a*b^4 - 3*(a^4*b
 - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

giac [**A**] time = 0.35, size = 253, normalized size = 1.70

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)\left(a^2+2\,b^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(a^2 + 2*b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2))/d
```

maple [**A**] time = 0.40, size = 184, normalized size = 1.23

$$\frac{-\frac{2\left(-\frac{(a+4b)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2}+\frac{\left(a^2+2b^2\right)\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a+4*b)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(a-4
*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1
/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.24, size = 204, normalized size = 1.37

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^2+4\,b\,a\right)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a\,b-a^2\right)}{(a+b)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,\left(a^2-2\right.}{2\,\sqrt{a+b}\,(a-b)^{5/2}}\right.}{d\,(a+b)^{5/2}\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*x
)/2)*(4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d
*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 +
 b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a
+ b)^(1/2)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{(a^2+2b^2)\tan(c+dx)}{2d(a^2-b^2)^2(a+b\sec(c+dx))}+\frac{a\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] -3*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b
)^(5/2)/d+1/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*(a^2+2*b^2)*t
an(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3836, 4003, 12, 3831, 2659, 208}

$$\frac{(a^2+2b^2)\tan(c+dx)}{2d(a^2-b^2)^2(a+b\sec(c+dx))}+\frac{a\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)
*(a + b)^(5/2)*d) + (a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((a^2 + 2*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
```

```
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2b+a\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{3ab\sec(c+dx)}{a+b\sec(c+dx)}}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3ab)\int \frac{\sec(c}{a+b\sec}}{2\left(a^2-b\right.}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3a)\int \frac{1}{1+\frac{a\cos(c}{b}}}{2\left(a^2-b^2\right.}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3a)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right.}{2\left(a^2-b^2\right)^2 d(}$$

Mathematica [A] time = 0.40, size = 115, normalized size = 0.86

$$\frac{\frac{\sin(c+dx)\left(a\left(2a^2+b^2\right)\cos(c+dx)+b\left(a^2+2b^2\right)\right)}{(a\cos(c+dx)+b)^2} + \frac{6ab\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b + a
*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

fricas [B] time = 0.53, size = 565, normalized size = 4.22

$$\left[\frac{3\left(a^3b\cos(dx+c)^2 + 2a^2b^2\cos(dx+c) + ab^3\right)\sqrt{a^2-b^2}\,\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\,\mathrm{s}}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right.}{4\left(\left(a^8-3a^6b^2+3a^4b^4-a^2b^6\right)d\cos(dx+c)^2 + 2\left(a^7b-3a^5b^3+3a^3b^5-ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^3*b*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(a^2 -
b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2)) + 2*(a^4*b + a^2*b^3 - 2*b^5 + (2*a^5 - a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b
^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x
+ c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), -1/2*(3*(a^3*b*cos(d*x +
 c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4*b + a^2*b^3
- 2*b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
]
```

giac [B] time = 0.32, size = 277, normalized size = 2.07

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan}{\left(a^4-2a^2b^2+b^4\right)\left(a\,\mathrm{ta}\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/2
*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

maple [A] time = 0.35, size = 195, normalized size = 1.46

$$\frac{\frac{-\frac{\left(2a^2+ab+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(2a^2-ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2}-\frac{3ab\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.37, size = 210, normalized size = 1.57

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(2a^2+ab+2b^2\right)}{(a+b)^2(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2a^2-ab+2b^2\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{d\left(2ab-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-2ab+b^2\right)+a^2+b^2\right)}-\frac{3ab\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2 ...}{2\sqrt{a+b}\,(...}\right)}{d\,(a+b)^{5/2}\,(a-b ...}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) - (tan(
c/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*
a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*
a*b + b^2) + a^2 + b^2)) - (3*a*b*atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^
2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(a - b
)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

```
[Out] (2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)
/(a+b)^(5/2)/d-1/2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/2*a*b*tan(
d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3833, 4003, 12, 3831, 2659, 208}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (b*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (3*a*b*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
```

```
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a+b\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\int \frac{\left(2a^2+b^2\right)\sec}{a+b\sec(c+}}{2\left(a^2-\right.}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\int}{2\left(a^2\right.}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\int}{2b\left(a^2\right.}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\mathrm{S}}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3}{2\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.47, size = 115, normalized size = 0.86

$$\frac{\frac{b\sin(c+dx)\left(\left(b^2-4a^2\right)\cos(c+dx)-3ab\right)}{(a\cos(c+dx)+b)^2} - \frac{2\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + (b*(-3*a*b + (-4*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b
+ a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

fricas [B] time = 0.51, size = 595, normalized size = 4.47

$$\left[\frac{\left(2a^2b^2+b^4+\left(2a^4+a^2b^2\right)\cos(dx+c)^2+2\left(2a^3b+ab^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx}{a^2\cos(a}\right.}{4\left(\left(a^8-3a^6b^2+3a^4b^4-a^2b^6\right)d\cos(dx+c)^2+2\left(a^7b-3a^5b^3+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b + a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(3*a^3*b^2 -
3*a*b^4 + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
d), 1/2*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b +
 a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*a^3*b^2 - 3*a*b^4 + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
 a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
s(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

giac [B] time = 0.33, size = 254, normalized size = 1.91

$$-\frac{\frac{\left(\pi \left\lfloor \frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}-\frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(2*a^2 + b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b*tan(1/2*
d*x + 1/2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2))/d
```

maple [A] time = 0.38, size = 186, normalized size = 1.40

$$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2}+\frac{(2a^2+b^2)\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*a
-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1
/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.18, size = 204, normalized size = 1.53

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(b^2+4ab\right)}{(a+b)^2(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(4ab-b^2\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{d\left(2ab-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-2ab+b^2\right)+a^2+b^2\right)}+\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)\left(a^2-2\right.}{2\sqrt{a+b}\,(a-b)^{5/2}}\right.}{d\,(a+b)^{5/2}\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*x
)/2)*(4*a*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d
*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 +
 b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a
+ b)^(1/2)*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$\frac{x}{a^3}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+
b*sec(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+}{2ad\left(a^2-b^2\right)(a+b\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)+2ab\sec(c+dx)-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{2\left(a^2-b^2\right)^2}{}}{2}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(b(6}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(6a^4}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(6a^4}{}$$

$$= \frac{x}{a^3} - \frac{b\left(6a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+}$$

Mathematica [A] time = 0.76, size = 205, normalized size = 1.18

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{3ab^2\left(2a^2-b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{2b\left(6a^4-5a^2b^2+2b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}\right.}{2a^3d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(2*(c + d*x)*(b + a*Cos[c + d*x])^2 +
(2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
```

```
^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sin[c + d*x])
/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[c + d
*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sec[c + d*x])^3)
```

fricas [B] time = 0.56, size = 919, normalized size = 5.31

$$\left[\frac{4\left(a^8-3a^6b^2+3a^4b^4-a^2b^6\right)dx\cos(dx+c)^2+8\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)dx\cos(dx+c)+4\left(a^6b^2-3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 8*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*x + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b - 5*a
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(5*a^5*b^3 - 7*a^3*b
^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x +
c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 +
3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x
*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c
) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (6*a^4*b^3 - 5*a^2*b^5
+ 2*b^7 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 -
 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (5*a^5*b^3 - 7*a^3*b^
5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x + c
))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3
*a^5*b^6 - a^3*b^8)*d)]
```

giac [B] time = 0.21, size = 322, normalized size = 1.86

$$\frac{\frac{\left(6a^4b-5a^2b^3+2b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}-\frac{6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^4\tan}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) - (6*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) -
5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x + 1/2*c) + 2*b^5*tan(1
/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (d*x + c)/a^3)/d
```

maple [B] time = 0.42, size = 664, normalized size = 3.84

$$-\frac{6b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2(a-b)\left(a^2+2ab+b^2\right)}-\frac{b^3\left(\tan\right.}{da\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d/a*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+2/d/a^2*b^4/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d/a*b^3/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/
2*c)-2/d/a^2*b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+
b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d/a*b^3/(
a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a
-b)*(a+b))^(1/2))-2/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arcta
nh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^3*arctan(tan(1/2*d*x
+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.22, size = 5090, normalized size = 29.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4
*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b +
a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
 - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32
*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b
^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3
*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b -
 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*
a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*
a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3 - ((((8*(12*a^14*b - 4*a^
15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*
b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8
*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (d*x)/2)*(8*a^15*b
 - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11
*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^
4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3
- (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 +
 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^
2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3
- 3*a^9*b^2))/a^3)/((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18
*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)
)/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 -
```

3*a^11*b^2) - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*
a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^
3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3
*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10
 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a
^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 -
a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 + (((((8*
(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a
^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7
- a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (
d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a
^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a
^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^
9*b^2)))*1i)/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^
10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^
7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7
*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 + (16*(12*a^8*b - 2*a*b^8 + 4*b^9 -
18*a^2*b^7 + 13*a^3*b^6 + 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2
))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3
- 3*a^11*b^2))))/(a^3*d) + ((tan(c/2 + (d*x)/2)^3*(a*b^3 - 2*b^4 + 6*a^2*b^
2))/((a^2*b - a^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(a*b^3 + 2*b^4 - 6*a^2*
b^2))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2
*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) +
 (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 -
32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3
 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4
- 3*a^8*b^3 - 3*a^9*b^2) + (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b
^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^
13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^1
0*b^3 - 3*a^11*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(
6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8
- 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^
14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b
^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3
- 3*a^9*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*
(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*((a
+ b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10
 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + (b*((8*tan(c/2 + (d
x)/2)(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*
a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^1
1 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (b
((8(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 +
 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6
*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*b*ta
n(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8
*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 +
48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 +
5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7
- a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*((a + b)^5*(a
- b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8
- 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^
4 + 2*b^4 - 5*a^2*b^2)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 1
0*a^9*b^4 - 5*a^11*b^2)))/((16*(12*a^8*b - 2*a*b^8 + 4*b^9 - 18*a^2*b^7 + 1
3*a^3*b^6 + 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2))/(a^12*b + a
^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
+ (b*((8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b
^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^
8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*
b^3 - 3*a^9*b^2) + (b*((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*

```
a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))
/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 -
3*a^11*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 +
2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9
*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))
/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^1
0*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*
b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 -
a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*((a + b)^5*(
a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^
8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) - (b*((8*tan(c/2 + (d*x)/2)*(4*a
^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 4
8*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7
 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (b*((8*(12*a^
14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^
5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*
b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*b*tan(c/2 + (d*
x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^15*b - 8
*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5
 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 -
 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 +
 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*((a + b)^5*(a - b)^5)^(1/
2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^
6 + 10*a^9*b^4 - 5*a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 -
 5*a^2*b^2))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*
a^11*b^2))))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*1i)/(d
*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3), x)
```

3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=223

$$-\frac{3bx}{a^4}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tanh^{-1}}{a^4d(a-b)^{5/2}(a+}$$

```
[Out] -3*b*x/a^4+3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/
2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(2*a^4-11*a^2*b^2+6*b^4
)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*
x+c))^2+3/2*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.62, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-11a^2b^2+2a^4+6b^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{3b^2\left(2a^2-b^2\right)\text{s}}{2a^2d\left(a^2-b^2\right)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*x)/a^4 + (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4 -
 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sin[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*b^2*(2*a^2 - b^2)*Sin[
c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2+3b^2+2ab\sec(c+dx)-2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2} d}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\cos(c+}{}}{}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\,(}{2a^2\left(a^2-\right.}$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

$$= -\frac{3bx}{a^4} + \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(2a^4-11a^2b^2+6b^4\right.}{2a^3\left(a^2-b\right.}$$

Mathematica [A] time = 0.88, size = 229, normalized size = 1.03

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(-\frac{ab^3\left(8a^2-5b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{6b^2\left(4a^4-5a^2b^2+2b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right)}{\left(a^2-b^2\right)^{5/2}}\right.}{2a^4d(a+b\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(-6*b*(c + d*x)*(b + a*Cos[c + d*x])^2
 - (6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^4*Sin[c +
d*x])/((a - b)*(a + b)) - (a*b^3*(8*a^2 - 5*b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^2*(a + b)^2) + 2*a*(b + a*Cos[c + d*x])^2*Sin[c + d*x]))/
(2*a^4*d*(a + b*Sec[c + d*x])^3)
```

fricas [B] time = 0.60, size = 1037, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 24
*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 12*(a^6*b^3 -
 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a
```

```
^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 +
 2*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*
b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7*b
^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a
^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*co
s(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(
d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) +
(a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8)*d*x*cos(d*x + c) + 6*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)
*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^
6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sqrt(
-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^
7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^
4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*
b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b
^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

giac [A] time = 0.31, size = 357, normalized size = 1.60

$$\frac{3\left(4a^4b^2-5a^2b^4+2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{-a^2+b^2}}-\frac{8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5ab^5\,\mathrm{ta}}{}$$

$$-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) - (8*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1/2*d
*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b^3*tan(1/2*d*x + 1/2*
c) - 7*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*a*b^5*tan(1/2*d*x + 1/2*c) + 4*b^6*
tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*(d*x + c)*b/a^4 - 2*tan(1/2*d*
x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

maple [B] time = 0.68, size = 702, normalized size = 3.15

$$\frac{8b^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2(a-b)\left(a^2+2ab+b^2\right)}+\frac{b^4\left(\tan^3\right.}{d\,a^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 8/d/a*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d/a^2*b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-4/d*b^5/a
^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3-8/d/a*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d/a^2*b^4/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2
```

```
*d*x+1/2*c)+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+12/d*b^2/(a^4-2*a^2*b^2+b^4)/((
a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-15/
d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^
(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^3*tan(1/2
*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^4*b*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.00, size = 5338, normalized size = 23.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5 + 6*b^5 - 12*a^2*b^3 - 4*a^
3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a*b^
4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2))/((a^3*b - a^4)*(a +
b)^2) + (2*tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2))/(
a*(a^2*b - a^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b
 - a^2 + 3*b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - ta
n(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) - (6*b*atan(((3*b*((8*tan(c/2 +
(d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 -
432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10
*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*
b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^
8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*
a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 -
3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8
*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*
b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^
6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a^4))/a^4 + (3*b
*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 +
 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^
9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^
9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b
^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 1
2*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5
+ 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*tan(c/2 + (d*x)/2)*(8*a^17*b -
 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13
*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 - a
^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)/a
^4))/a^4)/((48*(36*b^12 - 18*a*b^11 - 162*a^2*b^10 + 81*a^3*b^9 + 288*a^4*b
^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*a^8*b^4))/(a^15*b + a^16 -
 a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) -
(b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9
 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*
a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*
```

a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9
*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 -
12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^
5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b
- 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^
13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 -
a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)
/a^4)*3i)/a^4 + (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^
10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 +
36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6
+ 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((24*(4*a^17*b - 4*
a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5
+ 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10
*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*tan(c/2 + (d
x)/2)(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*
a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*
(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3
*a^11*b^2)))*3i)/a^4)*3i)/a^4)))/(a^4*d) - (b^2*atan(((b^2*((a + b)^5*(a -
b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 28
8*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*
b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8
*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (3*b^2*((24*(4*a^17*b - 4*a^8
*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 +
26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^
6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (12*b^2*tan(c/2 +
(d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b
- 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^1
3*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*
b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7
*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b)^5*(a - b
)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 1
0*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i)/(2*
(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b
^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11
- 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 28
8*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^
7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b^2*((2
4*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b
^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16
- a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) +
(12*b^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*
a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 4
8*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14
- a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a
^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2))
)*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b
^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4 + 2*b^4 -
5*a^2*b^2)*3i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4
- 5*a^12*b^2)))/((48*(36*b^12 - 18*a*b^11 - 162*a^2*b^10 + 81*a^3*b^9 + 288
*a^4*b^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*a^8*b^4))/(a^15*b +
a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b
^2) - (3*b^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*b^12 -
72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^
6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^1
3 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) +
(3*b^2*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 -
32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2))/(a^15
*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a
^14*b^2) - (12*b^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 +

```
2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^
11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2
))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(
a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*
a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^
14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(4*a^4
 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^
10*b^4 - 5*a^12*b^2)) + (3*b^2*((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d
*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 43
2*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b
^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^
3 - 3*a^11*b^2) - (3*b^2*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*
b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 -
8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4
- 3*a^13*b^3 - 3*a^14*b^2) + (12*b^2*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^
5)^(1/2)*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 3
2*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^1
5*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b
^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4
 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(4*a^4 + 2*b^4 -
5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*
a^12*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 -
10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))))*((a + b)^5*(a - b)^5)^(1/2)*(4*a^
4 + 2*b^4 - 5*a^2*b^2)*3i)/(d*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 1
0*a^10*b^4 - 5*a^12*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=296

$$\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{x\left(a^2+12b^2\right)}{2a^5}-\frac{3b\left(2a^4-7a^2b^2+4b^4\right)}{2a^4d\left(a^2-b^2\right.}$$

```
[Out] 1/2*(a^2+12*b^2)*x/a^5-b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*t
an(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d-3/2*b*(2*a^4-7
*a^2*b^2+4*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-10*a^2*b^2+6*b^4)*cos
(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b
^2)/d/(a+b*sec(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a
^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 1.00, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4+6b^4\right)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)^2}-\frac{b^3\left(-29a^2b^2+20a^4+12b\right.}{a^5d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 12*b^2)*x)/(2*a^5) - (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)
*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d
) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^
2)^2*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-2a^2+4b^2+2ab\sec(c+dx)-3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(7a^2-4b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \int \frac{\cos^2(}{}$$

$$= \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b}{}$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+d}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+d}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+d}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b\left(2a^4-7a\right.}{2a^4}$$

Mathematica [A] time = 2.04, size = 199, normalized size = 0.67

$$\frac{2\left(a^2+12b^2\right)(c+dx) + \frac{2ab^4\left(10a^2-7b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + a^2\sin(2(c+dx)) + \frac{4b^3\left(20a^4-29a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{}{(b}}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + 12*b^2)*(c + d*x) + (4*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh
[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*b*S
in[c + d*x] + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])
^2) + (2*a*b^4*(10*a^2 - 7*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*C
os[c + d*x])) + a^2*Sin[2*(c + d*x)])/(4*a^5*d)
```

fricas [A] time = 0.61, size = 1158, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(d
*x + c)^2 + 4*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*x*
```

```
cos(d*x + c) + 2*(a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*
d*x + (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2
*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^7*b^3 - 27*a^5*b^5 + 33*a^
3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3
+ 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2
- 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13
- 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b
^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^
6 - a^5*b^8)*d), 1/2*((a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*
b^8)*d*x*cos(d*x + c)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 -
12*a*b^9)*d*x*cos(d*x + c) + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8
 - 12*b^10)*d*x - (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*
b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) - (6*a^7*b^3 - 27*a^5*b^5 + 33*a^3*b^7 - 12*a*
b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3 + 4*(a^9*b -
3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2 - 43*a^6*b^4
+ 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2
+ 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^
5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*
d)]
```

giac [B] time = 0.83, size = 1723, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*b
^6)*sqrt(-a^2 + b^2)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^15 - a
^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^6
- 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11)*sqrt(
-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2
*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b - 2*a^
7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3
 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^8*b - 2*
a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/((a^9 - 2*a^7*b^2 + a^5*b^4)^2*
(a^2 - 2*a*b + b^2) + (a^10*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 - a^6*b^5 -
 2*a^5*b^6 + a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (a^15 - a^14*b + 8*
a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^6 - 158*a^8*
b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11 + a^6*abs(a^9 -
2*a^7*b^2 + a^5*b^4) - a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^4*b^2*ab
s(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^3*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) -
23*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*a*b^5*abs(a^9 - 2*a^7*b^2 + a
^5*b^4) + 12*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(pi*floor(1/2*(d*x + c)/pi
 + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 -
sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b
 - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^
5)^2))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/(a^8*b*
abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 2*a^6*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) +
 a^4*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - (a^9 - 2*a^7*b^2 + a^5*b^4)^2) +
2*(a^7*tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2
*tan(1/2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan
(1/2*d*x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*
d*x + 1/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*a^7*tan(1/2*d*x + 1/2*c)
^5 - 4*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26
```

```
*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*a^
2*b^5*tan(1/2*d*x + 1/2*c)^5 - 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*b^7*tan
(1/2*d*x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*x +
1/2*c)^3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x + 1/2*
c)^3 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^
3 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3 - a^7*t
an(1/2*d*x + 1/2*c) + 4*a^6*b*tan(1/2*d*x + 1/2*c) + 13*a^5*b^2*tan(1/2*d*x
 + 1/2*c) - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*a^3*b^4*tan(1/2*d*x + 1/2*c
) - 17*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*a*b^6*tan(1/2*d*x + 1/2*c) + 12*b^
7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c
)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

maple [**B**] time = 0.61, size = 827, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -10/d/a^2*b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(
a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*b
^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d/a^2*b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*b^5/a^3/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-20/d/a*b^3/(a^4-2*a^2*b^
2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2))+29/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a-
b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^4/(1+tan(1/2*d*x+1
/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
2*d*x+1/2*c)*b+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+1/d/a^
3*arctan(tan(1/2*d*x+1/2*c))+12/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 9.29, size = 5950, normalized size = 20.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1
104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 +
 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^
```

12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a
^12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 + 24
*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 27
6*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^
12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4
*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9
 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32
*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b
^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i)*1
i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^
14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6
*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 +
 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4
 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^1
1 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^
6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^1
9 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2
) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^
11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^
4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*
a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*
12i)*1i)/(2*a^5))/((8*(1728*b^15 - 864*a*b^14 - 7344*a^2*b^13 + 3456*a^3*b^
12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^6*b^9 + 2326*a^7*b^8 + 1314*a^
8*b^7 - 11*a^9*b^6 + 411*a^10*b^5 - 20*a^11*b^4 + 20*a^12*b^3))/(a^18*b + a
^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b
^2) - (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 11
04*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 +
872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^1
2*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^
12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 + 24*
a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276
*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^1
2*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*
tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9
+ 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*
a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^
5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i))/(
2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 -
 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8
 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*
a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3
*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11 +
24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 +
276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 -
a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) +
(4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b
^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 -
32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10
*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i + b^2*12i)
)/(2*a^5)))*(a^2*1i + b^2*12i)*1i)/(a^5*d) - ((tan(c/2 + (d*x)/2)^3*(18*a*b
^6 - 4*a^6*b + 3*a^7 + 36*b^7 - 67*a^2*b^5 - 29*a^3*b^4 + 26*a^4*b^3 + 5*a^
5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^5*(18*a
*b^6 + 4*a^6*b + 3*a^7 - 36*b^7 + 67*a^2*b^5 - 29*a^3*b^4 - 26*a^4*b^3 + 5*
a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^7*(6*
a*b^5 + 5*a^5*b + a^6 - 12*b^6 + 23*a^2*b^4 - 10*a^3*b^3 - 8*a^4*b^2))/((a^
4*b - a^5)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*a*b^5 + 5*a^5*b - a^6 + 12*b
^6 - 23*a^2*b^4 - 10*a^3*b^3 + 8*a^4*b^2))/((a + b)*(a^6 - 2*a^5*b + a^4*b^
2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + tan(c/2 + (d*x)/2)^
2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/

2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (b^3*atan(((b^3*((8*tan(c/2 + (d*x
)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^
11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6
- 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 -
a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b
^3*((a + b)^5*(a - b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 +
212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 -
120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13
*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*b^3*tan(c/2
+ (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^
19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 +
48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 +
5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^
7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4
+ 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a
^11*b^4 - 5*a^13*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a
^2*b^2)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*
a^13*b^2)) + (b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 28
8*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827
*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b
^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11
*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (b^3*((a + b)^5*(a - b)^5)^(1/2)*((4*(4*a
^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*
b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^
2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16
*b^3 - 3*a^17*b^2) + (4*b^3*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*
(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^
12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^
3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 -
5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 -
3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*
b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*((a + b)^5*(a -
b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7
*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))/((8*(1728*b^15 - 864*a*b^14
- 7344*a^2*b^13 + 3456*a^3*b^12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^
6*b^9 + 2326*a^7*b^8 + 1314*a^8*b^7 - 11*a^9*b^6 + 411*a^10*b^5 - 20*a^11*b
^4 + 20*a^12*b^3))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^
15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (b^3*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a
^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^
10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 +
74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^
6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b^3*((a + b)^5*(a
- b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 1
00*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 8
0*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^
5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*b^3*tan(c/2 + (d*x)/2)*((a +
b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*(8*a^19*b - 8*a^10*b^
10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 3
2*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*
a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*
a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(20*a^4 + 12*b^4 - 29*a^
2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13
b^2)))((a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^1
5 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^3*(
(8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b
^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*
b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2))/
(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 -
3*a^13*b^2) - (b^3*((a + b)^5*(a - b)^5)^(1/2)*((4*(4*a^21 - 48*a^10*b^11

```
+ 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6
+ 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^19
- a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2)
+ (4*b^3*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 -
29*a^2*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^
7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((
a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*
b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^1
3*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 -
10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(20*a^
4 + 12*b^4 - 29*a^2*b^2))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10
*a^11*b^4 - 5*a^13*b^2))))*((a + b)^5*(a - b)^5)^(1/2)*(20*a^4 + 12*b^4 - 2
9*a^2*b^2)*1i)/(d*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 -
 5*a^13*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=316

$$-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+$$

```
[Out] -4*a*arctanh(sin(d*x+c))/b^5/d+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*arc
tanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2
)/d+1/6*(12*a^4-23*a^2*b^2+6*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*sec(
d*x+c)^3*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-1/6*a^2*(4*a^2-9*b^2)*
sec(d*x+c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*a^3*(4*a^4
-11*a^2*b^2+12*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 1.11, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3845, 4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9\right.}{6b^2d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-4*a*ArcTanh[Sin[c + d*x]])/(b^5*d) + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^
4 - 20*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*b^5*(a + b)^(7/2)*d) + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6
*b^4*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^3) - (a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x
])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^
2 + 12*b^4)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(3a^2-3ab\sec(c+dx)-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{\int \text{se}}{}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{a^3\,(}{2b}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-\right.}{6b^2\left(a^2-\right.}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-\right.}{6b^2\left(a^2-\right.}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\,\mathrm{t}}{3b\left(a^2-b^2\right)d(a+b}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d} - \frac{a^2\sec^3(c+dx)\,\mathrm{t}}{3b\left(a^2-b^2\right)d(a+b}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

Mathematica [A] time = 6.23, size = 416, normalized size = 1.32

$$-\frac{a^3\sin(c+dx)}{3b^2d(b-a)(a+b)(a\cos(c+dx)+b)^3} + \frac{6a^5\sin(c+dx)-11a^3b^2\sin(c+dx)}{6b^3d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{-18a^7\sin(c+dx)+50a^5b^2\,\mathrm{si}}{6b^4d(b-a)^3(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((a^2*(-8*a^6 + 28*a^4*b^2 - 35*a^2*b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^3*d)) + (4*
a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^5*d) - (4*a*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])/(b^5*d) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) - (a^3*Sin[c + d*x])/(3*b^2*(-a + b)*(a + b)*d*(b + a*Cos[c +
 d*x])^3) + (6*a^5*Sin[c + d*x] - 11*a^3*b^2*Sin[c + d*x])/(6*b^3*(-a + b)^
2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (-18*a^7*Sin[c + d*x] + 50*a^5*b^2*
Sin[c + d*x] - 47*a^3*b^4*Sin[c + d*x])/(6*b^4*(-a + b)^3*(a + b)^3*d*(b +
a*Cos[c + d*x]))
```

fricas [B] time = 3.03, size = 2058, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6)*cos(d*x + c)^4 +
3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c)^3 + 3*(8*a
^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + (8*a^8*b^3
- 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((
2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*co
s(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + b^2)) - 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^
8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9
)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^
10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)
*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 -
 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 -
4*a^5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 -
 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 -
4*a^3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(6*a^8*b^4 - 2
4*a^6*b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 1
33*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a
^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^
3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(
d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos
(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)
*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^
15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^
16)*d*cos(d*x + c)), 1/6*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6
)*cos(d*x + c)^4 + 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(
d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x +
 c)^2 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sq
rt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*c
os(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*co
s(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*
cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos
(d*x + c))*log(sin(d*x + c) + 1) + 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a
^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^
5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a
^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^
3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*a^8*b^4 - 24*a^6*
b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 133*a^7
*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a^8*b^4
 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^3 - 16
9*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(d*x +
c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos(d*x +
 c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos
(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*
cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*
cos(d*x + c))]
```

giac [A] time = 0.39, size = 592, normalized size = 1.87

$$-\frac{\frac{3\left(8a^{8}-28a^{6}b^{2}+35a^{4}b^{4}-20a^{2}b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{6}b^{5}-3a^{4}b^{7}+3a^{2}b^{9}-b^{11}\right)\sqrt{-a^{2}+b^{2}}}+\frac{18a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-42a^{8}b\tan\left(\frac{1}{2}dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*s
qrt(-a^2 + b^2)) + (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x +
1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/
2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*a^9*tan(1/2*d*x + 1/2*c)^3 +
152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 1
20*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*
tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b
^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 - 12*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/b^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^
4))/d
```

maple [B] time = 0.43, size = 1481, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*a^6/b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-5/d*a^4/b/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+
b^3)*tan(1/2*d*x+1/2*c)^5-20/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+12/d*a^7
/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-116/3/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3+40/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^7/b^4/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+18/d*a^5/b^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)+5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-20/d*a^
3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-28
/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+35/d*a^4/b/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1
/2))-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^4/(tan(1/2*d*x+1/2*c)-1)
+4/d*a/b^5*ln(tan(1/2*d*x+1/2*c)-1)-1/d/b^4/(tan(1/2*d*x+1/2*c)+1)-4/d*a/b^
5*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 10.36, size = 7476, normalized size = 23.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*b^
5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3) +
(tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*b^5
 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2) - (
tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26*a^3*
b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2 + (d*x)/
2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 11*a^4*b
^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2
+ (d*x)/2)^4*(6*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*
b^3) - tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2
 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((8*tan(c/2 +
 (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12
 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7
 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*
b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14
- 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8
) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*
b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b
^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b
^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 1
0*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (32*a*tan(c
/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*
b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10
*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b
^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b
^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5)*
4i)/b^5 + (a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 -
128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2
025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4
+ 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 +
10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*
b^10 - a^10*b^9 - a^11*b^8) + (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^
21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16
+ 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 -
 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*
a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13
- a^11*b^12) + (32*a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^2
1 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16
 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11
 - 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^1
5 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^1
0*b^9 - a^11*b^8))))/b^5)*4i)/b^5)/((32*(128*a^16 - 64*a^15*b + 320*a^4*b^1
2 + 480*a^5*b^11 - 1520*a^6*b^10 - 1280*a^7*b^9 + 3088*a^8*b^8 + 1602*a^9*b
^7 - 3472*a^10*b^6 - 1088*a^11*b^5 + 2288*a^12*b^4 + 400*a^13*b^3 - 832*a^1
4*b^2))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^1
8 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*
```

b^12) - (4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (32*a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5))/b^5 + (4*a*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (32*a*tan(c/2 + (d*x)/2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5))/b^5))*8i)/(b^5*d) + (a^2*atan(((a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a^2*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*1i)/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)) + (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a^2*((16*(8*a*b^23 - 20*a^2*b^2

2 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a ^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7) ^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 16 0*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8 *a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2 *b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*((a + b)^7*(a - b)^7)^(1 /2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21* a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*1 i)/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^1 0*b^9 + 7*a^12*b^7 - a^14*b^5)))/((32*(128*a^16 - 64*a^15*b + 320*a^4*b^12 + 480*a^5*b^11 - 1520*a^6*b^10 - 1280*a^7*b^9 + 3088*a^8*b^8 + 1602*a^9*b^7 - 3472*a^10*b^6 - 1088*a^11*b^5 + 2288*a^12*b^4 + 400*a^13*b^3 - 832*a^14* b^2))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^ 12) - (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 12 8*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 202 5*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10 *a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^ 10 - a^10*b^9 - a^11*b^8) - (a^2*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8 *a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^ 5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 4 8*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 12 0*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8* a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b ^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20 *b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a ^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^ 2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)) + (a^2*((8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2 *b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7 *b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a ^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3 *b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a^2*((16*(8*a*b^23 - 20*a^2*b^22 - 3 6*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a ^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^1 3*b^11 - 8*a^14*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^ 19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^ 10*b^13 - a^11*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2)*(8*a*b^23 - 8*a^2*b^22 - 48*a^ 3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8 *b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13 *b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35* a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17

```
 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a
^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*((a + b)^7*(a - b)^7)^(1/2)*(
8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b
^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*((
a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*a^2*b^4 - 28*a^4*b^2))/(2*(b
^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 +
7*a^12*b^7 - a^14*b^5))))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 20*b^6 + 35*
a^2*b^4 - 28*a^4*b^2)*1i)/(d*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13
 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*sec(c + d*x))**4, x)
```

3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=259

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{6b^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/b^4/d-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*a^
2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a^3*(3*a^2-8
*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*a^2*(9*a^4-28*a^2
*b^2+34*b^4)*tan(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.75, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a^3\left(3a^2-\right.}{6b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*
ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
+ b)^(7/2)*d) - (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^3) + (a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2
*d*(a + b*Sec[c + d*x])^2) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x
])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-3ab\sec(c+dx)-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \int \frac{\sec}{}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{a^2\left(9\right.}{6b^3}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{a^2\left(9\right.}{6b^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan}{6b^3\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan}{6b^3\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} -$$

Mathematica [A] time = 4.14, size = 250, normalized size = 0.97

$$\frac{-\frac{a^2b\sin(c+dx)\left(11a^4b^2-32a^2b^4+a^2\left(6a^4-17a^2b^2+26b^4\right)\cos^2(c+dx)+15ab\left(a^4-3a^2b^2+4b^4\right)\cos(c+dx)+36b^6\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3} + \frac{6a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-}}{\left(a^2-b^2\right)^{7/2}}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(11*a^4*
b^2 - 32*a^2*b^4 + 36*b^6 + 15*a*b*(a^4 - 3*a^2*b^2 + 4*b^4)*Cos[c + d*x] +
 a^2*(6*a^4 - 17*a^2*b^2 + 26*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*b^4*d)
```

fricas [B] time = 3.20, size = 1822, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9 + (2*a^10 - 7*a^8*b^2
 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b
^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6 - 8*a
```

```
^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*(a^8*b^3 -
 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*(a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^
5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4
*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3
*b^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(11*a^8*b^3 - 43*a^
6*b^5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*
a^4*b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10
 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4
*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 -
4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4
*a^2*b^13 + b^15)*d), -1/6*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9
+ (2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b
- 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*
b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(a^8*b^3 - 4
*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^8*b^3 - 4*a^6*
b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b
^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b
^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*a^8*b^3 - 43*a^6*b^
5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*a^4*
b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a
^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^1
1 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^
3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2
*b^13 + b^15)*d)]
```

giac [B] time = 0.47, size = 559, normalized size = 2.16

$$\frac{3\left(2a^7-7a^5b^2+8a^3b^4-8ab^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6b^4-3a^4b^6+3a^2b^8-b^{10}\right)\sqrt{-a^2+b^2}}+\frac{6a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15a^7b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^6b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a
^2 + b^2)) + (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^
5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a
^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 12*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*a^6*b^2*
tan(1/2*d*x + 1/2*c)^3 - 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 72*a^2*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x +
1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*
```

```
b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*log(abs(tan(1/
2*d*x + 1/2*c) + 1))/b^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4)/d
```

maple [B] time = 0.36, size = 1429, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5-6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+4/d*a^3/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-4/d*a^6/b^3/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3-24/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a
*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+2/d*a^6/b^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*a^4/b/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)-4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+12/d*a^2*b/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+
b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+7/d*a^5/b^2
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*
(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*a*b^
2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2
*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b^4*ln(
tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 12.45, size = 7222, normalized size = 27.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^4),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b^4 + 4*a^3*b^3 - 6*a^4*b^
2))/((a*b^3 - b^4)*(a + b)^3) - (4*tan(c/2 + (d*x)/2)^3*(3*a^6 + 18*a^2*b^4
 - 11*a^4*b^2))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*b^3)) + (tan(c/2 + (d*x)/
2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4*b^2))/((a + b)*(3*a*b^5
- b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
- 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3)
 + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3))) - (atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^
18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13
+ 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*
b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*
b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (8*t
an(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*
a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*
a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a
*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6
*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 -
(8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12
+ 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164
*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16
 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11
 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4 - ((
((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5
*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^
11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 -
 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8
*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*tan(c/2 + (d*x)/2)*(8*a*b^2
1 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 -
160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 +
 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15
- 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^
8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 + (8*tan(c/2 + (d*x)/2)*(8*a
^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^1
0 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 1
17*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a
^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9
 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4)/((((8*(16*a*b^20 - 4*b^21 + 1
2*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a
^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*
b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 +
 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b
^10 - a^11*b^9) - (8*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^1
9 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14
 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9
- 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13
+ 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^
7 - a^11*b^6)))/b^4 - (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 +
 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b
^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 -
 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*
a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a
^11*b^6))/b^4 + (((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a
^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*
b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^
20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10
*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*tan(c/2 +
(d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17
- 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12
 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b
```

^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 1
0*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))/b^4 + (8*tan(c/
2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*
b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6
- 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 -
5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))/b^4 - (16*(16*a*b^12
- 2*a^12*b + 4*a^13 + 48*a^2*b^11 - 64*a^3*b^10 - 64*a^4*b^9 + 110*a^5*b^8
+ 66*a^6*b^7 - 110*a^7*b^6 - 34*a^8*b^5 + 70*a^9*b^4 + 11*a^10*b^3 - 26*a^1
1*b^2))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^1
5 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*
b^9)))*2i)/(b^4*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b
- 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9
+ 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 4
8*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^
4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 -
a^10*b^7 - a^11*b^6) - (a*((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^
18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13
+ 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*
b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*
b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (4*a
*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4
- 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b
^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*
b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a
^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^
6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a
^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^
11*b^6)))*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^
2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^
10*b^8 + 7*a^12*b^6 - a^14*b^4)))*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^
6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6
*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)) + (a*((8*tan(c/
2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*
b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6
- 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 -
5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (a*((8*(16*a*b^20 -
4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^
15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10
- 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*
a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^1
1 - a^10*b^10 - a^11*b^9) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(
1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^
3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8
*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13
*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^
8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 -
5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8
*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*((a + b)^7*(a - b)^7)^(1/2)*(2*a^
6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 3
5*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*((a + b)^
7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(2*(b^18 - 7
*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*
b^6 - a^14*b^4)))/((16*(16*a*b^12 - 2*a^12*b + 4*a^13 + 48*a^2*b^11 - 64*a^
3*b^10 - 64*a^4*b^9 + 110*a^5*b^8 + 66*a^6*b^7 - 110*a^7*b^6 - 34*a^8*b^5 +
70*a^9*b^4 + 11*a^10*b^3 - 26*a^11*b^2))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a
^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^1
2 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (a*((8*tan(c/2 + (d*x)/2)*(8*a^14

```
- 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 -
120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a
^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b
^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5
*a^9*b^8 - a^10*b^7 - a^11*b^6) - (a*((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19
- 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 3
0*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^1
3*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^1
5 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*
b^9) - (4*a*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 +
 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18
+ 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13
- 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(
(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8
+ 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*
b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^
10*b^7 - a^11*b^6)))*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4
 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b
^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*((a + b)^7*(a - b)^7)^(1/2)*(2
*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14
- 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)) - (a*((
8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b^14 + 44*a^2*b^12 +
 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*
a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 - 48*a^12*b^2))/(a*b^16
+ b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11
- 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (a*((8*(16*a
*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 3
0*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a
^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^
17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5
*a^9*b^11 - a^10*b^10 - a^11*b^9) + (4*a*tan(c/2 + (d*x)/2)*((a + b)^7*(a -
 b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^21 - 8*a^2*b^20
 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 +
 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10
+ 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12
 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^
2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10
 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*((a + b)^7*(a - b)^7)^(1/
2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*
b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*(
(a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^1
8 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*
a^12*b^6 - a^14*b^4))))*((a + b)^7*(a - b)^7)^(1/2)*(2*a^6 - 8*b^6 + 8*a^2*
b^4 - 7*a^4*b^2)*1i)/(d*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35
*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=222

$$-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx)}$$

```
[Out] -b*(3*a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*
x+c))^3-1/6*a^2*(2*a^2-7*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec(d*x
+c))
```

Rubi [A] time = 0.42, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4080, 4003, 12, 3831, 2659, 208}

$$-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
(a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(6*b
^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4
)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(a^2-3ab\sec(c+dx)-\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,d}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec}{}}{}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a\right.}{6b^2\left(\right.}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a\right.}{6b^2\left(\right.}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a\right.}{6b^2\left(\right.}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a\right.}{6b^2\left(\right.}$$

$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{}{6}$$

Mathematica [A] time = 1.03, size = 158, normalized size = 0.71

$$\frac{\frac{6b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{a\sin(c+dx)\left(2a^4+a^2\left(4a^2+11b^2\right)\cos^2(c+dx)+3ab\left(a^2+9b^2\right)\cos(c+dx)-5a^2b^2+18b^4\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^2 - b^2)^(7/2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4 + 3*a*b*(a^2 + 9*b^2)*C
os[c + d*x] + a^2*(4*a^2 + 11*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.61, size = 903, normalized size = 4.07

$$\left[-\frac{3\left(3a^2b^4+2b^6+\left(3a^5b+2a^3b^3\right)\cos(dx+c)^3+3\left(3a^4b^2+2a^2b^4\right)\cos(dx+c)^2+3\left(3a^3b^3+2ab^5\right)\cos(\right.}{12\left(\left(a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8\right)d\cos(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3*
a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2
```

```
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7 - 7*a^5*b^2 + 23*a^3*b^4
 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b +
8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^
3*b^3)*cos(d*x + c)^3 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7 - 7*a^5*b^2 + 23*a^3
*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*
b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
+ 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^
4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

giac [A] time = 0.32, size = 403, normalized size = 1.82

$$-\frac{\frac{3\left(3a^2b+2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^3b^2\tan\left(\frac{1}{2}dx\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/
2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*
c)^5 - 4*a^5*tan(1/2*d*x + 1/2*c)^3 - 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 3
6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1
/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x +
 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

maple [A] time = 0.41, size = 285, normalized size = 1.28

$$\frac{\frac{-\frac{\left(2a^2+3ab+6b^2\right)a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{4\left(a^2+9b^2\right)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^2-3ab+6b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3b^2a-b^3\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^3}-\frac{b\left(3a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^6-3a^4b^2+3b^4a^2-b^6\right)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5+2/3*(a^2+9*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-b*(3*a^2+
2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*
x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.41, size = 378, normalized size = 1.70

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+3\,a^2\,b+6\,a\,b^2\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^3+9\,a\,b^2\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3-\dots\right.}{\left(a+b\right)\,\left(a^3-3\,a^2\,b\dots\right.}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b)) - (
4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) +
 (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b
^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 +
3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
) - (b*atanh((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(3*
a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=206

$$\frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{(\ldots}{6\ldots}$$

```
[Out] a*(a^2+4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/
2)/(a+b)^(7/2)/d-1/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a*
(a^2-6*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(a^4-10*a^2*b
^2-6*b^4)*tan(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.35, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 4003, 12, 3831, 2659, 208}

$$\frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{(\ldots}{6\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^3) + (a*(a^2 - 6*b^2)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a +
b*Sec[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d*x])/(6*b*(a^2 -
b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
```

```
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(-3ab-\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-\cdots\right.}{6b\left(a^2\cdots\right.}$$

$$= \frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\cdots}{6b\left(\cdots\right.}$$

Mathematica [A] time = 1.10, size = 165, normalized size = 0.80

$$\frac{\frac{\sin(c+dx)\left(-a^2b\left(13a^2+2b^2\right)\cos^2(c+dx)+3a\left(a^4-9a^2b^2-2b^4\right)\cos(c+dx)+b\left(a^4-10a^2b^2-6b^4\right)\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3} - \frac{6a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-6*a*(a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(7/2) + ((b*(a^4 - 10*a^2*b^2 - 6*b^4) + 3*a*(a^4 - 9*a^2*b^2 -
```

```
2*b^4)*Cos[c + d*x] - a^2*b*(13*a^2 + 2*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])
/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.56, size = 902, normalized size = 4.38

$$\left[-\frac{3\left(a^3b^3+4ab^5+\left(a^6+4a^4b^2\right)\cos\left(dx+c\right)^3+3\left(a^5b+4a^3b^3\right)\cos\left(dx+c\right)^2+3\left(a^4b^2+4a^2b^4\right)\cos\left(dx+c\right)\right)\sqrt{}}{12\left(\left(a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8\right)d\cos\left(dx+c\right)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b
+ 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7
 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5*b^2
 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^
2)*cos(d*x + c)^3 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a
^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6
*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5
*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b
^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

giac [B] time = 0.38, size = 431, normalized size = 2.09

$$-\frac{\frac{3\left(a^3+4ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2a-2b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{3a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 - 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

maple [A] time = 0.35, size = 294, normalized size = 1.43

$$-\frac{\frac{2\left(-\frac{(a^3+6a^2b+2b^2a+2b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3a^2b+3b^2a+b^3)}+\frac{2(7a^2+3b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}+\frac{(a^3-6a^2b+2b^2a-2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^3}+\frac{a(a^2+4b^2)\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^6-3a^4b^2+3b^4a^2-b^6)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+2/3*(7*a^2+3*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arcta
nh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.38, size = 380, normalized size = 1.84

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(7\,a^2\,b+3\,b^3\right)}{3\,(a+b)^2\left(a^2-2\,a\,b+b^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(a^3+6\,a^2\,b+2\,a\,b^2+2\,b^3\right)}{(a+b)^3\,(a-b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^3-\right.}{(a+b)\left(a^3-3\,\right.}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(7*a^2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^2*b + a^3 + 2*b^3))/((a + b)^3*(a
 - b)) + (tan(c/2 + (d*x)/2)*(2*a*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
 - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3
) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))) + (a*atanh((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a - 2*b)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a*b^2 +
a^3)))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{}{3a}$$

```
[Out] -b*(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d+1/3*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*(2*a^
2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*a*(2*a^2+13*b^2)*t
an(d*x+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3836, 4003, 12, 3831, 2659, 208}

$$-\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((b*(4*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Se
c[c + d*x])^3) + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
```

```
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3b+2a\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)}{\cdots}}{\cdots}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2\cdots\right.}{6\left(a^2-b^2\cdots\right.}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2\cdots\right.}{6\left(a^2-b^2\cdots\right.}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2\cdots\right.}{6\left(a^2-b^2\cdots\right.}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2\cdots\right.}{6\left(a^2-b^2\cdots\right.}$$

$$= -\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\cdots}{6(\cdots}$$

Mathematica [A] time = 1.28, size = 164, normalized size = 0.85

$$\frac{\frac{6b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{\sin(c+dx)\left(2a^3b^2+a\left(6a^4+10a^2b^2-b^4\right)\cos^2(c+dx)-3b\left(-2a^4-9a^2b^2+b^4\right)\cos(c+dx)+13ab^4\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + ((2*a^3*b^2 + 13*a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2 + b^4)*
```

```
Cos[c + d*x] + a*(6*a^4 + 10*a^2*b^2 - b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/(
(a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.58, size = 901, normalized size = 4.69

$$\left[-\frac{3\left(4a^2b^4+b^6+\left(4a^5b+a^3b^3\right)\cos\left(dx+c\right)^3+3\left(4a^4b^2+a^2b^4\right)\cos\left(dx+c\right)^2+3\left(4a^3b^3+ab^5\right)\cos\left(dx+c\right)\right)\sqrt{\cdots}}{12\left(\left(a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8\right)d\cos\left(dx+c\right)\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4*
b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6 + (6
*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*
b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b
^3)*cos(d*x + c)^3 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3
+ a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6
 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7
*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^
3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

giac [B] time = 0.35, size = 431, normalized size = 2.24

$$-\frac{\frac{3\left(4a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12a^3b^2\tan\left(\frac{1}{2}dx\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/2*
d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x +
1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2*c
)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*a^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^3
*b^2*tan(1/2*d*x + 1/2*c)^3 + 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1
/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*c) - 3
*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

maple [A] time = 0.43, size = 297, normalized size = 1.55

$$\frac{\frac{-\frac{(2a^3+2a^2b+6b^2a+b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3b^2a+b^3)}+\frac{4(3a^2+7b^2)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(2a^3-2a^2b+6b^2a-b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3b^2a-b^3)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^3}-\frac{b(4a^2+b^2)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^6-3a^4b^2+3b^4a^2-b^6)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5+2/3*(3*a^2+7*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan
h(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.34, size = 382, normalized size = 1.99

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^3+2\,a^2\,b+6\,a\,b^2+b^3\right)}{(a+b)^3\,(a-b)}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^3+7\,a\,b^2\right)}{3\,(a+b)^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^3\right.}{(a+b)\left(a^3-3\,a\right.}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a - b
)) - (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b +
 b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b
 - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3
) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a - 2*b)*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a^2*b +
b^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{5ab\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{}{3d}$$

```
[Out] a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d-1/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-5/6*a*b*
tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*tan(d*x+c)
/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3833, 4003, 12, 3831, 2659, 208}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{5ab\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^3) - (5*a*b*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]
)^2) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sec[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a+2b\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+d}{}}{}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b\right.}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b\right.}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b\right.}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a\right.}{6\left(a^2-b\right.}$$

$$= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{}{6\left(a\right.}$$

Mathematica [A] time = 1.17, size = 163, normalized size = 0.89

$$-\frac{\frac{12a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{b\sin(c+dx)\left(18a^4+6ab\left(9a^2+b^2\right)\cos(c+dx)+17a^2b^2+\left(18a^4-5a^2b^2+2b^4\right)\cos(2(c+dx))+10b^4\right)}{(a\cos(c+dx)+b)^3}}{12d(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -1/12*((12*a*(2*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/Sqrt[a^2 - b^2] + (b*(18*a^4 + 17*a^2*b^2 + 10*b^4 + 6*a*b*(9*a^2 +
```

```
b^2)*Cos[c + d*x] + (18*a^4 - 5*a^2*b^2 + 2*b^4)*Cos[2*(c + d*x)])*Sin[c +
d*x])/(b + a*Cos[c + d*x])^3)/((a - b)^3*(a + b)^3*d)
```

fricas [B] time = 0.59, size = 905, normalized size = 4.92

$$\left[-\frac{3\left(2a^3b^3+3ab^5+\left(2a^6+3a^4b^2\right)\cos(dx+c)^3+3\left(2a^5b+3a^3b^3\right)\cos(dx+c)^2+3\left(2a^4b^2+3a^2b^4\right)\cos(dx\right.}{12\left(\left(a^{11}-4a^9b^2+6a^7b^4-4a^5b^6+a^3b^8\right)d\cos(dx+c)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2*
a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7
 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^
2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4
*b^2)*cos(d*x + c)^3 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^
2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*a^4*b^3 - 7*a^2*b^5 - 4*
b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5
*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4
 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

giac [B] time = 0.36, size = 403, normalized size = 2.19

$$-\frac{\frac{3\left(2a^3+3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{18a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^3\tan\left(\frac{1}{2}d\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
+ arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (18*a^4*b*tan(1
/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1/2*d
*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)
^5 - 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 32*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*t
an(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x
+ 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

maple [A] time = 0.35, size = 284, normalized size = 1.54

$$-\frac{\frac{2\left(-\frac{(6a^2+3ab+2b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3a^2b+3b^2a+b^3)}+\frac{2(9a^2+b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3b^2a-b^3)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^3}+\frac{a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a^6-3a^4b^2+3b^4a^2-b^6)\sqrt{(a-b)(a+b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(2*a^2
+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.32, size = 378, normalized size = 2.05

$$\frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(2\,a^2+3\,b^2\right)\left(2\,a-2\,b\right)\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\left(2\,a^3+3\,a\,b^2\right)\sqrt{a+b}\,\left(a-b\right)^{7/2}}\right)\left(2\,a^2+3\,b^2\right)}{d\,\left(a+b\right)^{7/2}\,\left(a-b\right)^{7/2}}-\frac{}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)
```

```
[Out] (a*atanh((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2
*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^2
+ 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(3*a*b^2
 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) - (4*tan(c/2 + (d*x)/2)^3*(9*a^2*b
 + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(6*a^2*b -
 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 +
 (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a
*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 +
 (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

3.520 $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=242

$$\frac{x}{a^4}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tanh^{-}}{a^4d(a-b)^{7/2}(a+b)}$$

```
[Out] x/a^4-b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*b^2*tan(d*x+c)/a/(a^2-
b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(a^2-
b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\tan(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{b^2\left(8a^2\right.}{6a^2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-4),x]
```

```
[Out] x/a^4 - (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*Tan
[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*(8*a^2 - 3*b^2
)*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a
^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
+ d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3\left(a^2-b^2\right)+3ab\sec(c+dx)-2b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{6(a^2\cdots}{\cdots}}{\cdots}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(26\cdots\right.}{6a^3\left(\cdots\right.}$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \cdots$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \cdots$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \cdots$$

$$= \frac{x}{a^4} - \frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{b^2\tan(\cdots}{3a\left(a^2-b^2\right)d(a\cdots}$$

Mathematica [A] time = 1.50, size = 268, normalized size = 1.11

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b)\left(-\frac{ab^3\left(12a^2-7b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{ab^2\left(36a^4-32a^2b^2+11b^4\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^3(a+b)^3} - \cdots\right.}{6a^4d(a+b\sec(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(6*(c + d*x)*(b + a*Cos[c + d*x])^3 -
(6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (2*a*b^
4*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^3*(12*a^2 - 7*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + (a*b^2*(36*a^4 - 32*a^2*b^2 + 1
1*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3)))/(6*a^4*
d*(a + b*Sec[c + d*x])^4)
```

fricas [B] time = 0.65, size = 1456, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x
+ c)^3 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(
d*x + c)^2 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*
cos(d*x + c) + 12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x
+ 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*a^9*b - 8*a^7*b^3 + 7*
a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6
- 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^
9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(26*a^7*b^4 - 4
3*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 -
 11*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b
^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^
6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8
*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4
*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 -
4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6
+ a^3*b^8)*d*x*cos(d*x + c)^3 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*d*x*cos(d*x + c) + 6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 -
4*a^2*b^9 + b^11)*d*x - 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*
a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 -
8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^
5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (26*a^7*b^4 - 43*a
^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11
*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)
*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 +
 a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^
7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^
7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a
^6*b^9 + a^4*b^11)*d)]
```

giac [B] time = 0.27, size = 532, normalized size = 2.20

$$\frac{\frac{3\left(8a^6b-8a^4b^3+7a^2b^5-2b^7\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^{10}-3a^8b^2+3a^6b^4-a^4b^6\right)\sqrt{-a^2+b^2}}-\frac{36a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-60a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a
^2 + b^2)) - (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*
c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 +
 6*b^8*tan(1/2*d*x + 1/2*c)^5 - 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*a^4
*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1/2*d*x + 1
/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x + 1/2*c) + 6*
b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(d*x + c)/a^4)
/d
```

maple [B] time = 0.53, size = 1408, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -12/d*b^2*a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-4/d*b^3/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5+6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+1/d*b^5/a^2/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5-2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+24/d*b^2*a/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-44/3/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3+4/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-12/d*b^2*a/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)+4/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+6/d*b^4/a/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)-1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-2/d*b^6/a^3/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)-8/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*b^3/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(
a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b^7/a
^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/
2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 12.79, size = 7234, normalized size = 29.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^4,x)
```

```
[Out] (2*atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11
 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 +
110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a
^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 +
 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (tan(c/2 + (d*x)/2)
*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^
12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a
^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 -
 a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 +
10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 + (8*tan(c/2
 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b
^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6
- 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 -
a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 1
0*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 - ((((8*(16*a^20*b
 - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12
*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^
4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a
^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^
4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 +
 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 +
160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 4
8*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*
a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4
 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 - (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a
^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*
a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b
^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a
^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4
- 5*a^14*b^3 - 5*a^15*b^2))/a^4)/((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 -
 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110
*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^1
9*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 1
0*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*
b^2) - (tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^
12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*
b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)
*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10
*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b
^2)))*1i)/a^4 + (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^1
4 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 +
160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^
12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*
a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^
2))*1i)/a^4 + (((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^1
0*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*
b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^
20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14
*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d
*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 -
```

```
120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 +
 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b +
a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*
b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 - (8*t
an(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48
*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^
8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a
^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b
^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 + (16*(1
6*a^12*b - 2*a*b^12 + 4*b^13 - 26*a^2*b^11 + 11*a^3*b^10 + 70*a^4*b^9 - 34*
a^5*b^8 - 110*a^6*b^7 + 66*a^7*b^6 + 110*a^8*b^5 - 64*a^9*b^4 - 64*a^10*b^3
 + 48*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^1
2*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3
- 5*a^18*b^2))))/(a^4*d) - ((tan(c/2 + (d*x)/2)^5*(2*b^6 - a*b^5 - 6*a^2*b^
4 + 4*a^3*b^3 + 12*a^4*b^2))/((a^3*b - a^4)*(a + b)^3) + (4*tan(c/2 + (d*x)
/2)^3*(3*b^6 - 11*a^2*b^4 + 18*a^4*b^2))/(3*(a + b)^2*(a^5 - 2*a^4*b + a^3*
b^2)) + (tan(c/2 + (d*x)/2)*(a*b^5 + 2*b^6 - 6*a^2*b^4 - 4*a^3*b^3 + 12*a^4
*b^2))/((a + b)*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))/(d*(tan(c/2 + (d*x)
/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 +
 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)
/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (b*atan(((b*((8*tan(c/2 + (d*x)/2
)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*
a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*
b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 -
 a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5
 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (b*((a + b)^7*(a - b)^7)^(1/2)*
((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*
b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^
5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a
^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^
5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a +
b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*(8*a^21*b - 8
*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*
a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a
^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^
8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^
17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^
6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(8*a^6 - 2*b^6 +
 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 3
5*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*((a + b)^7*(a - b)^7
)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(2*(a^18 - a^4*b^14 + 7
*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*
b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 -
48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*
a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b
^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10
*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) -
 (b*((a + b)^7*(a - b)^7)^(1/2)*((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^
9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14
*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2
))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^1
3*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2)
+ (4*b*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^
2*b^4 - 8*a^4*b^2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*
a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 12
0*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18
 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^1
4*b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^
9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3
```

```
- 5*a^15*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^1
4 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*
a^16*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*
b^2)*1i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*
a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))/((16*(16*a^12*b - 2*a*b^12 + 4*b^13
- 26*a^2*b^11 + 11*a^3*b^10 + 70*a^4*b^9 - 34*a^5*b^8 - 110*a^6*b^7 + 66*a^
7*b^6 + 110*a^8*b^5 - 64*a^9*b^4 - 64*a^10*b^3 + 48*a^11*b^2))/(a^19*b + a^
20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14
*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (b*((8*tan(c/
2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*
b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6
 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^16*b + a^17 -
 a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 +
10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (b*((a + b)^7*(a - b
)^7)^(1/2)*((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11
 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 +
110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a
^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 +
 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (4*b*tan(c/2 + (d*x
)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*(8
*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*
b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17
*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b
^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(
a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 -
 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(8*a^
6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a
^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*((a + b)^
7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*
b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 -
 7*a^16*b^2)) - (b*((8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8
*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^
8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 4
4*a^12*b^2))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 -
 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^1
5*b^2) - (b*((a + b)^7*(a - b)^7)^(1/2)*((8*(16*a^20*b - 4*a^21 + 4*a^8*b^1
3 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 -
110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*
a^19*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8
- 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^
18*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^
6 + 7*a^2*b^4 - 8*a^4*b^2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^
12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*
b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)
)/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6
+ 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^
9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a
^14*b^3 - 5*a^15*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 -
 a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*
b^4 - 7*a^16*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6 + 7*a^2*b^4
- 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8
- 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))))*((a + b)^7*(a - b)^7)^(1/2)*(8
*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(d*(a^18 - a^4*b^14 + 7*a^6*b^12
- 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sec(c + dx))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-4), x)
```

3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=299

$$-\frac{4bx}{a^5}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\text{s}}{6a^4d\left(a^2-b^2\right)^3}$$

```
[Out] -4*b*x/a^5+b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d+1/6*(6*a^6-65*a^
4*b^2+68*a^2*b^4-24*b^6)*sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b^2*sin(d*x+c)/a/
(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*sin(d*x+c)/a^2/(a^2-b^
2)^2/d/(a+b*sec(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^4)*sin(d*x+c)/a^3/
(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 1.04, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b}{-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-4*b*x)/a^5 + (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d
) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Sin[c + d*x])/(6*a^4*(a^2 -
 b^2)^3*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
+ (b^2*(9*a^2 - 4*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b
^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
```

```
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-3a^2+4b^2+3ab\sec(c+dx)-3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)}{}}{}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(12a^4\right.}{2a^3\left(a^2\right.}$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + -$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{4bx}{a^5} + \frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\left(6a^6-65a^4\right.}{}$$

Mathematica [A] time = 1.70, size = 293, normalized size = 0.98

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b)\left(\frac{5ab^4\left(3a^2-2b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{ab^3\left(60a^4-71a^2b^2+26b^4\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^3(a+b)^3} + \frac{6b^2(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(-24*b*(c + d*x)*(b + a*Cos[c + d*x])^
3 + (6*b^2*(-20*a^6 + 35*a^4*b^2 - 28*a^2*b^4 + 8*b^6)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2)
+ (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)) + (5*a*b^4*(3*a^2 - 2*b^2)*(b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) - (a*b^3*(60*a^4 - 71*
a^2*b^2 + 26*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3
) + 6*a*(b + a*Cos[c + d*x])^3*Sin[c + d*x]))/(6*a^5*d*(a + b*Sec[c + d*x])
^4)
```

fricas [B] time = 0.68, size = 1603, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(48*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d
*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d
*x*cos(d*x + c)^2 + 144*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^
11)*d*x*cos(d*x + c) + 48*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b
^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11 + (20*a^9*b^2 -
 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a
^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20*a^7*b^4 - 35*a^5*b^
6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2)) - 2*(6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6
*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18
*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c
)^2 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*
cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 +
 a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^
7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a
^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*
a^7*b^9 + a^5*b^11)*d), -1/6*(24*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^
7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*
a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^
7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c) + 24*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*
b^8 - 4*a^2*b^10 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*
b^11 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 +
3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20
*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6*(a^1
2 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18*a^11
*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c)^2 +
 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d
*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*
b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a
^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^
8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b
^9 + a^5*b^11)*d)]
```

giac [A] time = 0.32, size = 564, normalized size = 1.89

$$-\frac{\frac{3\left(20\,a^{6}b^{2}-35\,a^{4}b^{4}+28\,a^{2}b^{6}-8\,b^{8}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{11}-3\,a^{9}b^{2}+3\,a^{7}b^{4}-a^{5}b^{6}\right)\sqrt{-a^{2}+b^{2}}}-\frac{60\,a^{6}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a^{5}b^{4}\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*s
qrt(-a^2 + b^2)) - (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2
*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d
*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 - 120*a^6*b^3*tan(1/2*d*x + 1/2*c)
^3 + 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*a^2*b^7*tan(1/2*d*x + 1/2*c)^
3 + 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c) + 105*a
^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 117*a^3*b^6
```

```
*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b^8*tan(1/2*
d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4
- a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*(d*x + c)*b/a^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*a^4))/d
```

maple [B] time = 0.68, size = 1448, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 20/d*b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5-18/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/d*b^6/a^3/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5+6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-40/d*b^3/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+116/3/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3-12/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^
2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+20/d*b^3/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)-5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-18/d*b^5/a^2/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+6/d*b^7/a^4
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a
-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-35/d
*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*
x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
))-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(ta
n(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^4*tan(1/2*d*x+1/2*c)/(1+t
an(1/2*d*x+1/2*c)^2)-8/d/a^5*b*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 10.38, size = 7534, normalized size = 25.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^4,x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 -
11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2))/((a^4*b - a^5)*(a + b)^3) + (tan(c/2
+ (d*x)/2)^3*(12*a*b^7 - 18*a^8 - 72*b^8 + 236*a^2*b^6 - 47*a^3*b^5 - 273*a
^4*b^4 + 60*a^5*b^3 + 72*a^6*b^2))/(3*(a + b)^2*(3*a^6*b - a^7 + a^4*b^3 -
3*a^5*b^2)) - (tan(c/2 + (d*x)/2)*(4*a*b^6 - 2*a^6*b - 2*a^7 + 8*b^7 - 24*a
^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2))/((a + b)*(3*a^6*b - a^7 + a^
4*b^3 - 3*a^5*b^2)) + (tan(c/2 + (d*x)/2)^5*(12*a*b^7 + 18*a^8 + 72*b^8 - 2
36*a^2*b^6 - 47*a^3*b^5 + 273*a^4*b^4 + 60*a^5*b^3 - 72*a^6*b^2))/(3*(a^4*b
 - a^5)*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6
*a^2*b - 6*b^3) + tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3) + tan(c/2
+ (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 + (d*x)/2)^8*(
3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*b*atan(((4*b*((8*tan(c/2 + (d*x)/2)*(
128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*
a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824
*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18
*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 1
0*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b*((16
*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*
a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^
19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^
11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a
^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b*tan(c/2 + (d*x)/2)*(8
*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^1
4*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^
19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)*32i)/(a^5*(a^18*b + a^19 -
 a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6
+ 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*4i)/a^5))/a^5 + (4
*b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b
^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a
^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^1
3*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5
*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*
b^3 - 5*a^17*b^2) - (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12
*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17
*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2
))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^
16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2)
 + (b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^
12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*
b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)
*32i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 -
 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^1
7*b^2)))*4i)/a^5))/a^5)/((32*(128*b^16 - 64*a*b^15 - 832*a^2*b^14 + 400*a^3
*b^13 + 2288*a^4*b^12 - 1088*a^5*b^11 - 3472*a^6*b^10 + 1602*a^7*b^9 + 3088
*a^8*b^8 - 1280*a^9*b^7 - 1520*a^10*b^6 + 480*a^11*b^5 + 320*a^12*b^4))/(a^
22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7
 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b*
((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13
 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*
b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b
^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^
11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3
 - 5*a^17*b^2) + (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^
12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^
7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/
(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*
b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) -
(b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12
```

- 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7
- 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)*32
i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10
*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b
^2)))*4i)/a^5)*4i)/a^5 + (b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 -
768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^1
0 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^
5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 -
a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^
5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b*((16*(8*a^23*b - 8*a^10*b^1
4 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9
+ 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 -
36*a^21*b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14
*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 -
5*a^20*b^3 - 5*a^21*b^2) + (b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 +
8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 +
160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 -
48*a^21*b^3 - 8*a^22*b^2)*32i)/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 +
5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15
*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*4i)/a^5)*4i)/a^5)))/(a^5*d) - (b^2*atan((
(b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3
*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025
*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a
^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 +
5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^1
6*b^3 - 5*a^17*b^2) + (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*
a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*
a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22
*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 1
0*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*
b^2) - (4*b^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^
6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^1
2*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a
^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*
b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*
b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^
10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4
- 5*a^16*b^3 - 5*a^17*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 +
28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 +
35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*((a + b)^7*(a - b)
^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*1i)/(2*(a^19 - a^5*b^1
4 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*
a^17*b^2)) + (b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b
^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a
^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^1
2*b^4 - 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 +
5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^1
5*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^
11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a
^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*
b^3 - 20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5
*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*
b^3 - 5*a^21*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(
20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*
b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^1
6*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*
b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b
^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^
9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5

+ 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(20*
a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 -
21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*((a +
b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*1i)/(2*(a
^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*
a^15*b^4 - 7*a^17*b^2)))/((32*(128*b^16 - 64*a*b^15 - 832*a^2*b^14 + 400*a^
3*b^13 + 2288*a^4*b^12 - 1088*a^5*b^11 - 3472*a^6*b^10 + 1602*a^7*b^9 + 308
8*a^8*b^8 - 1280*a^9*b^7 - 1520*a^10*b^6 + 480*a^11*b^5 + 320*a^12*b^4))/(a
^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^
7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (b
^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b
^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a
^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^1
3*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5
*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*
b^3 - 5*a^17*b^2) + (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^
12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^
17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b
^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*
a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^
2) - (4*b^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6
+ 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*
b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^1
7*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^
2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^
6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10
*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 -
5*a^16*b^3 - 5*a^17*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 2
8*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 3
5*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*((a + b)^7*(a - b)^7
)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7
*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*
b^2)) + (b^2*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 +
768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^
9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4
- 128*a^13*b^3 + 64*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^
10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4
- 5*a^16*b^3 - 5*a^17*b^2) - (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^
13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b
^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 -
20*a^22*b^2))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15
*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 -
5*a^21*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(20*a^
6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13
+ 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8
- 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 -
8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 -
35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^1
0 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*
a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 -
8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^
9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*((a + b)^7
(a - b)^7)^(1/2)(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2))/(2*(a^19 - a^
5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4
- 7*a^17*b^2))))*((a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4
- 35*a^4*b^2)*1i)/(d*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*
b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=387

$$\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{x\left(a^2+20b^2\right)}{2a^6}+\frac{\left(a^6-23a^4b^2+27a\right.}{}$$

```
[Out] 1/2*(a^2+20*b^2)*x/a^6-b^3*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)*arctanh((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d-1/6
*b*(24*a^6-146*a^4*b^2+167*a^2*b^4-60*b^6)*sin(d*x+c)/a^5/(a^2-b^2)^3/d+1/2
*(a^6-23*a^4*b^2+27*a^2*b^4-10*b^6)*cos(d*x+c)*sin(d*x+c)/a^4/(a^2-b^2)^3/d
+1/3*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+5/6*b^2*(2*
a^2-b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*b^2
*(48*a^4-53*a^2*b^2+20*b^4)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*se
c(d*x+c))
```

Rubi [A] time = 1.46, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(-146a^4b^2+167a^2b^4+24a^6-60b^6\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}+\frac{\left(-23a^4b^2+27a^2b^4+a^6-10b^6\right)\sin(c+dx)\cos(c+dx)}{2a^4d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2 + 20*b^2)*x)/(2*a^6) - (b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20*b^
6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*
(a + b)^(7/2)*d) - (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[c +
 d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6 - 23*a^4*b^2 + 27*a^2*b^4 - 10*b^6)*
Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b^2*Cos[c + d*x]*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (5*b^2*(2*a^2 - b^2)
*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2)
+ (b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a^2+5b^2+3ab\sec(c+dx)-4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{5b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int}{}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{5b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2(}{}$$

$$= \frac{\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\cos(c+dx)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\cos(c+dx)\sin(c}{3a\left(a^2-b^2\right)d(a+b\sec}$$

$$= -\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} + \frac{\left(a^6-23a^4b^2+27a^2b^4-10\right.}{2a^4\left(a^2-\right.}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} + \frac{\left(a^6-23a^4b^2\right.}{}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} + \frac{\left(a^6-23a^4b^2\right.}{}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} + \frac{\left(a^6-23a^4b^2\right.}{}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{7/2}(a+b)^{7/2}d} -$$

Mathematica [A] time = 6.38, size = 326, normalized size = 0.84

$$-\frac{b^6\sin(c+dx)}{3a^5d(b-a)(a+b)(a\cos(c+dx)+b)^3} - \frac{4b\sin(c+dx)}{a^5d} + \frac{\sin(2(c+dx))}{4a^4d} + \frac{\left(a^2+20b^2\right)(c+dx)}{2a^6d} + \frac{13b^7\sin(c+}{6a^5d(b-a)^2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2 + 20*b^2)*(c + d*x))/(2*a^6*d) + (b^3*(-40*a^6 + 84*a^4*b^2 - 69*a^2*
b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sq
rt[a^2 - b^2]*(-a^2 + b^2)^3*d) - (4*b*Sin[c + d*x])/(a^5*d) - (b^6*Sin[c +
 d*x])/(3*a^5*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x])^3) + (-18*a^2*b^5*Sin
[c + d*x] + 13*b^7*Sin[c + d*x])/(6*a^5*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c
 + d*x])^2) + (-90*a^4*b^4*Sin[c + d*x] + 122*a^2*b^6*Sin[c + d*x] - 47*b^8
*Sin[c + d*x])/(6*a^5*(-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^4*d)
```

fricas [B] time = 0.75, size = 1767, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^13 + 16*a^11*b^2 - 74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a
^3*b^10)*d*x*cos(d*x + c)^3 + 18*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a
^6*b^7 - 79*a^4*b^9 + 20*a^2*b^11)*d*x*cos(d*x + c)^2 + 18*(a^11*b^2 + 16*a
^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x +
c) + 6*(a^10*b^3 + 16*a^8*b^5 - 74*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20
*b^13)*d*x + 3*(40*a^6*b^6 - 84*a^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b
^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 -
 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84
*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 - 227*a^3*b^10 + 6
0*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 + a^5*b^8)*cos(d*x
+ c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*cos(d*x
 + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a^5*b^8 + 110*a^3*
b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*a^6*b^7 - 190*a^4
*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^1
3*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6
*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b
^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12
*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d), 1/6*(3*(a^13 + 16*a^11*b^2 -
74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a^3*b^10)*d*x*cos(d*x + c)^3 + 9
*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a^6*b^7 - 79*a^4*b^9 + 20*a^2*b^1
1)*d*x*cos(d*x + c)^2 + 9*(a^11*b^2 + 16*a^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8
 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x + c) + 3*(a^10*b^3 + 16*a^8*b^5 - 7
4*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20*b^13)*d*x - 3*(40*a^6*b^6 - 84*a
^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20
*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 - 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2
*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11
)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) - (24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 -
 227*a^3*b^10 + 60*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 +
a^5*b^8)*cos(d*x + c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 +
 a^4*b^9)*cos(d*x + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a
^5*b^8 + 110*a^3*b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*
a^6*b^7 - 190*a^4*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4
*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b
 - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^
15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (
a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

giac [A] time = 0.35, size = 615, normalized size = 1.59

$$\frac{6\left(40a^6b^3-84a^4b^5+69a^2b^7-20b^9\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^{12}-3a^{10}b^2+3a^8b^4-a^6b^6\right)\sqrt{-a^2+b^2}}-\frac{2\left(90a^6b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-162a^5b^5\tan\left(\frac{1}{2}d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*(40*a^6*b^3 - 84*a^4*b^5 + 69*a^2*b^7 - 20*b^9)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*
sqrt(-a^2 + b^2)) - 2*(90*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*a^5*b^5*tan(
```

```
1/2*d*x + 1/2*c)^5 - 48*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*a^3*b^7*tan(1/
2*d*x + 1/2*c)^5 - 48*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*a*b^9*tan(1/2*d*x
 + 1/2*c)^5 + 36*b^10*tan(1/2*d*x + 1/2*c)^5 - 180*a^6*b^4*tan(1/2*d*x + 1/
2*c)^3 + 392*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*a^2*b^8*tan(1/2*d*x + 1/2
*c)^3 + 72*b^10*tan(1/2*d*x + 1/2*c)^3 + 90*a^6*b^4*tan(1/2*d*x + 1/2*c) +
162*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*a^4*b^6*tan(1/2*d*x + 1/2*c) - 213*a^
3*b^7*tan(1/2*d*x + 1/2*c) - 48*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*a*b^9*tan
(1/2*d*x + 1/2*c) + 36*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^
7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a -
 b)^3) + 3*(a^2 + 20*b^2)*(d*x + c)/a^6 - 6*(a*tan(1/2*d*x + 1/2*c)^3 + 8*b
*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 8*b*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

maple [B] time = 0.80, size = 1576, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -30/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-6/d*b^5/a^2/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5+34/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+3/d*b^7/a^4/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5-12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+60/
d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^
2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-212/3/d*b^6/a^3/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3+24/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-30/d*b^4/a/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)+6/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+34/d*b^6/
a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*
b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-3/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
-12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b
^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))+84/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arcta
nh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-69/d*b^7/a^4/(a^6-3*a^4*b^
2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1
/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d/a^4/(1+tan(1/
2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)^3*b+1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)
-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b+1/d/a^4*arctan(tan
(1/2*d*x+1/2*c))+20/d/a^6*arctan(tan(1/2*d*x+1/2*c))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 10.80, size = 8133, normalized size = 21.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^9*(7*a^7*b - 10*a*b^7 + a^8 + 20*b^8 - 59*a^2*b^6 + 27
*a^3*b^5 + 57*a^4*b^4 - 21*a^5*b^3 - 11*a^6*b^2))/(a^5*(a + b)^3*(a - b)) +
 (2*tan(c/2 + (d*x)/2)^3*(30*a*b^8 + 21*a^8*b - 6*a^9 + 120*b^9 - 364*a^2*b
^7 - 71*a^3*b^6 + 369*a^4*b^5 + 45*a^5*b^4 - 111*a^6*b^3 - 3*a^7*b^2))/(3*a
^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(21*a^8*b - 30*a*b^8 + 6*
a^9 + 120*b^9 - 364*a^2*b^7 + 71*a^3*b^6 + 369*a^4*b^5 - 45*a^5*b^4 - 111*a
^6*b^3 + 3*a^7*b^2))/(3*a^5*(a + b)^3*(a - b)^2) + (2*tan(c/2 + (d*x)/2)^5*
(9*a^10 + 180*b^10 - 611*a^2*b^8 + 740*a^4*b^6 - 324*a^6*b^4 + 36*a^8*b^2))
/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)*(10*a*b^7 - 7*a^7*b + a^
8 + 20*b^8 - 59*a^2*b^6 - 27*a^3*b^5 + 57*a^4*b^4 + 21*a^5*b^3 - 11*a^6*b^2
))/(a^5*(a + b)*(a - b)^3))/(d*(tan(c/2 + (d*x)/2)^2*(9*a*b^2 + 3*a^2*b - a
^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - t
an(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a*b^2 + 3*a^2*
b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^3 - b^3) + tan
(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) - (atan((((((((4*(4*a^
27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^1
6*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724
*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^2
5*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7
- 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*t
an(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13
+ 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8
 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 -
 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*
a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b
^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) - (8*tan(c/2 + (d*x)/2)*(800
*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 1152
2*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b
^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^1
3*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^
11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a
^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i)*1i)/(2
*a^6) - (((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*
a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 -
 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3
+ 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^1
8*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3
- 5*a^24*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12
*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^1
7*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22
*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^
10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10
*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) + (8*tan
(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 -
4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^
7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 4
5*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^2
0*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7
```

- 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i)*1i)/(2*a^6))/((((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) - (8*tan(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i))/(2*a^6) - (8*(8000*b^19 - 4000*a*b^18 - 50800*a^2*b^17 + 24400*a^3*b^16 + 135260*a^4*b^15 - 62030*a^5*b^14 - 193689*a^6*b^13 + 82337*a^7*b^12 + 155991*a^8*b^11 - 57345*a^9*b^10 - 64479*a^10*b^9 + 16999*a^11*b^8 + 8281*a^12*b^7 + 204*a^13*b^6 + 1396*a^14*b^5 - 40*a^15*b^4 + 40*a^16*b^3))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*20i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(a^2*1i + b^2*20i))/(2*a^6) + (8*tan(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2))*(a^2*1i + b^2*20i))/(2*a^6))*(a^2*1i + b^2*20i)*1i)/(a^6*d) - (b^3*atan(((b^3*((8*tan(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (b^3*((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*b^3*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10

*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 1
0*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*((a + b)^7*(a - b)^7)
^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2))/(2*(a^20 - a^6*b^14 + 7
*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18
b^2)))((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*
b^2)*1i)/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35
*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)) + (b^3*((8*tan(c/2 + (d*x)/2)*(800*a
*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*
a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^1
0 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*
b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 - a^10*b^11
 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^1
6*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) + (b^3*((4*(4*a^27 - 80*a^12
*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*b^11 + 640
*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 +
816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 -
 a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^
6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (4*b^3*tan(c/2 +
 (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^
4*b^2)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11
- 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6
 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^1
4 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7
*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8
 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a
^19*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a
^4*b^2))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35
*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6
 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2)*1i)/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 -
 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))/((8
*(8000*b^19 - 4000*a*b^18 - 50800*a^2*b^17 + 24400*a^3*b^16 + 135260*a^4*b^
15 - 62030*a^5*b^14 - 193689*a^6*b^13 + 82337*a^7*b^12 + 155991*a^8*b^11 -
57345*a^9*b^10 - 64479*a^10*b^9 + 16999*a^11*b^8 + 8281*a^12*b^7 + 204*a^13
*b^6 + 1396*a^14*b^5 - 40*a^15*b^4 + 40*a^16*b^3))/(a^25*b + a^26 - a^15*b^
11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a
^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (b^3*((8*tan(c/2 + (d*x)
/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 + 4720*a^2*b^16 - 4720*a^3*b^1
5 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^6*b^12 - 14812*a^7*b^11 - 103
85*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 - 3640*a^11*b^7 + 45*a^12*b^6 +
 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 35*a^16*b^2))/(a^20*b + a^21 -
 a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^
6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (b^3*((4*(4*a^27
 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15*b^12 - 1404*a^16*
b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 1764*a^20*b^7 + 724*a
^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 + 52*a^25*b^2))/(a^25*
b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 -
10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*b^3
*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b
^4 - 84*a^4*b^2)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*
a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 12
0*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20
 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^
16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 +
5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18
*b^3 - 5*a^19*b^2)))*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*
b^4 - 84*a^4*b^2))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^1
2*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*((a + b)^7*(a - b)^7)^(1/
2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2))/(2*(a^20 - a^6*b^14 + 7*a^8
*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2

```
)) - (b^3*((8*tan(c/2 + (d*x)/2)*(800*a*b^17 + 2*a^17*b - a^18 - 800*b^18 +
 4720*a^2*b^16 - 4720*a^3*b^15 - 11522*a^4*b^14 + 11522*a^5*b^13 + 14837*a^
6*b^12 - 14812*a^7*b^11 - 10385*a^8*b^10 + 10430*a^9*b^9 + 3325*a^10*b^8 -
3640*a^11*b^7 + 45*a^12*b^6 + 350*a^13*b^5 - 209*a^14*b^4 + 68*a^15*b^3 - 3
5*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b
^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5
*a^19*b^2) + (b^3*((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13
 - 248*a^15*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^1
9*b^8 - 1764*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^
24*b^3 + 52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9
+ 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^
23*b^3 - 5*a^24*b^2) + (4*b^3*tan(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2
)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2)*(8*a^25*b - 8*a^12*b^14 + 8*a
^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160
*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a
^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a
^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^1
1 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^
16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*((a + b)^7*(a - b)^7)^(1/
2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2))/(2*(a^20 - a^6*b^14 + 7*a^8
*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2
)))*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*b^6 + 69*a^2*b^4 - 84*a^4*b^2)
)/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b
^6 + 21*a^16*b^4 - 7*a^18*b^2))))*((a + b)^7*(a - b)^7)^(1/2)*(40*a^6 - 20*
b^6 + 69*a^2*b^4 - 84*a^4*b^2)*1i)/(d*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^
10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

```
[Out] -1/12*x+5/6*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3783, 2657}

$$\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-1),x]
```

```
[Out] -x/12 + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(6*d)
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\sec(c+dx)}\,dx &= \frac{x}{3}-\frac{1}{3}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx\\ &= -\frac{x}{12}+\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.97

$$\frac{2(c+dx)+5\tan^{-1}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-1),x]
```

```
[Out] (2*(c + d*x) + 5*ArcTan[2*Cot[(c + d*x)/2]])/(6*d)
```

fricas [A] time = 0.47, size = 33, normalized size = 1.06

$$\frac{4\,dx+5\,\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d*x + 5*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)))/d
```

giac [A] time = 0.16, size = 30, normalized size = 0.97

$$-\frac{dx + c - 10 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(d*x + c - 10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

maple [A] time = 0.40, size = 34, normalized size = 1.10

$$-\frac{5 \arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6d} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c)),x)
```

```
[Out] -5/6/d*arctan(1/2*tan(1/2*d*x+1/2*c))+2/3/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.78, size = 47, normalized size = 1.52

$$\frac{4 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 5 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)/
(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.82, size = 21, normalized size = 0.68

$$\frac{x}{3} - \frac{5 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3),x)
```

```
[Out] x/3 - (5*atan(tan(c/2 + (d*x)/2)/2))/(6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{5 \sec(c + dx) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x)
```

```
[Out] Integral(1/(5*sec(c + d*x) + 3), x)
```

3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{288d}+\frac{29x}{576}$$

```
[Out] 29/576*x+35/288*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/48*tan(d*x+c)/d/(3+5
*sec(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3785, 3919, 3831, 2657}

$$-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{288d}+\frac{29x}{576}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (29*x)/576 + (35*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(288*d) - (25*Tan
[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\sec(c+dx))^2}\,dx &= -\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}+\frac{1}{48}\int \frac{16+15\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\ &= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{35}{144}\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\ &= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{7}{144}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx\\ &= \frac{29x}{576}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 1.30

$$\frac{160(c+dx)-150\sin(c+dx)+96(c+dx)\cos(c+dx)+35(3\cos(c+dx)+5)\tan^{-1}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{288d(3\cos(c+dx)+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (160*(c + d*x) + 96*(c + d*x)*Cos[c + d*x] + 35*ArcTan[2*Cot[(c + d*x)/2]]*
(5 + 3*Cos[c + d*x]) - 150*Sin[c + d*x])/(288*d*(5 + 3*Cos[c + d*x]))
```

fricas [A] time = 0.47, size = 73, normalized size = 1.30

$$\frac{192\,dx\cos\left(dx+c\right)+320\,dx+35\left(3\,\cos\left(dx+c\right)+5\right)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)-300\,\sin\left(dx+c\right)}{576\left(3\,d\cos\left(dx+c\right)+5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/576*(192*d*x*cos(d*x + c) + 320*d*x + 35*(3*cos(d*x + c) + 5)*arctan(1/4*
(5*cos(d*x + c) + 3)/sin(d*x + c)) - 300*sin(d*x + c))/(3*d*cos(d*x + c) +
5*d)
```

giac [A] time = 0.17, size = 59, normalized size = 1.05

$$\frac{29\,dx+29\,c-\frac{300\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4}+70\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/576*(29*d*x + 29*c - 300*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 4
) + 70*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

maple [A] time = 0.50, size = 63, normalized size = 1.12

$$-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288d}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^2,x)
```

```
[Out] -25/48/d*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)-35/288/d*arctan(1/2*ta
n(1/2*d*x+1/2*c))+2/9/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.45, size = 88, normalized size = 1.57

$$-\frac{\frac{150 \sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)} - 64 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 35 \arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{288\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/288*(150*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*
x + c) + 1)) - 64*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 35*arctan(1/2*s
in(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.86, size = 52, normalized size = 0.93

$$\frac{x}{9} - \frac{\frac{35 \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{288} + \frac{25 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^2,x)
```

```
[Out] x/9 - ((35*atan(tan(c/2 + (d*x)/2)/2))/288 + (25*tan(c/2 + (d*x)/2))/(48*(t
an(c/2 + (d*x)/2)^2 + 4)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sec(c+dx)+3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**2,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-2), x)
```

3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$-\frac{125\tan(c+dx)}{4608d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{27648d}-\frac{1007x}{55296}$$

```
[Out] -1007/55296*x+3055/27648*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/96*tan(d*x+
c)/d/(3+5*sec(d*x+c))^2-125/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3785, 4060, 3919, 3831, 2657}

$$-\frac{125\tan(c+dx)}{4608d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{27648d}-\frac{1007x}{55296}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-3),x]
```

```
[Out] (-1007*x)/55296 + (3055*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(27648*d)
- (25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) - (125*Tan[c + d*x])/(460
8*d*(3 + 5*Sec[c + d*x]))
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
```

```
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} + \frac{1}{96}\int \frac{32+30\sec(c+dx)-25\sec^2(c+dx)}{(3+5\sec(c+dx))^2}\,dx$$
$$= -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} - \frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))} + \frac{\int \frac{512-165\sec(c+dx)}{3+5\sec(c+dx)}\,dx}{4608}$$
$$= \frac{x}{27} - \frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} - \frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))} - \frac{3055\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}\,dx}{13824}$$
$$= \frac{x}{27} - \frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} - \frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))} - \frac{611\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx}{13824}$$
$$= -\frac{1007x}{55296} + \frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d} - \frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2} - \frac{125\tan(c+dx)}{4608d(3+5\sec(c-$$

Mathematica [A] time = 0.35, size = 108, normalized size = 1.33

$$\frac{-3750\sin(c+dx) - 4725\sin(2(c+dx)) + 30720(c+dx)\cos(c+dx) + 4608c\cos(2(c+dx)) + 4608dx\cos(2(c+}{27648d(3\cos(c+dx)+5)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-3),x]
```

```
[Out] (30208*c + 30208*d*x + 30720*(c + d*x)*Cos[c + d*x] + 3055*ArcTan[2*Cot[(c
+ d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 4608*c*Cos[2*(c + d*x)] + 4608*d*x*Cos[
2*(c + d*x)] - 3750*Sin[c + d*x] - 4725*Sin[2*(c + d*x)])/(27648*d*(5 + 3*C
os[c + d*x])^2)
```

fricas [A] time = 0.54, size = 116, normalized size = 1.43

$$\frac{18432\,dx\cos(dx+c)^2 + 61440\,dx\cos(dx+c) + 51200\,dx + 3055\left(9\cos(dx+c)^2 + 30\cos(dx+c) + 25\right)\text{arcta}}{55296\left(9\,d\cos(dx+c)^2 + 30\,d\cos(dx+c) + 25\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(18432*d*x*cos(d*x + c)^2 + 61440*d*x*cos(d*x + c) + 51200*d*x + 30
55*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) + 3
)/sin(d*x + c)) - 300*(63*cos(d*x + c) + 25)*sin(d*x + c))/(9*d*cos(d*x + c
)^2 + 30*d*cos(d*x + c) + 25*d)
```

giac [A] time = 0.18, size = 75, normalized size = 0.93

$$-\frac{1007\,dx + 1007\,c - \frac{300\left(19\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 44\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2} - 6110\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{55296\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/55296*(1007*d*x + 1007*c - 300*(19*tan(1/2*d*x + 1/2*c)^3 - 44*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 6110*arctan(sin(d*x + c)/(cos
(d*x + c) + 3)))/d
```

maple [A] time = 0.43, size = 94, normalized size = 1.16

$$\frac{475\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4608d\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}-\frac{275\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1152d\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648d}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^3,x)
```

```
[Out] 475/4608/d/(tan(1/2*d*x+1/2*c)^2+4)^2*tan(1/2*d*x+1/2*c)^3-275/1152/d/(tan(
1/2*d*x+1/2*c)^2+4)^2*tan(1/2*d*x+1/2*c)-3055/27648/d*arctan(1/2*tan(1/2*d*
x+1/2*c))+2/27/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.47, size = 131, normalized size = 1.62

$$-\frac{\frac{150\left(\frac{44\sin(dx+c)}{\cos(dx+c)+1}-\frac{19\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{\frac{8\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+16}-2048\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+3055\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{27648\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/27648*(150*(44*sin(d*x + c)/(cos(d*x + c) + 1) - 19*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 16) - 2048*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 3
055*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.91, size = 79, normalized size = 0.98

$$\frac{x}{27}-\frac{3055\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{27648\,d}-\frac{\frac{275\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1152}-\frac{475\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4608}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^3,x)
```

```
[Out] x/27 - (3055*atan(tan(c/2 + (d*x)/2)/2))/(27648*d) - ((275*tan(c/2 + (d*x)/
2))/1152 - (475*tan(c/2 + (d*x)/2)^3)/4608)/(d*(8*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 16))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(5\sec(c+dx)+3)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**3,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-3), x)
```

3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$-\frac{16925\tan(c+dx)}{221184d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{4608d(5\sec(c+dx)+3)^2}-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1327104d}+$$

```
[Out] 21553/2654208*x+11215/1327104*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/144*ta
n(d*x+c)/d/(3+5*sec(d*x+c))^3-25/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))^2-16925
/221184*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 4060, 3919, 3831, 2657}

$$-\frac{16925\tan(c+dx)}{221184d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{4608d(5\sec(c+dx)+3)^2}-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1327104d}+$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (21553*x)/2654208 + (11215*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(132710
4*d) - (25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) - (25*Tan[c + d*x])
/(4608*d*(3 + 5*Sec[c + d*x])^2) - (16925*Tan[c + d*x])/(221184*d*(3 + 5*Se
c[c + d*x]))
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}+\frac{1}{144}\int \frac{48+45\sec(c+dx)-50\sec^2(c+dx)}{(3+5\sec(c+dx))^3}\,dx$$

$$= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}+\frac{\int \frac{1536-870\sec(c+dx)-75\sec}{(3+5\sec(c+dx))^2}}{13824}$$

$$= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+}$$

$$= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+}{221184d(3+5\sec}$$

$$= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+}{221184d(3+5\sec}$$

$$= \frac{21553x}{2654208}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(}{4608d(3+5\sec}$$

Mathematica [A] time = 0.53, size = 141, normalized size = 1.33

$$\frac{-5660475\sin(c+dx)-3082500\sin(2(c+dx))-582975\sin(3(c+dx))+8036352(c+dx)\cos(c+dx)+221}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (6307840*c + 6307840*d*x + 8036352*(c + d*x)*Cos[c + d*x] + 22430*ArcTan[2*
Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^3 + 2211840*c*Cos[2*(c + d*x)] + 221
1840*d*x*Cos[2*(c + d*x)] + 221184*c*Cos[3*(c + d*x)] + 221184*d*x*Cos[3*(c
 + d*x)] - 5660475*Sin[c + d*x] - 3082500*Sin[2*(c + d*x)] - 582975*Sin[3*(
c + d*x)])/(2654208*d*(5 + 3*Cos[c + d*x])^3)
```

fricas [A] time = 0.47, size = 159, normalized size = 1.50

$$\frac{884736\,dx\cos(dx+c)^3+4423680\,dx\cos(dx+c)^2+7372800\,dx\cos(dx+c)+4096000\,dx+11215\big(27\cos}{2654208\big(27\,d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2654208*(884736*d*x*cos(d*x + c)^3 + 4423680*d*x*cos(d*x + c)^2 + 7372800
*d*x*cos(d*x + c) + 4096000*d*x + 11215*(27*cos(d*x + c)^3 + 135*cos(d*x +
c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)
```

```
) - 300*(7773*cos(d*x + c)^2 + 20550*cos(d*x + c) + 16925)*sin(d*x + c))/(2
7*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)
```

giac [A] time = 0.19, size = 88, normalized size = 0.83

$$\frac{21553\,dx + 21553\,c - \frac{300\left(1037\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4576\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+11312\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\right)^{3}} + 22430\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2654208\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2654208*(21553*d*x + 21553*c - 300*(1037*tan(1/2*d*x + 1/2*c)^5 + 4576*ta
n(1/2*d*x + 1/2*c)^3 + 11312*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 4)^3 + 22430*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

maple [A] time = 0.55, size = 125, normalized size = 1.18

$$-\frac{25925\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{221184d\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{3575\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6912d\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824d\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}\right.}{2}\right.}{1327104d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^4,x)
```

```
[Out] -25925/221184/d/(tan(1/2*d*x+1/2*c)^2+4)^3*tan(1/2*d*x+1/2*c)^5-3575/6912/d
/(tan(1/2*d*x+1/2*c)^2+4)^3*tan(1/2*d*x+1/2*c)^3-17675/13824/d/(tan(1/2*d*x
+1/2*c)^2+4)^3*tan(1/2*d*x+1/2*c)-11215/1327104/d*arctan(1/2*tan(1/2*d*x+1/
2*c))+2/81/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 1.13, size = 171, normalized size = 1.61

$$-\frac{\frac{150\left(\frac{11312\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{4576\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1037\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{\frac{48\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{12\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+64} - 32768\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 11215\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{1327104\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1327104*(150*(11312*sin(d*x + c)/(cos(d*x + c) + 1) + 4576*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 1037*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 32768*arctan(sin(d*x + c)/(cos(
d*x + c) + 1)) + 11215*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

mupad [B] time = 1.09, size = 105, normalized size = 0.99

$$\frac{x}{81} - \frac{11215\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1327104\,d} - \frac{\frac{25925\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{221184}+\frac{3575\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6912}+\frac{17675\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{13824}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+48\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/cos(c + d*x) + 3)^4,x)
```

```
[Out] x/81 - (11215*atan(tan(c/2 + (d*x)/2)/2))/(1327104*d) - ((17675*tan(c/2 + (
d*x)/2))/13824 + (3575*tan(c/2 + (d*x)/2)^3)/6912 + (25925*tan(c/2 + (d*x)/
2)^5)/221184)/(d*(48*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + tan(c
/2 + (d*x)/2)^6 + 64))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\sec(c+dx)+3)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**4,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-4), x)
```

3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

```
[Out] 1/5*x+3/20*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/20*ln(2*cos(1/2*
d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3783, 2659, 206}

$$\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-1),x]
```

```
[Out] x/5 + (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20*d) - (3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5+3\sec(c+dx)}\,dx &= \frac{x}{5}-\frac{1}{5}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx\\ &= \frac{x}{5}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{5d}\\ &= \frac{x}{5}+\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+\right.\right.}{20d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.99

$$\frac{4(c+dx)+3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-1),x]
```

```
[Out] (4*(c + d*x) + 3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[2*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

fricas [A] time = 0.51, size = 52, normalized size = 0.74

$$\frac{8\,dx-3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right)-2\,\sin\left(dx+c\right)+\frac{5}{2}\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*d*x - 3*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 3*log(3/2*co
s(d*x + c) - 2*sin(d*x + c) + 5/2))/d
```

giac [A] time = 0.21, size = 43, normalized size = 0.61

$$\frac{4\,dx+4\,c-3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/20*(4*d*x + 4*c - 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 3*log(abs(tan(1/
2*d*x + 1/2*c) - 2)))/d
```

maple [A] time = 0.42, size = 51, normalized size = 0.73

$$\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c)),x)
```

```
[Out] 3/20/d*ln(tan(1/2*d*x+1/2*c)-2)-3/20/d*ln(tan(1/2*d*x+1/2*c)+2)+2/5/d*arcta
n(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.48, size = 70, normalized size = 1.00

$$\frac{8\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)-3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)+3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 2) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

mupad [B] time = 0.88, size = 21, normalized size = 0.30

$$\frac{x}{5}-\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5),x)
[Out] x/5 - (3*atanh(tan(c/2 + (d*x)/2)/2))/(10*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3\sec\left(c+dx\right)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x)
[Out] Integral(1/(3*sec(c + d*x) + 5), x)
```

3.528 $\int \frac{1}{(5+3\sec(c+dx))^2}\,dx$

Optimal. Leaf size=95

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c-\right.\right.}{1600d}$$

```
[Out] 1/25*x+123/1600*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-123/1600*ln(2
*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/80*tan(d*x+c)/d/(5+3*sec(d*x+c)
)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 206}

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c-\right.\right.}{1600d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] x/25 + (123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(1600*d) - (123*Log
[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(1600*d) + (9*Tan[c + d*x])/(80*d*
(5 + 3*Sec[c + d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))} - \frac{1}{80}\int \frac{-16+15\sec(c+dx)}{5+3\sec(c+dx)}\,dx$$
$$= \frac{x}{25} + \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))} - \frac{123}{400}\int \frac{\sec(c+dx)}{5+3\sec(c+dx)}\,dx$$
$$= \frac{x}{25} + \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))} - \frac{41}{400}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx$$
$$= \frac{x}{25} + \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))} - \frac{41\,\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{200d}$$
$$= \frac{x}{25} + \frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d} - \frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\text{s}\right.}{1600d}$$

Mathematica [A] time = 0.16, size = 162, normalized size = 1.71

$$\frac{5\cos(c+dx)\left(64(c+dx)+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] (5*Cos[c + d*x]*(64*(c + d*x) + 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - 123*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(64*c + 64*d*x +
123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 123*Log[2*Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 60*Sin[c + d*x]))/(1600*d*(3 + 5*Cos[c + d*x]))
```

fricas [A] time = 0.46, size = 102, normalized size = 1.07

$$\frac{640\,dx\cos(dx+c)+384\,dx-123\,(5\,\cos(dx+c)+3)\log\left(\frac{3}{2}\,\cos(dx+c)+2\,\sin(dx+c)+\frac{5}{2}\right)+123\,(5\,\cos(dx}{3200\,(5\,d\cos(dx+c)+3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3200*(640*d*x*cos(d*x + c) + 384*d*x - 123*(5*cos(d*x + c) + 3)*log(3/2*c
os(d*x + c) + 2*sin(d*x + c) + 5/2) + 123*(5*cos(d*x + c) + 3)*log(3/2*cos(
d*x + c) - 2*sin(d*x + c) + 5/2) + 360*sin(d*x + c))/(5*d*cos(d*x + c) + 3*
d)
```

giac [A] time = 0.19, size = 69, normalized size = 0.73

$$\frac{64\,dx+64\,c-\frac{180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4}-123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1600*(64*d*x + 64*c - 180*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
4) - 123*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 123*log(abs(tan(1/2*d*x + 1/2
*c) - 2)))/d
```

maple [A] time = 0.48, size = 87, normalized size = 0.92

$$-\frac{9}{160d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600d}-\frac{9}{160d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600d}+\frac{2}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^2,x)
```

```
[Out] -9/160/d/(tan(1/2*d*x+1/2*c)-2)+123/1600/d*ln(tan(1/2*d*x+1/2*c)-2)-9/160/d
/(tan(1/2*d*x+1/2*c)+2)-123/1600/d*ln(tan(1/2*d*x+1/2*c)+2)+2/25/d*arctan(t
an(1/2*d*x+1/2*c))
```

maxima [A] time = 0.47, size = 111, normalized size = 1.17

$$-\frac{\frac{180\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)}-128\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1600*(180*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(d
*x + c) + 1)) - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 123*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 2) - 123*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 2))/d
```

mupad [B] time = 0.88, size = 52, normalized size = 0.55

$$\frac{x}{25}-\frac{\frac{123\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{800}+\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{80\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^2,x)
```

```
[Out] x/25 - ((123*atanh(tan(c/2 + (d*x)/2)/2))/800 + (9*tan(c/2 + (d*x)/2))/(80*
(tan(c/2 + (d*x)/2)^2 - 4)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sec(c+dx)+5)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**2,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-2), x)
```

3.529 $\int \frac{1}{(5+3\sec(c+dx))^3} dx$

Optimal. Leaf size=120

$$\frac{963\tan(c+dx)}{12800d(3\sec(c+dx)+5)}+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(\right.}{}$$

```
[Out] 1/125*x+8361/256000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-8361/2560
00*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/160*tan(d*x+c)/d/(5+3*se
c(d*x+c))^2+963/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 3919, 3831, 2659, 206}

$$\frac{963\tan(c+dx)}{12800d(3\sec(c+dx)+5)}+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] x/125 + (8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(256000*d) - (836
1*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(256000*d) + (9*Tan[c + d*x])
/(160*d*(5 + 3*Sec[c + d*x])^2) + (963*Tan[c + d*x])/(12800*d*(5 + 3*Sec[c
+ d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
```

```
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} - \frac{1}{160}\int \frac{-32+30\sec(c+dx)-9\sec^2(c+dx)}{(5+3\sec(c+dx))^2}\,dx$$

$$= \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} + \frac{\int \frac{512-1365\sec(c+dx)}{5+3\sec(c+dx)}\,dx}{12800}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{8361\int \frac{\sec(c+dx)}{5+3\sec(c+d}}{64000}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\int \frac{1}{1+\frac{5}{3}\cos(c+}}{64000}$$

$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\,\text{Subst}\left(\int \frac{}{\frac{8}{3}}\right.}{}$$

$$= \frac{x}{125} + \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} - \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+d\right.\right.}{2560}$$

Mathematica [B] time = 0.32, size = 241, normalized size = 2.01

$$115560\sin(c+dx)+110700\sin(2(c+dx))+359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] (88064*c + 88064*d*x + 359523*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
60*Cos[c + d*x]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 25*Cos[2*(c + d
*x)]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 83
61*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 359523*Log[2*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 115560*Sin[c + d*x] + 110700*Sin[2*(c + d*x)])/(5
12000*d*(3 + 5*Cos[c + d*x])^2)
```

fricas [A] time = 0.49, size = 155, normalized size = 1.29

$$102400\,dx\cos(dx+c)^2+122880\,dx\cos(dx+c)+36864\,dx-8361\left(25\cos(dx+c)^2+30\cos(dx+c)+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/512000*(102400*d*x*cos(d*x + c)^2 + 122880*d*x*cos(d*x + c) + 36864*d*x -
 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) + 2*si
n(d*x + c) + 5/2) + 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*
cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(205*cos(d*x + c) + 107)*sin(d*
x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

giac [A] time = 0.17, size = 85, normalized size = 0.71

$$\frac{2048\,dx + 2048\,c - \frac{540\left(49\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 156\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^2} - 8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) + 8361\,\log\left(\left|\tan\left(\frac{1}{2}\,d\right.\right.\right.}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/256000*(2048*d*x + 2048*c - 540*(49*tan(1/2*d*x + 1/2*c)^3 - 156*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^2 - 8361*log(abs(tan(1/2*d*x + 1
/2*c) + 2)) + 8361*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

maple [A] time = 0.43, size = 123, normalized size = 1.02

$$-\frac{27}{2560d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000d}+\frac{27}{2560d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^3,x)
```

```
[Out] -27/2560/d/(tan(1/2*d*x+1/2*c)-2)^2-1323/25600/d/(tan(1/2*d*x+1/2*c)-2)+836
1/256000/d*ln(tan(1/2*d*x+1/2*c)-2)+27/2560/d/(tan(1/2*d*x+1/2*c)+2)^2-1323
/25600/d/(tan(1/2*d*x+1/2*c)+2)-8361/256000/d*ln(tan(1/2*d*x+1/2*c)+2)+2/12
5/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.49, size = 155, normalized size = 1.29

$$-\frac{\frac{540\left(\frac{156\sin(dx+c)}{\cos(dx+c)+1}-\frac{49\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16} - 4096\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/256000*(540*(156*sin(d*x + c)/(cos(d*x + c) + 1) - 49*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 16) - 4096*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) +
 8361*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 8361*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 2))/d
```

mupad [B] time = 0.95, size = 78, normalized size = 0.65

$$\frac{x}{125}-\frac{8361\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{128000\,d}+\frac{\frac{1053\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3200}-\frac{1323\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12800}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^3,x)
```

```
[Out] x/125 - (8361*atanh(tan(c/2 + (d*x)/2)/2))/(128000*d) + ((1053*tan(c/2 + (d
*x)/2))/3200 - (1323*tan(c/2 + (d*x)/2)^3)/12800)/(d*(tan(c/2 + (d*x)/2)^4
- 8*tan(c/2 + (d*x)/2)^2 + 16))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sec(c+dx)+5)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**3,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-3), x)
```

3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$\frac{38733\tan(c+dx)}{1024000d(3\sec(c+dx)+5)}+\frac{519\tan(c+dx)}{12800d(3\sec(c+dx)+5)^2}+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx\right.\right.}{2048000}$$

```
[Out] 1/625*x+278151/20480000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-27815
1/20480000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+3/80*tan(d*x+c)/d/
(5+3*sec(d*x+c))^3+519/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))^2+38733/1024000*
tan(d*x+c)/d/(5+3*sec(d*x+c))
```

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 3919, 3831, 2659, 206}

$$\frac{38733\tan(c+dx)}{1024000d(3\sec(c+dx)+5)}+\frac{519\tan(c+dx)}{12800d(3\sec(c+dx)+5)^2}+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx\right.\right.}{2048000}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-4),x]
```

```
[Out] x/625 + (278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20480000*d) -
(278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20480000*d) + (3*Tan[c
 + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + (519*Tan[c + d*x])/(12800*d*(5 + 3
*Sec[c + d*x])^2) + (38733*Tan[c + d*x])/(1024000*d*(5 + 3*Sec[c + d*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} - \frac{1}{240}\int \frac{-48+45\sec(c+dx)-18\sec^2(c+dx)}{(5+3\sec(c+dx))^3}\,dx$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{\int \frac{1536-4230\sec(c+dx)+1557}{(5+3\sec(c+dx))^2}}{38400}$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(}{1024000d(5+3}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(}{1024000d(5+3}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(}{1024000d(5+3}$$

$$= \frac{x}{625} + \frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d} - \frac{278151\log\left(2\cos\left(\frac{1}{2}(\right.\right.}{204}$$

Mathematica [B] time = 0.52, size = 344, normalized size = 2.37

$$\frac{52174260\sin(c+dx)+51462000\sin(2(c+dx))+24286500\sin(3(c+dx))+4096000c\cos(3(c+dx))+409}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-4),x]
```

```
[Out] (18284544*c + 18284544*d*x + 4096000*c*Cos[3*(c + d*x)] + 4096000*d*x*Cos[3
*(c + d*x)] + 155208258*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 347688
75*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 915*Cos[c
+ d*x]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 450*Cos[2*(c + d*x)
]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 27
8151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 155208258*Log[2*Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] - 34768875*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x
```

```
)/2] + Sin[(c + d*x)/2]] + 52174260*Sin[c + d*x] + 51462000*Sin[2*(c + d*x)
] + 24286500*Sin[3*(c + d*x)])/(81920000*d*(3 + 5*Cos[c + d*x])^3)
```

fricas [A] time = 0.53, size = 208, normalized size = 1.43

$$\frac{8192000\,dx\cos(dx+c)^3 + 14745600\,dx\cos(dx+c)^2 + 8847360\,dx\cos(dx+c) + 1769472\,dx - 278151\,\big(125\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/40960000*(8192000*d*x*cos(d*x + c)^3 + 14745600*d*x*cos(d*x + c)^2 + 8847
360*d*x*cos(d*x + c) + 1769472*d*x - 278151*(125*cos(d*x + c)^3 + 225*cos(d
*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) +
5/2) + 278151*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c) +
 27)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(44975*cos(d*x + c
)^2 + 47650*cos(d*x + c) + 12911)*sin(d*x + c))/(125*d*cos(d*x + c)^3 + 225
*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) + 27*d)
```

giac [A] time = 0.19, size = 98, normalized size = 0.68

$$\frac{32768\,dx + 32768\,c - \frac{540\left(2559\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 16032\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 26384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^3} - 278151\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.\right.}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/20480000*(32768*d*x + 32768*c - 540*(2559*tan(1/2*d*x + 1/2*c)^5 - 16032*
tan(1/2*d*x + 1/2*c)^3 + 26384*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 - 4)^3 - 278151*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 278151*log(abs(tan(1
/2*d*x + 1/2*c) - 2)))/d
```

maple [A] time = 0.55, size = 159, normalized size = 1.10

$$-\frac{27}{10240d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^3} - \frac{1431}{102400d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2} - \frac{69093}{2048000d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)} + \frac{278151\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{20480000d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^4,x)
```

```
[Out] -27/10240/d/(tan(1/2*d*x+1/2*c)-2)^3-1431/102400/d/(tan(1/2*d*x+1/2*c)-2)^2
-69093/2048000/d/(tan(1/2*d*x+1/2*c)-2)+278151/20480000/d*ln(tan(1/2*d*x+1/
2*c)-2)-27/10240/d/(tan(1/2*d*x+1/2*c)+2)^3+1431/102400/d/(tan(1/2*d*x+1/2*
c)+2)^2-69093/2048000/d/(tan(1/2*d*x+1/2*c)+2)-278151/20480000/d*ln(tan(1/2
*d*x+1/2*c)+2)+2/625/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.52, size = 194, normalized size = 1.34

$$-\frac{\frac{540\left(\frac{26384\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{16032\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2559\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{12\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - 64} - 65536\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 278151\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 278151}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20480000*(540*(26384*sin(d*x + c)/(cos(d*x + c) + 1) - 16032*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 2559*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) - 65536*arctan(sin(d*x + c)/(co
s(d*x + c) + 1)) + 278151*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 278151
*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

mupad [B] time = 1.10, size = 105, normalized size = 0.72

$$\frac{x}{625}-\frac{278151\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10240000\,d}-\frac{\frac{69093\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024000}-\frac{13527\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000}+\frac{44523\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64000}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+48\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/cos(c + d*x) + 5)^4,x)
```

```
[Out] x/625 - (278151*atanh(tan(c/2 + (d*x)/2)/2))/(10240000*d) - ((44523*tan(c/2
 + (d*x)/2))/64000 - (13527*tan(c/2 + (d*x)/2)^3)/32000 + (69093*tan(c/2 +
(d*x)/2)^5)/1024000)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 64))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\sec\left(c+dx\right)+5\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**4,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-4), x)
```

3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{2(a-b)\sqrt{a+b}\,\left(2a^2-9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-b)\sqrt{a}}{}$$

```
[Out] 2/15*(a-b)*(2*a^2-9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/15*(a-b)*(2*a+9*b)*cot(d*x+c)*EllipticF(
(a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-s
ec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/5*(a+b*sec(
d*x+c))^(3/2)*tan(d*x+c)/b/d-4/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.44, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\,\left(2a^2-9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-b)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(a - b
)*Sqrt[a + b]*(2*a + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*d) - (4*a*Sqrt[a + b*Sec[c
+ d*x]]*Tan[c + d*x])/(15*b*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(5*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c+dx)\left(\frac{3b}{2} - a\sec(c+ \cdots\right.}{5b}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+ \cdots}{5bd}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+ \cdots}{5bd}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(2a^2-9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right) \cdots}{15b^3d}$$

Mathematica [A] time = 13.50, size = 401, normalized size = 1.37

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2\left(9b^2-2a^2\right)\sin(c+dx)}{15b^2} + \frac{2a\tan(c+dx)}{15b} + \frac{2}{5}\tan(c+dx)\sec(c+dx)\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+ \cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(2*a^3
 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*b*(-2*a^2 + 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - 9*b^2)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b
^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]) + (S
qrt[a + b*Sec[c + d*x]]*((2*(-2*a^2 + 9*b^2)*Sin[c + d*x])/(15*b^2) + (2*a*
Tan[c + d*x])/(15*b) + (2*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c) + a} \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

maple [B] time = 1.79, size = 1584, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-cos(d*x+c)^2*a^2*b+3*b^3-9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+9*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+5*a*b^2*c
os(d*x+c)^3-2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^3+9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-9*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-2*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3+4*cos(d*x+c)*a*b^2+2*a^2*cos(d*x+c)^3*b-cos(d*x+c)^4*a^2*b-9*cos(d
*x+c)^4*a*b^2+2*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+9
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a*b^2+2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b+9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)^3*a^3+6*cos(d*x+c)^2*b^3-9*cos(d*x+c)^
3*b^3+2*cos(d*x+c)^4*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx + c) + a} \sec(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\cos(c + d\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(c + dx)} \sec^3(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b}}{3d}$$

```
[Out] -2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d-2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/
d
```

Rubi [A] time = 0.28, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3835, 4005, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*(a - b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(3*b*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
```

```
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}\int\frac{\sec(c+dx)(b+a\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{1}{3}a\int\frac{\sec(c+dx)(1+\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{3b^2d}$$

Mathematica [A] time = 10.79, size = 293, normalized size = 1.22

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(a\cos(c+dx)\tan\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*
d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*
b) + (2*Tan[c + d*x])/3))/d
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

maple [B] time = 1.45, size = 913, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-s
in(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+cos(
d*x+c)^3*a^2+cos(d*x+c)^3*a*b-cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b+cos(d*x+c)^
2*b^2-2*a*b*cos(d*x+c)-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+
c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\cos (c + d\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec (c + dx)} \sec^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=209

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\,\cot(c}{}$$

[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d

Rubi [A] time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3829, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\,\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = (a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{bd}$$

Mathematica [A] time = 10.34, size = 232, normalized size = 1.11

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d} - \frac{2\sqrt{a+b\sec(c+dx)}\left(\tan\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)+\frac{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}}{}\right.}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d - (2*Sqrt[a + b*Sec[c + d*x]]*(
((a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b + a*Co
s[c + d*x])*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2
]^2]*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

maple [B] time = 1.36, size = 814, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
```

```
c),((a-b)/(a+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+a*cos(d*x+c)^2-a*cos(
d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c) + a} \sec(dx+c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(c+dx)} \sec(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

3.534 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}\,(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(
b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}\,(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}\,(a+}{\sqrt{a+b}\,d}$$

Mathematica [A] time = 1.61, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\sqrt{a+b\sec(c+dx)}\left((b-a)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2\right.}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.42, size = 215, normalized size = 1.72

$$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(1+\cos\left(dx+c\right)\right)^2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)a-\mathrm{Ellip}\right.}{d\left(b+a\cos\left(dx+c\right)\right)\sin\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x)
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}$$

```
[Out] (a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d-b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3857, 4059, 3921, 3784, 3832, 4004}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/d
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
```

```
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{2}\int \frac{b-b\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{1}{2}\int \frac{b+b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx-\frac{1}{2}b\int$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{bd}$$

Mathematica [C] time = 18.25, size = 2713, normalized size = 8.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
(2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))
/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(
c + d*x)/2]^4]*((Sec[(c + d*x)/2]^2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*T
an[(c + d*x)/2]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
```

```
c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/(Sqrt[(-a +
 b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]
) + (a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(I*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[
-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqr
t[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Co
s[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(
a + b)]*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]) -
 (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)
/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c
 + d*x)/2])*(-(Sec[(c + d*x)/2]^4*Sin[c + d*x]) + 2*Cos[c + d*x]*Sec[(c + d
*x)/2]^4*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(a +
 b)]*Sqrt[b + a*Cos[c + d*x]]*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^(3/2)) + (S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-((Sqrt[(-a
 + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/Sqrt[2]) + Sqrt[2]*a*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sin[c + d*x]*Tan[(c + d*x)/2] - (Sqrt[(-a + b)/(a
 + b)]*(b + a*Cos[c + d*x])*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]) + ((I/2)*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*
Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ (I*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a +
 b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))
/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (b*Sqrt[(-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)])/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b))*Sqrt[1 + ((-a + b)*
Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)
]) - ((a - b)*Sqrt[(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)
/(a - b)])/(2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])))/(Sqrt[(-a
+ b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4
]) + ((I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]
^2)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2
*Sec[c + d*x]*Tan[c + d*x]))/(2*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]])))
```

fricas [F] time = 2.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c)+a} \cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

maple [B] time = 1.66, size = 826, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b
*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b+a*cos(d*x+c)^3-a*cos(d*x+c)^2+cos(d*x+c)^2*b-b*cos(d*x+c))
*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c)+a} \cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec (c + dx)} \cos (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=396

$$-\frac{\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{b\sin(c+d}{}$$

```
[Out] 1/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a/d+1/4*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(4*a^2-b^2)*cot(d*x+c)*EllipticPi((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/4*b*si
n(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/2*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d
```

Rubi [A] time = 0.60, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3857, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{b\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(2*a + b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^
2*d) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (Cos[c + d*x]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{4}\int \frac{\cos(c+dx)\left(b+2a\,\mathrm{se}\right.}{\sqrt{a+b}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c}{2d}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c}{2d}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{4ad}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{4ad}$$

Mathematica [C] time = 18.77, size = 1173, normalized size = 2.96

$$\frac{\sqrt{a+b\sec(c+dx)}\,\sin(2(c+dx))}{4d}+\frac{\sqrt{a+b\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d*x
]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)
*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2
*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*
a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*
Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

maple [B] time = 1.43, size = 1254, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-2*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-4*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*a^2+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+8*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2
*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x
+c)+a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(
d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*s
in(d*x+c)+2*cos(d*x+c)^4*a^2+3*cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2-cos(d*x+
c)^2*a*b+cos(d*x+c)^2*b^2-2*a*b*cos(d*x+c)-cos(d*x+c)*b^2)*(1+cos(d*x+c))^2
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c) + a} \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2 \sqrt{a + \frac{b}{\cos(c+d\,x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(c+dx)} \cos^2(c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=405

$$\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}}{}$$

```
[Out] -2/315*(a-b)*(8*a^4+33*a^2*b^2+147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/315*(a-b)*(8*a^3+6*a^2*b+
39*a*b^2-147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b^3/d+2/315*(8*a^2+49*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d
*x+c)/b^2/d-8/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/9*sec(d*x+c)*(
a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*(a+b*sec(d*x+c)
)^(1/2)*tan(d*x+c)/b^2/d
```

Rubi [A] time = 0.84, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3865, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b
^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(315*b^3*d) + (2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(315*b^2*d) + (2*(8*a^2 + 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(315*b^2*d) - (8*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^
2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)(a \cdots}{\cdots}$$

$$= -\frac{8a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d} + \frac{2\sec(c+dx)(a+b\sec(c+ \cdots}{9bd}$$

$$= \frac{2\left(8a^2+49b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315b^2d} - \frac{8a(a+b\sec(c+ \cdots}{63 \cdots}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a \cdots}{\cdots}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a \cdots}{\cdots}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(8a^4+33a^2b^2+147b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b \cdots}}{\cdots}\right.\right.}{315b^4d}$$

Mathematica [A] time = 17.83, size = 550, normalized size = 1.36

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{8\sec(c+dx)\left(22ab^2\sin(c+dx)-a^3\sin(c+dx)\right)}{315b^2}+\frac{2\sec^2(c+dx)\left(3a^2\sin(c+dx)+49b^2\sin(c+dx)\right)}{315b}+\frac{2\left(8a^4+\right.}{}\right.}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(8*
a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 +
2*a^3*b + 33*a^2*b^2 + 186*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 + 33*a^2*b^2 + 147*b^4)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315
*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2))
+ (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(8*a^4 + 33*a^2*b^2 + 147*b^
4)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^2*(3*a^2*Sin[c + d*x] + 49*b^2
*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(-(a^3*Sin[c + d*x]) + 22*a*b^2*S
in[c + d*x]))/(315*b^2) + (20*a*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*b*Sec[
c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x]))
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^5 + a\sec(dx+c)^4\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

maple [B] time = 2.12, size = 2522, normalized size = 6.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^5-4*cos(d*x+c)^6*a^4*b-147*cos(d*x+c)^5*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5-53*cos(d*
x+c)^2*a^2*b^3+8*cos(d*x+c)^6*a^5-8*cos(d*x+c)^5*a^5+147*cos(d*x+c)^5*b^5-9
```

```
8*cos(d*x+c)^4*b^5-14*cos(d*x+c)^2*b^5+cos(d*x+c)^3*a^3*b^2-52*cos(d*x+c)^3
*a*b^4-35*b^5+33*cos(d*x+c)^6*a^3*b^2+88*cos(d*x+c)^6*a^2*b^3+147*cos(d*x+c
)^6*a*b^4+8*cos(d*x+c)^5*a^4*b-34*cos(d*x+c)^5*a^3*b^2+33*cos(d*x+c)^5*a^2*
b^3-10*cos(d*x+c)^5*a*b^4-4*cos(d*x+c)^4*a^4*b-68*cos(d*x+c)^4*a^2*b^3-85*c
os(d*x+c)*a*b^4+8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+2*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2+33*cos(d*x+c)^5*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^
2*b^3+186*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^4-8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b-33*cos(d*x+c)^5*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2-33*c
os(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a^2*b^3-147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+8*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+2*cos(d*x+c)^4
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^3*b^2+33*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^2*b^3+186*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-8*cos(d*x+c)^4*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b-33
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^3*b^2-33*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-147*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+147*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*b^5-8*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^5-147*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+147*cos(d*x+c)^5*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5)/(b+a*
cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**4, x)
```

3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=342

$$-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\,\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{105b^2d}$$

[Out] 4/105*a*(a-b)*(3*a^2-41*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(a-b)*(6*a^2+57*a*b-25*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-4/35*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/7*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d

Rubi [A] time = 0.60, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4002, 4005, 3832, 4004}

$$-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\,\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{105b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]

[Out] (4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 25*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
```

```
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \sec(c+dx)\left(\frac{5b}{2} - a\sec(c+dx\right.}{7b}$$

$$= -\frac{4a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+}{7bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c+dx)}{35bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c+dx)}{35bd}$$

$$= \frac{4a(a-b)\sqrt{a+b}\,\left(3a^2-41b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^3d}$$

Mathematica [A] time = 14.05, size = 471, normalized size = 1.38

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(-\frac{4a\left(3a^2-41b^2\right)\sin(c+dx)}{105b^2} + \frac{2\sec(c+dx)\left(3a^2\sin(c+dx)+25b^2\sin(c+dx)\right)}{105b} + \frac{16}{35}a\tan(c+dx)\sec(\right.}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(3
*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + b*(-6*a^3 + 51*a^2*b + 82*a*b^2 + 25*b^3
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*(3*a^2 - 41*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]
*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-4*a*(3*a
```

```
^2 - 41*b^2)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]*(3*a^2*Sin[c + d*x]
+ 25*b^2*Sin[c + d*x]))/(105*b) + (16*a*Sec[c + d*x]*Tan[c + d*x])/35 + (2*
b*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

maple [B] time = 1.68, size = 1852, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x)
[Out] 2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(27*cos(d*x+c)^2*a^2*b^2+55*cos(d*x+c)^4*a^2*b^2-25*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+15*
b^4-6*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^4+6*cos(d*x+c)^4*a^3*b-82*cos(d*x+c)^4*a*b^3+68*cos(d*x
+c)^3*a*b^3+39*cos(d*x+c)*a*b^3-3*cos(d*x+c)^5*a^3*b-82*cos(d*x+c)^5*a^2*b^
2-25*cos(d*x+c)^5*a*b^3-3*cos(d*x+c)^3*a^3*b-6*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4-25*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*b^4+6*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^3*b-51*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-82*cos(d*x+c)^4*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-6*co
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^3*b+82*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+82*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+6*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
```

```
^3*b-51*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a^2*b^2-82*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-6*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+82*cos
(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a^2*b^2+82*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-6*cos(d*x+c)^4*a^4-25*b^4*cos(d*x+c)^
4+10*cos(d*x+c)^2*b^4+6*cos(d*x+c)^5*a^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin
(d*x+c)^5/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos (c + d\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^{\frac{3}{2}} \sec^3 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^2d}+\frac{2\tan(c\cdots}{}$$

```
[Out] -2/5*(a-b)*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/5*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*(a+b*sec(d*x+c))
^(3/2)*tan(d*x+c)/d+2/5*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.41, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3835, 4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^2d}+\frac{2\tan(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^2*d) - (2*(a - 3*b
)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b*d) + (2*a*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(5*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{3}{5}\int \sec(c+dx)(b+a\sec(c+dx$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx}{5d}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx}{5d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\left(a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{5b^2d}$$

Mathematica [A] time = 13.26, size = 408, normalized size = 1.45

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{5b}+\frac{4}{5}a\tan(c+dx)+\frac{2}{5}b\tan(c+dx)\sec(c+dx)\right)}{d(a\cos(c+dx)+b)} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a^
3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b*d*(b +
 a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c +
d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(a^2 + 3*b^2)*Sin[c + d*x])/(5*b) + (4*
a*Tan[c + d*x])/5 + (2*b*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d
*x]))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^3 + a\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

maple [B] time = 1.45, size = 1566, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b+4*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3
-cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2*b-3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+
4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b^2+3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a*b^2-3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+cos(d*x+c)^4*a^3+2*cos(d*x+c)^4*a^2*b+3*
cos(d*x+c)^4*a*b^2-cos(d*x+c)^3*a^3+a^2*cos(d*x+c)^3*b+3*cos(d*x+c)^3*b^3-3
*cos(d*x+c)^2*a^2*b-2*cos(d*x+c)^2*b^3-3*cos(d*x+c)*a*b^2-b^3)/(b+a*cos(d*x
+c))/cos(d*x+c)^2/sin(d*x+c)^5/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2(a-b)(3a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\right.}{3bd}$$

```
[Out] -8/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d+2/3*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*b*(a+b*sec(d*x+c))^(1/2)*tan
(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3830, 4005, 3832, 4004}

$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2(a-b)(3a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*(a - b)*(3*a - b)*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(3*b*d) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d)
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\sec(c+dx)\left(\frac{3a^2}{2}+\frac{b^2}{2}+2ab\,\text{se}\right.}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}((a-b)(3a-b))\int \frac{\sec(c+d}{\sqrt{a+b\sec(c}}$$

$$= -\frac{8a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{3bd}$$

Mathematica [A] time = 10.51, size = 304, normalized size = 1.22

$$-\frac{2\sqrt{a+b\sec(c+dx)}\left(-2\left(3a^2+4ab+b^2\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*(8*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*(3*a^2 + 4*a*
b + b^2)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 5*a*b*Sin[c + d*x] - 2*a^2*Sin[2*(c + d*x)] + 4*
a*b*Cos[c + d*x]*Tan[(c + d*x)/2] + 4*a^2*Cos[c + d*x]^2*Tan[(c + d*x)/2] -
 b^2*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x]))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^2+a\sec\left(dx+c\right)\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

maple [B] time = 1.27, size = 1106, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))^2*(3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+4*cos(d*x+c)^3*a^2+cos(d*x+c)^3*a*b-4*cos(d*x+c)^2*a^2+4*cos(d*x+c)^2*a*b+cos(d*x+c)^2*b^2-5*a*b*cos(d*x+c)-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x),x)

[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)

[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=309

$$\frac{2(2a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\,\cot(c+}{}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/d+2*(2*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/d-2*a*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3781, 3921, 3784, 3832, 4004}

$$\frac{2(2a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\,\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rule 3781

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[(Csc[
c + d*x]*(1 + Csc[c + d*x]))/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\,dx = b^2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(1+}{}}}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{-\frac{b(1+}{}}}{d}$$

Mathematica [C] time = 18.20, size = 882, normalized size = 2.85

$$\frac{2b\cos(c+dx)\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d(b+a\cos(c+dx))} + \frac{2\left(-b^2\sqrt{\frac{b-a}{a+b}}\,\tan^5\left(\frac{1}{2}(c+dx)\right)+ab\sqrt{\frac{b-a}{a+b}}\,\tan^5\left(\frac{1}{2}(c+dx)\right)-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c
+ d*x])) + (2*(a + b*Sec[c + d*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (2*I)*a^2*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*T
an[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/
```

```
2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.35, size = 1199, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*
(EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
sin(d*x+c)*a^2-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-2*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*a^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*sin(d*x+c)-2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b^2*sin(d*x+c)+a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-cos(d*x+c)^2*a*b+a*b*cos(d*x+c)
-cos(d*x+c)*b^2+b^2)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2), x)
```

3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=334

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\,(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{d}$$

```
[Out] a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d+(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/d-3*b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.33, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3864, 4058, 3921, 3784, 3832, 4004}

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\,(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a + 2*b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (a*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/d
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
```

```
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab-2b^2\sec(c+dx)+ab}{\sqrt{a+b\sec(c+dx}}$$

$$= \frac{a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab+\left(-ab-2b^2\right)\sec(c+}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{bd}$$

$$= \frac{a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{bd}$$

Mathematica [C] time = 12.00, size = 439, normalized size = 1.31

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(a\sqrt{\frac{b-a}{a+b}}\,\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*I)*a*(a -
b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)] + (4*I)*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*
```

```
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (12*I)
*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + a*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 2.26, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

maple [B] time = 1.26, size = 1026, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+6*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b-4*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*sin(d*x+c)+a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*a*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^3*a^2-cos(
d*x+c)^2*a^2+cos(d*x+c)^2*a*b-a*b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{3}{2}}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+5b\sin(c+dx$$

```
[Out] 5/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/d+1/4*(2*a+5*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*EllipticPi((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+5/4*b*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/d
```

Rubi [A] time = 0.54, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3864, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+5b\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a + 5*b)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a +
b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*
a*d) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*Cos[c + d*x]*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
```

```
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} - \frac{1}{4}\int \frac{\cos(c+dx)\left(-5\right.}{}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx}}{2d}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx}}{2d}$$

$$= \frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{4d}$$

$$= \frac{5(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{4d}$$

Mathematica [C] time = 18.59, size = 1159, normalized size = 2.97

$$\frac{a\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(2(c+dx))}{4d(b+a\cos(c+dx))}-\frac{(a+b\sec(c+dx))^{3/2}\left(-5b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+5ab\sqrt{\frac{b-a}{a+b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*Co
s[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*b*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] + 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 10*a*b*Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 5*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]^5 - 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (5*I)*(a - b)*b*EllipticE[I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2 - a*b - b^2)*EllipticF[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-
1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a\cos\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

maple [B] time = 1.23, size = 1440, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(5*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+5*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-4*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+2
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^2+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+6*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+5*a*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)
+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d
*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
sin(d*x+c)+2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b*sin(d*x+c)-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^2*sin(d*x+c)+8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*sin(d*x+c)*a^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*cos(d*x+c)^4*a^2+7*cos(d*x+c)^3*a*
b-2*cos(d*x+c)^2*a^2-5*cos(d*x+c)^2*a*b+5*cos(d*x+c)^2*b^2-2*a*b*cos(d*x+c)
-5*cos(d*x+c)*b^2)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+
a*cos(d*x+c))/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=463

$$\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{693b^2d}+\frac{2\left(8a^4+5\right.}{}$$

```
[Out] -2/693*a*(a-b)*(8*a^4+51*a^2*b^2+741*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/693*(a-b)*(8*a^4+6*a^3*
b+57*a^2*b^2-606*a*b^3+135*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/693*a*(8*a^2+67*b^2)*(a+b*sec(d*x
+c))^(3/2)*tan(d*x+c)/b^2/d+2/693*(8*a^2+81*b^2)*(a+b*sec(d*x+c))^(5/2)*tan
(d*x+c)/b^2/d-8/99*a*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*sec(d*x+c
)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d+2/693*(8*a^4+57*a^2*b^2+135*b^4)*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d
```

Rubi [A] time = 1.04, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3865, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{693b^2d}+\frac{2\left(57a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(693
*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3
+ 135*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) - (8*a*(a + b*Sec[c + d*
x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(
7/2)*Tan[c + d*x])/(11*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd} + \frac{2\int \sec(c+dx)(a+b$$

$$= -\frac{8a(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d} + \frac{2\sec(c+dx)(a+b\sec(c+dx)}{11bd}$$

$$= \frac{2\left(8a^2+81b^2\right)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{693b^2d} - \frac{8a(a+b\sec(c+dx)}{99b^2}$$

$$= \frac{2a\left(8a^2+67b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{693b^2d} + \frac{2\left(8a^2+81b^2\right)(a}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{693b^2d} + \frac{2a\left(8a^2+\right.}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{693b^2d} + \frac{2a\left(8a^2+\right.}{}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\left(8a^4+51a^2b^2+741b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a-}}\right.\right.}{693b^4d}$$

Mathematica [A] time = 16.96, size = 615, normalized size = 1.33

$$\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2\sec^2(c+dx)\left(3a^3\sin(c+dx)+229ab^2\sin(c+dx)\right)}{693b}+\frac{2}{693}\sec^3(c+dx)\left(113a^2\sin(c+dx)+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
8*a^5 + 8*a^4*b + 51*a^3*b^2 + 51*a^2*b^3 + 741*a*b^4 + 741*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^5
+ 2*a^4*b + 51*a^3*b^2 + 663*a^2*b^3 + 741*a*b^4 + 135*b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4 + 51*a^
2*b^2 + 741*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(693*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Se
c[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(8*a^4
 + 51*a^2*b^2 + 741*b^4)*Sin[c + d*x])/(693*b^3) + (2*Sec[c + d*x]^3*(113*a
^2*Sin[c + d*x] + 81*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(3*a^3*Sin[
c + d*x] + 229*a*b^2*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]*(-4*a^4*Sin[c
 + d*x] + 205*a^2*b^2*Sin[c + d*x] + 135*b^4*Sin[c + d*x]))/(693*b^2) + (46
*a*b*Sec[c + d*x]^3*Tan[c + d*x])/99 + (2*b^2*Sec[c + d*x]^4*Tan[c + d*x])/
11))/(d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^6+2\,ab\sec(dx+c)^5+a^2\sec(dx+c)^4\right)\sqrt{b\sec(dx+c)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^6 + 2*a*b*sec(d*x + c)^5 + a^2*sec(d*x + c)^4)*s
qrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

maple [B] time = 2.30, size = 2807, normalized size = 6.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/693/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-135*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*b^6+8*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
```

((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6-8*cos(d*x+c)^6*a^5*b+63*b^6+307
*cos(d*x+c)^6*a^2*b^4-51*cos(d*x+c)^6*a^3*b^3-741*cos(d*x+c)^6*a*b^5+4*cos(
d*x+c)^5*a^5*b+140*cos(d*x+c)^5*a^3*b^3+566*cos(d*x+c)^5*a*b^5-cos(d*x+c)^4
*a^4*b^2+160*cos(d*x+c)^4*a^2*b^4+116*cos(d*x+c)^3*a^3*b^3+86*cos(d*x+c)^3*
a*b^5+274*cos(d*x+c)^2*a^2*b^4+224*cos(d*x+c)*a*b^5+4*cos(d*x+c)^7*a^5*b-51
*cos(d*x+c)^7*a^4*b^2-205*cos(d*x+c)^7*a^3*b^3-741*cos(d*x+c)^7*a^2*b^4-135
*cos(d*x+c)^7*a*b^5+52*cos(d*x+c)^6*a^4*b^2-135*cos(d*x+c)^6*b^6+54*cos(d*x
+c)^4*b^6-8*cos(d*x+c)^7*a^6+8*cos(d*x+c)^6*a^6+18*cos(d*x+c)^2*b^6+8*cos(d
x+c)^5(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^6-135*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^6-2*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-51*cos(d*x+c)^6*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3
-663*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^2*b^4-741*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+8*cos(d*x+c)^5*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+51*cos(d
x+c)^5(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^4*b^2+51*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+741*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+741*cos(d*x+c
)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^5-8*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^5*b-2*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-51*cos(d*x+c)^5*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3-6
63*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b^4-741*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^5+8*cos(d*x+c)^6*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+51*cos(d*x
+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^4*b^2+51*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3+741*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+741*cos(d*x+c)^
6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b^5-8*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b

```
))^(1/2))*sin(d*x+c)*a^5*b)/(b+a*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)^5/b^3
```

maxima [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315bd}+\frac{2(a-b)\sqrt{a\cdots}}{}$$

```
[Out] 2/315*(a-b)*(10*a^4-279*a^2*b^2-147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*(a-b)*(10*a^3+165*a^
2*b-114*a*b^2+147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/315*(10*a^2-49*b^2)*(a+b*sec(d*x+c))^(3/2)
*tan(d*x+c)/b/d-4/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/9*(a+b*sec(d
*x+c))^(7/2)*tan(d*x+c)/b/d-4/315*a*(5*a^2-57*b^2)*(a+b*sec(d*x+c))^(1/2)*t
an(d*x+c)/b/d
```

Rubi [A] time = 0.78, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3840, 4002, 4005, 3832, 4004}

$$-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315bd}+\frac{2(a-b)\sqrt{a\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*
b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(315*b^2*d) - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(315*b*d) - (2*(10*a^2 - 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(315*b*d) - (4*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*
b*d) + (2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)\left(\frac{7b}{2} - a\sec(c+\right.}{9}$$

$$= -\frac{4a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2(a+b\sec(c+dx))^{7/2}\tan(}{9bd}$$

$$= -\frac{2\left(10a^2-49b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315bd} - \frac{4a(a+b\sec(c}{}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)}{}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(10a^4-279a^2b^2-147b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}}{}\right.\right.}{315b^3d}$$

Mathematica [A] time = 16.57, size = 552, normalized size = 1.38

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2\sec(c+dx)\left(5a^3\sin(c+dx)+163ab^2\sin(c+dx)\right)}{315b} + \frac{2}{315}\sec^2(c+dx)\left(75a^2\sin(c+dx)+\right.\right.}{d(a\cos(c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(10*
a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
```

```
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-10*a
^4 + 155*a^3*b + 279*a^2*b^2 + 261*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (10*a^4 - 279*a^2*b^2 -
147*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*
x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-10*a^4 + 279*a
^2*b^2 + 147*b^4)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^2*(75*a^2*Sin[c
 + d*x] + 49*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(5*a^3*Sin[c + d*x] +
 163*a*b^2*Sin[c + d*x]))/(315*b) + (38*a*b*Sec[c + d*x]^2*Tan[c + d*x])/63
 + (2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 2.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b^{2} \sec\left(dx+c\right)^{5} + 2\,ab \sec\left(dx+c\right)^{4} + a^{2} \sec\left(dx+c\right)^{3}\right)\sqrt{b \sec\left(dx+c\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3)*s
qrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(dx+c\right) + a\right)^{\frac{5}{2}} \sec\left(dx+c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

maple [B] time = 1.88, size = 2523, normalized size = 6.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-10*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^5-5*cos(d*x+c)^6*a^4*b+147*cos(d*x+c)^5*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5+170*cos(d
*x+c)^2*a^2*b^3+10*cos(d*x+c)^6*a^5-10*cos(d*x+c)^5*a^5-147*cos(d*x+c)^5*b^
5+98*cos(d*x+c)^4*b^5+14*cos(d*x+c)^2*b^5+80*cos(d*x+c)^3*a^3*b^2+82*cos(d*
x+c)^3*a*b^4+35*b^5-279*cos(d*x+c)^6*a^3*b^2-163*cos(d*x+c)^6*a^2*b^3-147*c
os(d*x+c)^6*a*b^4+10*cos(d*x+c)^5*a^4*b+199*cos(d*x+c)^5*a^3*b^2-279*cos(d*
x+c)^5*a^2*b^3-65*cos(d*x+c)^5*a*b^4-5*cos(d*x+c)^4*a^4*b+272*cos(d*x+c)^4*
a^2*b^3+130*cos(d*x+c)*a*b^4+10*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b-155*cos(d*x+c)^5*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2-2
79*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b^3-261*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
```

```
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4-10*cos(d*x+c)^5*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b+279*cos(d
*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^3*b^2+279*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^4+10*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a^4*b-155*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^3*b^2-279*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3-261*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^
4-10*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^4*b+279*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^2+279*cos(d*x+c)^4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^3+147*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*a*b^4-147*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^5-10*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5+147*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
5-147*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*b^5)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5/b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=333

$$\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{21bd}$$

```
[Out] -2/21*a*(a-b)*(3*a^2+29*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/21*(a-b)*(3*a^2-24*a*b+5*b^2)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+
2/7*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+b*sec(d*x+c))^(5/2)*tan(d*
x+c)/d+2/21*(3*a^2+5*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.57, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3835, 4002, 4005, 3832, 4004}

$$\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 29*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b^2*d) - (2*(a
 - b)*Sqrt[a + b]*(3*a^2 - 24*a*b + 5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) + (2*(3*a
^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + b*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(7*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{5}{7}\int \sec(c+dx)(b+a\sec(c+dx$$

$$=\frac{2a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(a+b\sec(c+dx))^{5/2}\tan(c+d}{7d}$$

$$=\frac{2\left(3a^2+5b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2a(a+b\sec(c+dx))^{3/}}{7d}$$

$$=\frac{2\left(3a^2+5b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2a(a+b\sec(c+dx))^{3/}}{7d}$$

$$=-\frac{2a(a-b)\sqrt{a+b}\left(3a^2+29b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+l}{a-l}\right.}{21b^2d}$$

Mathematica [A] time = 13.96, size = 474, normalized size = 1.42

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a\left(3a^2+29b^2\right)\sin(c+dx)}{21b}+\frac{2}{21}\sec(c+dx)\left(9a^2\sin(c+dx)+5b^2\sin(c+dx)\right)+\frac{6}{7}ab\right.}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(3*a^3 + 27*a^2*b + 29*a*b^2 + 5*b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ a*(3*a^2 + 29*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(21*b*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*S
ec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(3*a^
2 + 29*b^2)*Sin[c + d*x])/(21*b) + (2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] + 5*
b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d*x]*Tan[c + d*x])/7 + (2*b^2*Sec[c
+ d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

maple [B] time = 1.53, size = 1852, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-18*cos(d*x+c)^2*a^2*b^2-11*cos(d*x+c)^4*a^2*b^2+5*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-3*b
^4-3*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^4+3*cos(d*x+c)^4*a^3*b+29*cos(d*x+c)^4*a*b^3-22*cos(d*x+
c)^3*a*b^3-12*cos(d*x+c)*a*b^3+9*cos(d*x+c)^5*a^3*b+29*cos(d*x+c)^5*a^2*b^2
+5*cos(d*x+c)^5*a*b^3-12*cos(d*x+c)^3*a^3*b-3*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+5*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^4+3*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a^3*b+27*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+29*cos(d*x+c)^4*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-3*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^3*b-29*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-29*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+3*cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3
*b+27*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b^2+29*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

```
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-3*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-29*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^2*b^2-29*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-3*cos(d*x+c)^4*a^4+5*b^4*cos(d*x+c)^4-2
*cos(d*x+c)^2*b^4+3*cos(d*x+c)^5*a^4)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x
+c)^5/b
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=296

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}\ 2$$

```
[Out] -2/15*(a-b)*(23*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/15*(a-b)*(15*a^2-8*a*b+9*b^2)*cot(d*x+c)
*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*
b*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+16/15*a*b*(a+b*sec(d*x+c))^(1/2)*tan(
d*x+c)/d
```

Rubi [A] time = 0.45, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3830, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}\ 2$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (2*(a - b
)*Sqrt[a + b]*(15*a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (16*a*b*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*b*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(5*d)
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+d}$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\ \tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+}{5d}$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\ \tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+}{5d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\ (23a^2+9b^2)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}$$

Mathematica [A] time = 16.25, size = 440, normalized size = 1.49

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}\left(23a^2+9b^2\right)\sin(c+dx)+\frac{22}{15}ab\tan(c+dx)+\frac{2}{5}b^2\tan(c+dx)\sec(c+dx)\right)}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^(5/2)*(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*(
15*a^3 + 23*a^2*b + 17*a*b^2 + 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] - (23*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[c + d*x
])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^(5/2)*((2*(23*a^2 + 9*b^2)*Sin[c + d*x])/15 + (22*a*b*Tan[c + d*x])/15 +
 (2*b^2*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

maple [B] time = 1.35, size = 1775, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-34*cos(d*x+c)^2*a^2*b-3*b^3+9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-9*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+5*a*b
^2*cos(d*x+c)^3-23*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+9*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-23*c
os(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a^3-14*cos(d*x+c)*a*b^2+23*a^2*cos(d*x+c)^3*b+11*cos(d*x+c)^4*a^
2*b+9*cos(d*x+c)^4*a*b^2+15*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+15*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+23*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^2*b+17*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a*b^2-23*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+23*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*a^2*b+17*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-23*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
```

```
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*
b^2-23*cos(d*x+c)^3*a^3-6*cos(d*x+c)^2*b^3+9*cos(d*x+c)^3*b^3+23*cos(d*x+c)
^4*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{5}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

3.548 $\int (a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=352

$$\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}}{}$$

```
[Out] -14/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/d+2/3*(9*a^2-7*a*b+b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a^2*cot(d*x+c)*EllipticPi((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*b^2*(a+b
*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3782, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/d + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^3}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\sec(c+dx)+\frac{7}{2}ab^2\,\mathrm{se}}{\sqrt{a+b\sec(c+dx)}}$$

$$=\frac{2b^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{3a^3}{2}+\left(-\frac{7ab^2}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\right)\sec(c+dx}{\sqrt{a+b\sec(c+dx)}}$$

$$=-\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{s}}{}}}{3d}$$

$$=-\frac{14a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{s}}{}}}{3d}$$

Mathematica [C] time = 17.92, size = 713, normalized size = 2.03

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{14}{3}ab\sin(c+dx)+\frac{2}{3}b^2\tan(c+dx)\right)}{d(a\cos(c+dx)+b)^2}+\frac{2(a+b\sec(c+dx))^{5/2}\left(6ia^3\sqrt{1-\tan^2\left(\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(5/2)*((-7*I)*a*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
```

```
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - I*(3*a^3 - 9*a^2*b + 7*a*b^2 - b^3)*EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^3*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 7*a*b*Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^
2)^2)))/(3*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]
^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^
(5/2)*((14*a*b*Sin[c + d*x])/3 + (2*b^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c +
 d*x])^2)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \sec\left(dx+c\right)^2 + 2\,ab\sec\left(dx+c\right) + a^2\right)\sqrt{b\sec\left(dx+c\right)+a}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.28, size = 1514, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(7*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+7*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+3*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^3-9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
```

```
(1/2))*a^3+7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b+7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b-7*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-7*a^2*cos(d*x+c)^
3*b-a*b^2*cos(d*x+c)^3+7*cos(d*x+c)^2*a^2*b-7*cos(d*x+c)^2*a*b^2-cos(d*x+c)
^2*b^3+8*cos(d*x+c)*a*b^2+b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d
*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(5/2), x)
```

3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=353

$$\frac{\sqrt{a+b}\left(a^2+6ab-2b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+}}{}$$

```
[Out] (a-b)*(a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b/d+(a^2+6*a*b-2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+c)*EllipticPi((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*
(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a^2*sin(d*
x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3841, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(a^2+6ab-2b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a^2
+ 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (5*a*b*Sqrt[a + b]*Cot[c + d*x]*Elliptic
Pi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/d + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{\frac{5a^2b}{2} + 3ab^2\sec(c+dx) - \frac{1}{2}b\left(a^2 \right.}{\sqrt{a+b\sec(c+}}$$

$$= \frac{a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}\left(b\left(a^2-2b^2\right)\right)\int \frac{\sec(c+dx)(1+}{\sqrt{a+b\sec(}}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{bd}$$

Mathematica [B] time = 17.09, size = 780, normalized size = 2.21

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\,(a+b\sec(c+dx))^{5/2}\left(a^3\tan^5\left(\frac{1}{2}(c+dx)\right)-2a^3\tan^3\left(\frac{1}{2}(c+dx)\right)+a^3\tan\left(\frac{1}{2}(c+dx)\right)+2b\left(-3a^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*Co
s[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*(a^3*Tan[(c + d*x)/2] + a^2*b*Tan[(c + d*x)/2] - 2*a*b^2*Tan[(c + d*x
)/2] - 2*b^3*Tan[(c + d*x)/2] - 2*a^3*Tan[(c + d*x)/2]^3 + 4*a*b^2*Tan[(c +
 d*x)/2]^3 + a^3*Tan[(c + d*x)/2]^5 - a^2*b*Tan[(c + d*x)/2]^5 - 2*a*b^2*Ta
n[(c + d*x)/2]^5 + 2*b^3*Tan[(c + d*x)/2]^5 + 10*a^2*b*EllipticPi[-1, ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 10*a^2*b*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] + 2*b*(-3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(b + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 25.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)\sec(dx+c)^2 + 2\,ab\cos(dx+c)\sec(dx+c) + a^2\cos(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

maple [B] time = 1.36, size = 1640, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3-6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b+6*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
```

```
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+10*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a^2*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^3*sin(d*x+c)+a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*b-2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*b^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+10*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+cos(d*x+c)^
3*a^3-cos(d*x+c)^2*a^3+cos(d*x+c)^2*a^2*b+2*cos(d*x+c)^2*a*b^2-cos(d*x+c)*a
^2*b-2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-2*b^3)*(1+cos(d*x+c))^2*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$\frac{\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\,(4}{}$$

```
[Out] 9/4*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d+1/4*(2*a^2+9*a*b+8*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+15*b^2)*cot(d*x+c)*E
llipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(
a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.63, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\,(4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (9*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a^2 + 9*a*b
+ 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]*E
llipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(4*d) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (
a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
```

```
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(c+dx)\left(9\right.}{}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{4d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{4d}$$

Mathematica [C] time = 23.17, size = 4588, normalized size = 11.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[2*(c + d*x)])/(4*d*(b +
a*Cos[c + d*x])^2) + ((a^3/(2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
+ (3*a*b^2)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (11*a^2*b*Sqrt[
Sec[c + d*x]])/(8*Sqrt[b + a*Cos[c + d*x]]) + (b^3*Sqrt[Sec[c + d*x]])/Sqrt
[b + a*Cos[c + d*x]] + (9*a^2*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqr
t[b + a*Cos[c + d*x]]))*(a + b*Sec[c + d*x])^(5/2)*((18*I)*a*(a - b)*b*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)
/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
 b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c +
 d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-1/4*(Sqrt[Sec[(c + d*x)/2]^2]*T
an[(c + d*x)/2]*((18*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 -
 a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (a*Sin[c + d*x]*((18*I)*a*(a - b)*
b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
```

```
x)/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((
a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*(b + a*Cos
[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*(-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*((18*I)*a*(a - b)*b*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b
)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-
1 + Tan[(c + d*x)/2]^2)) + ((-9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((9*I)*a*(a - b)*b*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] - ((2*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x]
)/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])] + ((2*I)*a*(4*a^2 + 15*b^2)*Sqrt[(b + a*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[
c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])] + ((9*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - ((2*I)*(2*a^3 - a^2*b + 3*a*
b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + ((2*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) +
((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 9*a^2*b*Sqrt[(-a + b)/(
a + b)]*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + 9*a
*b*Sqrt[(-a + b)/(a + b)]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d
*x]*Tan[(c + d*x)/2] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (2*Sqrt[(-a + b)/(a + b)
]*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/
(Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c
+ d*x)/2]^2)/(a + b)]) - (2*a*Sqrt[(-a + b)/(a + b)]*(4*a^2 + 15*b^2)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*Sec[(c + d*x)/2]^2)/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a -
b))*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[
```

```
(c + d*x)/2]^2)/(a + b)]) - (9*a*(a - b)*b*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)
])/Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(4*Sqrt[(-a + b)/(a + b
)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - (((18*I)*a*(a - b)*b*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a -
 b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]
- 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2
]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(8*Sqrt[(-a + b)/(a + b
)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Se
c[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2))))
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^2\right)\sqrt{b\sec\left(dx+c\right)+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

maple [B] time = 1.23, size = 1646, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(9*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-4*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b-24*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
```

```
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3
+30*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a*b^2+9*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b+9*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-4*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-24*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+8*b^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+8*a^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+30*
b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a+2*cos(d*x+c)^4*a^3+11*a^2*cos(d*x+c)^3*b-2*cos(d*x+c)^2*a^3-9*cos(
d*x+c)^2*a^2*b+9*cos(d*x+c)^2*a*b^2-2*cos(d*x+c)*a^2*b-9*cos(d*x+c)*a*b^2)*
(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d
*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=460

$$\frac{(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\,(16a^2+26ab+33b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(se}{}}}{24d}$$

```
[Out] 1/24*(a-b)*(16*a^2+33*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/24*(16*a^2+26*a*b+33*b^2)*cot(d*x+c)*Ell
ipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*
(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5/8*b*(4*a
^2+b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((
a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d+1/24*(16*a^2+33*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/
2)/d+13/12*a*b*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/3*a^2*cos(d
*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.93, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{(16a^2+33b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\,(16a^2+26ab+33b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(se}{}}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d) + (Sqrt[a + b
]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(4*a^2
 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2 + 33*b^2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (13*a*b*Cos[c + d*x]*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(12*d) + (a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\cos^2(c+dx)}{}$$

$$= \frac{13ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d} + \frac{a^2\cos^2(c+dx)\sqrt{}}{}$$

$$= \frac{\left(16a^2+33b^2\right)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx)\sqrt{}}{}$$

$$= \frac{\left(16a^2+33b^2\right)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx)\sqrt{}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(16a^2+33b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{24bd}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(16a^2+33b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}\right.}{24bd}$$

Mathematica [B] time = 17.49, size = 1018, normalized size = 2.21

$$\frac{\cos^2(c+dx)\left(\frac{1}{12}\sin(c+dx)a^2+\frac{1}{12}\sin(3(c+dx))a^2+\frac{13}{24}b\sin(2(c+dx))a\right)(a+b\sec(c+dx))^{5/2}}{d(b+a\cos(c+dx))^2} + \sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*Sin[c + d*x])/12 + (13*a*b
*Sin[2*(c + d*x)])/24 + (a^2*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x])
^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a
^3*Tan[(c + d*x)/2] + 16*a^2*b*Tan[(c + d*x)/2] + 33*a*b^2*Tan[(c + d*x)/2]
 + 33*b^3*Tan[(c + d*x)/2] - 32*a^3*Tan[(c + d*x)/2]^3 - 66*a*b^2*Tan[(c +
d*x)/2]^3 + 16*a^3*Tan[(c + d*x)/2]^5 - 16*a^2*b*Tan[(c + d*x)/2]^5 + 33*a*
b^2*Tan[(c + d*x)/2]^5 - 33*b^3*Tan[(c + d*x)/2]^5 + 120*a^2*b*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^
3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 120*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^3*EllipticPi[-1, ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (16*a^3 + 16*a^2*b + 33*a*b^2 + 33*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(
38*a^2 - 13*a*b + 24*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d*(b + a*Cos[c + d
```

```
*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 28.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3\right)\sqrt{b\sec\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

maple [B] time = 1.28, size = 1881, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(18*cos(d*x+c)^2*a^2*b-16*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-30*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(
d*x+c)*b^3-33*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^3-59*a*b^2*cos(d*x+c)^3-8*cos(d*x+c)^5*a^3+48*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^3-33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^3*sin(d*x+c)+48*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))+33*cos(d*x+c)^2*a*b^2+26*cos(d*x+c)*a*b^2-34*cos(d*x
+c)^4*a^2*b-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*sin(d*x+c)-16*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-33*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+76*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-
26*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^2+16*cos(d*x+c)*a^2*b-120*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-30*EllipticP
```

```
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-16
*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b-33*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a+76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b*sin(d*x+c)-26*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^2*sin(d*x+c)-120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-8*cos(d*x+c)^3*a^3-33*cos(d*x+c)
^2*b^3+16*cos(d*x+c)^2*a^3+33*cos(d*x+c)*b^3)*(1+cos(d*x+c))^2*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=530

$$\frac{b(284a^2+15b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{(a-}{}$$

```
[Out] 1/192*(a-b)*(284*a^2+15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/192*(72*a^3+284*a^2*b+118*a*b^2+15*b^3
)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d-1/64*(48*a^4+120*a^2*b^2-5*b^4)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/192*b*(284*a^2+1
5*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/96*(36*a^2+59*b^2)*cos(d*x+c
)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+17/24*a*b*cos(d*x+c)^2*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2
)/d
```

Rubi [A] time = 1.30, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{b(284a^2+15b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{(36a^2+59b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a +
 b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sq
rt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2*
d) + (b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d)
 + ((36*a^2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
96*d) + (17*a*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}\int \frac{\cos^3(c+dx)\left(\frac{17}{\ldots}\right.}{\ldots}$$

$$= \frac{17ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d} + \frac{a^2\cos^3(c+dx)\sqrt{a\ldots}}{\ldots}$$

$$= \frac{\left(36a^2+59b^2\right)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{96d} + \frac{17ab\cos^2(\ldots}{\ldots}$$

$$= \frac{b\left(284a^2+15b^2\right)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{192ad} + \frac{\left(36a^2+59b^2\right)\cos(\ldots}{\ldots}$$

$$= \frac{b\left(284a^2+15b^2\right)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{192ad} + \frac{\left(36a^2+59b^2\right)\cos(\ldots}{\ldots}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\ldots}{192ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\ldots}{192ad}$$

Mathematica [C] time = 16.86, size = 1688, normalized size = 3.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((17*a*b*Sin[c + d*x])/96 + ((48
*a^2 + 59*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (a^2*
Sin[4*(c + d*x)])/32))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(
5/2)*(-284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 284*a^2*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2] - 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 568*a^3*b*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 30*a*b^3*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]^3 - 284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 2
84*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*a*b^3*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]^5 + (288*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*
I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (288*I)*a^4*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*I)*a^2*b^2*EllipticP
i[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (
a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
```

```
], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(284*a^3
 - 284*a^2*b + 15*a*b^2 - 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - (2*I)*(72*a^4 - 36*a^3*b + 38*a^2*b^2 - 59*a*b^3 - 15*b^4)*Elli
pticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*a*Sqrt[(-a + b)/(a +
b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)])
```

fricas [F] time = 34.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)^4\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^4\right)\sqrt{b\sec\left(dx+c\right)+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

maple [B] time = 1.47, size = 2330, normalized size = 4.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^2*(-30*cos(d*x+c)^2*a^2*b^2-254*cos(d*x+c)^4*a^2*b^
2-288*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a^4+30*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^4+284*cos(d*x+c)^2*a^3*b+15*cos(
d*x+c)^2*a*b^3+284*cos(d*x+c)*a^2*b^2+72*cos(d*x+c)*a^3*b-133*cos(d*x+c)^3*
a*b^3+118*cos(d*x+c)*a*b^3-184*cos(d*x+c)^5*a^3*b-172*cos(d*x+c)^3*a^3*b+14
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*
x+c)-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*
sin(d*x+c)-288*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^
(1/2))*a^4*sin(d*x+c)+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*b^4*sin(d*x+c)-48*cos(d*x+c)^6*a^4+72*cos(d*x+c)^2*a^4+15*
cos(d*x+c)*b^4-72*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
```

```
*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*b+644*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-118*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-284*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-284*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*a^2*sin(d*x+c)-15*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*a*sin(d*x+c)-720*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*
sin(d*x+c)+144*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)-15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)*b^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-72*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b+644*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x
+c)*a^2*b^2-118*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)*a*b^3-284*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-284*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2-1
5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*a-720*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-24*cos(d*x+c)^4*a^4-15*cos(d*x+
c)^2*b^4)*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x
+c))/sin(d*x+c)^5/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.553 $\int (a + b\sec(c + dx))^{7/2}\, dx$

Optimal. Leaf size=403

$$-\frac{2a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\,(58a^2}{}$$

```
[Out] -2/15*(a-b)*(58*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/15*(60*a^3-58*a^2*b+22*a*b^2-9*b^3)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*
a^3*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+26/15*a*b^2*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.50, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3782, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(-58a^2b+60a^3+22ab^2-9b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) + (2*Sqrt[a +
 b]*(60*a^3 - 58*a^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^3*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d
)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\sec(c+dx)}\left(\frac{5a^3}{2}+\frac{3}{2}b\left(5a^2\right.\right.$$

$$= \frac{26ab^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + $$

$$= \frac{26ab^2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + $$

$$= -\frac{2(a-b)\sqrt{a+b}\left(58a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{15d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(58a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{15d}$$

Mathematica [C] time = 15.70, size = 1150, normalized size = 2.85

$$2\left(-9b^4\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+9ab^3\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-58a^2b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+58a^3b\sqrt{\frac{b-a}{a+b}}\tan^5\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(7/2)*(58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2] + 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*a*b^3*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2] - 116*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 18*a*b^3*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]^5 - 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 9*
a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 9*b^4*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]^5 + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + I*b*(-58*a^3 + 58*a^2*b - 9*a*b^2 + 9*b^3)*EllipticE[I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(15*a^4 - 60*a^3*b + 58*a^2*b^2 - 22*a
*b^3 + 9*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*Sqrt
[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(1
- Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]
^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + T
an[(c + d*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(7/2)*((2*b*(58
*a^2 + 9*b^2)*Sin[c + d*x])/15 + (32*a*b^2*Tan[c + d*x])/15 + (2*b^3*Sec[c
+ d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^3)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3ab^2\sec(dx+c)^2+3a^2b\sec(dx+c)+a^3\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

maple [B] time = 1.50, size = 2185, normalized size = 5.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(74*cos(d*x+c)^2*a^2*b^2-16*cos(d*x+c)^4*a^2*b^2-9*cos(d*x+c)^3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+3*b^4
+15*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^4-58*cos(d*x+c)^4*a^3*b-9*cos(d*x+c)^4*a*b^3-10*cos(d*x+c
)^3*a*b^3+19*cos(d*x+c)*a*b^3+58*cos(d*x+c)^3*a^3*b-58*cos(d*x+c)^3*a^2*b^2
-60*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^3*b-58*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-22*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+58*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b+58*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+9*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+9*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-30
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^4+15*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-9*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+9*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4
-30*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*a^4-9*cos(d*x+c)^3*b^4-60*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-58*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2*b^2-22*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+58*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+58*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^
2+9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a*b^3+6*cos(d*x+c)^2*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin
(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(7/2),x)
[Out] Timed out
```

3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=359

$$\frac{8a(a-b)\sqrt{a+b}\left(12a^2+11b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^5d}+\frac{2(24\cdots}{}$$

```
[Out] 8/105*a*(a-b)*(12*a^2+11*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*
(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^5/d+2/105*(48*a^3-12*a^2*b+44*a*b^2+25*b^
3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(
1/2)/b^4/d+2/105*(24*a^2+25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/d-12
/35*a*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/7*sec(d*x+c)^2*(
a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.67, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3860, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^3d}+\frac{2\sqrt{a+b}\left(-12a^2b+48a^3+44ab^2+25b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\text{se}\cdots}{\cdots}}}{105b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (8*a*(a - b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^5*d) + (2*S
qrt[a + b]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d
) + (2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d)
 - (12*a*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (
2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
```

```
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{7bd} + \frac{\int \frac{\sec^2(c+dx)\left(4a+5b\sec(c+dx)-6a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}}{7b}$$

$$= -\frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{35b^2d} + \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$= \frac{2\left(24a^2+25b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{35b^2d}$$

$$= \frac{2\left(24a^2+25b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{35b^2d}$$

$$= \frac{8a(a-b)\sqrt{a+b}\,\left(12a^2+11b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{105b^5d}$$

Mathematica [A] time = 14.67, size = 463, normalized size = 1.29

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(-\frac{8a\left(12a^2+11b^2\right)\sin(c+dx)}{105b^4} + \frac{2\sec(c+dx)\left(24a^2\sin(c+dx)+25b^2\sin(c+dx)\right)}{105b^3} - \frac{12a\tan(c+dx)\sec(c+dx)}{35b^2} +\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*a*(12*a^3 +
12*a^2*b + 11*a*b^2 + 11*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3 - 12*a^2*b - 44*a*b^2 + 25*b^3)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*
(12*a^2 + 11*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(105*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]
) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-8*a*(12*a^2 + 11*b^2)*Sin[c + d*x
])/(105*b^4) + (2*Sec[c + d*x]*(24*a^2*Sin[c + d*x] + 25*b^2*Sin[c + d*x]))
/(105*b^3) - (12*a*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (2*Sec[c + d*x]^2*
Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{5}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{5}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.76, size = 1852, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(6*cos(d*x+c)^2*a^2*b^2-50*cos(d*x+c)^4*a^2*b^2-25*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+15*b
^4-48*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^4+48*cos(d*x+c)^4*a^3*b+44*cos(d*x+c)^4*a*b^3-16*cos(d*
x+c)^3*a*b^3-3*cos(d*x+c)*a*b^3-24*cos(d*x+c)^5*a^3*b+44*cos(d*x+c)^5*a^2*b
^2-25*cos(d*x+c)^5*a*b^3-24*cos(d*x+c)^3*a^3*b-48*cos(d*x+c)^3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4-25*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*b^4+48*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
```

```
(a+b))^(1/2))*sin(d*x+c)*a^3*b+12*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+44*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-
48*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^3*b-44*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-44*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+48*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3*b+12*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+44*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-48*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b
-44*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b^2-44*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-48*cos(d*x+c)^4*a^4-25*b^4*cos
(d*x+c)^4+10*cos(d*x+c)^2*b^4+48*cos(d*x+c)^5*a^4)/(b+a*cos(d*x+c))/cos(d*x
+c)^3/sin(d*x+c)^5/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=301

$$-\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}-\frac{2\sqrt{a+}}{}$$

```
[Out] -2/15*(a-b)*(8*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*
(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/15*(8*a^2-2*a*b+9*b^2)*cot(d*x+c)*Ellip
ticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-8/15*a*(
a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.42, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3860, 4082, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) - (2*Sqrt[
a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (8*a*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
```

```
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5bd} + \frac{\int \frac{\sec(c+dx)\left(2a+3b\sec(c+dx)-4a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,d}{5b}$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5bd} +$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5bd} +$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\left(8a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{15b^4d}$$

Mathematica [A] time = 14.59, size = 365, normalized size = 1.21

$$\frac{2\sqrt{\sec(c+dx)}\left(\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+b)\left(\left(8a^2+9b^2\right)\sin(c+dx)+b\tan(c+dx)(3b\sec(c+dx)-4a)\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^3 +
 8*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 +
 Sec[c + d*x]))] - 2*b*(8*a^2 + 2*a*b + 9*b^2)*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))] + (8*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]
) + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 + 9*b^2)*Sin[c + d*x] +
 b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*Sqrt[a + b*Sec[c + d
*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.66, size = 1584, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(4*cos(d*x+c)^2*a^2*b+3*b^3-9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+9*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+10*a*b^2
*cos(d*x+c)^3+8*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^3+9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-9*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+8*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a^3-cos(d*x+c)*a*b^2-8*a^2*cos(d*x+c)^3*b+4*cos(d*x+c)^4*a^2*b-9*cos
(d*x+c)^4*a*b^2-8*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-2*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+8*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b
+9*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b^2-8*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-2*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+8*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
```

```
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^2*b+9*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2+8*cos(d*x+c)^3*a^3+6*cos(d*x+c)^2*b^3-9*cos(d*x+c
)^3*b^3-8*cos(d*x+c)^4*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{4a(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d}+\frac{2\sqrt{a+b}\,(2a+b)\,\mathrm{co}}{}$$

```
[Out] 4/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/b^3/d+2/3*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*x+
c)/b/d
```

Rubi [A] time = 0.27, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3840, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}\,(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{4a(a-b)\sqrt{a+b}\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(2*a + b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d
)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{b}{2}-a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} - \frac{(2a)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b} + \frac{(2a+b)\int \frac{\sec(c}{\sqrt{a+b\,\mathrm{se}}}}{3b}$$

$$= \frac{4a(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{3b^3d}$$

Mathematica [A] time = 13.65, size = 341, normalized size = 1.40

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2\tan(c+dx)}{3b}-\frac{4a\sin(c+dx)}{3b^2}\right)}{d\sqrt{a+b\sec(c+dx)}} + \frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(a\cos(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(a + b)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (2*
a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*((-4*a*Sin[c + d*x])/(3*b^2) + (2*Tan[c + d*x]
)/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.61, size = 919, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
[Out] 2/3/d*(-1+cos(d*x+c))^2*(-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*sin(d*x+c)*
cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^2-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+2*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b-sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2+2*cos(d*x+c)^3*a^2-cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2+2*cos(d*x+c)
^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x+c)+b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(
1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx}{}$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3837, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{b^2d}$$

Mathematica [B] time = 19.30, size = 2189, normalized size = 10.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((-(
1/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (a*Sqrt[Sec[c + d*x]])/(
b*Sqrt[b + a*Cos[c + d*x]]) - (a*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*Sq
rt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Se
c[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] -
 (b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b*d*((1 + Cos[
c + d*x])^(-1))^(3/2)*Sqrt[1 + Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*((a*S
in[c + d*x]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Se
c[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(2*b*((1 + Cos[c + d*x])^(-1))^(3/2)*(b + a*Cos
[c + d*x])^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (3*Sin[c + d*x]*(-2*(a + b)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c +
 d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*
x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Se
c[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/
(2*b*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]]) - (Sec[c + d*x]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*S
qrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x])/(2*b*((1 + Cos[c + d*
x])^(-1))^(3/2)*Sqrt[b + a*Cos[c + d*x]]*(1 + Sec[c + d*x])^(3/2)) + (-1/2*
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sec[c + d*x]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2
- Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] -
 ((a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*(-((a*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + a*Sec[(
c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*Sec[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[(1
+ Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x])
)])/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)]) - ((a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
```

```
d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[1
 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2] - 2*
(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Sec[c + d*x]*Tan[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c
 + d*x])/((a + b)*(1 + Sec[c + d*x]))]*Tan[c + d*x] - b*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]^2*((1 + Sec[c + d*x])^(-1))^
(3/2)*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]*Tan[c + d*x]
+ (b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt
[(1 + Sec[c + d*x])^(-1)]*((b*Sec[c + d*x]*Tan[c + d*x])/((a + b)*(1 + Sec[
c + d*x])) - (Sec[c + d*x]*(a + b*Sec[c + d*x])*Tan[c + d*x])/((a + b)*(1 +
 Sec[c + d*x])^2)))/Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
)/(b*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[1 + Sec[
c + d*x]])))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.40, size = 639, normalized size = 3.13

$$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\left(1+\cos\left(dx+c\right)\right)^2\left(-1+\cos\left(dx+c\right)\right)^2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin\left(dx+c\right)\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d
```

```
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+a*cos(d*x+c)^2-a*cos(d*x+c)+b*cos(
d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] 2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b/d
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3832}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

Mathematica [A] time = 1.66, size = 93, normalized size = 0.94

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\,F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.12, size = 143, normalized size = 1.44

$$\frac{2\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(-1+\cos\left(dx+c\right)\right)\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,\left(1+\cos\left(dx\right.\right.}{d\left(b+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-1+cos(
d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c
))/sin(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{ad}$$

Mathematica [A] time = 1.57, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\right.\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 25.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.29, size = 178, normalized size = 1.68

$$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,(1+\cos(dx+c))^2\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{Ellip}\right.}{d\,(b+a\cos(dx+c))\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(d*x+c))/(b+a*cos(d*x+c))/sin(d
*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=338

$$\frac{b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{a+b\sec}}{ad}$$

```
[Out] (a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d+b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/
d
```

Rubi [A] time = 0.27, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3861, 4059, 3921, 3784, 3832, 4004}

$$\frac{b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{a+b\sec}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*
d) + (b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]),
 x_Symbol] :> -Simp[(Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(a*f), x] - Dis
t[b/(2*a), Int[(1 + Csc[e + f*x]^2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{b\int \frac{1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{b\int \frac{1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a} - \frac{b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{abd}$$

$$= \frac{(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{abd}$$

Mathematica [C] time = 24.42, size = 5060, normalized size = 14.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] integral(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)

maple [B] time = 1.42, size = 649, normalized size = 1.92

$$-\frac{(-1+\cos(dx+c))^2\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)

[Out] -1/d*(-1+cos(d*x+c))^2*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+a*cos(d*x+c)^3-a*cos(d*x+c)^2+cos(d*x+c)^2*b-b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=401

$$-\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4a^2d}+\frac{(2a-3b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+}}{}\right.\right.}{4a^2d}$$

```
[Out] -3/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/d+1/4*(2*a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*Ellip
ticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)
^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
-3/4*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.51, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3863, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}-\frac{3b\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((2*a - 3*b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*a^3*d) - (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (
Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
```

```
b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(3b-2a\sec(c+dx)-b\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2ad} + \cdots$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2ad} + \cdots$$

$$= -\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c\cdots}{a-b}}\cdots}{4a^2d}$$

$$= -\frac{3(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c\cdots}{a-b}}\cdots}{4a^2d}$$

Mathematica [C] time = 19.38, size = 1195, normalized size = 2.98

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\sin(2(c+dx))}{4ad\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Sec[
c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a*b*
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] - 6*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 3*a*b*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 + (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (
6*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a^2*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*b^2*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*I)*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(2*a^2 - a*
b + 3*b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt
[(-a + b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqr
t[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)
/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.39, size = 1259, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(8*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+6*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-3*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b-3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+
c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+8*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+6*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-3*
a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+
c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin
(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*sin(d*x+c)+2*a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*b*sin(d*x+c)+2*cos(d*x+c)^4*a^2-cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2+3*
cos(d*x+c)^2*a*b-3*cos(d*x+c)^2*b^2-2*a*b*cos(d*x+c)+3*cos(d*x+c)*b^2)*(1+c
os(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c
)^5/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c)^2}{\sqrt{b\sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=399

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2(4a+3b)\left(4a^2+b^2\right)}{}$$

```
[Out] -2/5*(16*a^4-8*a^2*b^2-3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b^5/d/(a+b)^(1/2)-2/5*(4*a+3*b)*(4*a^2+b^2)*cot(d*x+c)*
EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)-2*a
^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/5*a*(8*a^
2-3*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2-b^2)/d+2/5*(6*a^2-b^2)*
sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A] time = 0.77, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2-3b^2\right)\tan(c}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^5*Sqrt[a + b]*d) - (2*(
4*a + 3*b)*(4*a^2 + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(
8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(a^2 - b^2)*d)
 + (2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b^2*(a^2 - b^2)*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(6a^2-b^2\right)\sec^2(c+d}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(6a^2-b^2\right)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+a}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+a}{5b^3\left(a^2-b^2\right)d}$$

$$= -\frac{2\left(16a^4-8a^2b^2-3b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{5b^5\sqrt{a+b}\,d}$$

Mathematica [A] time = 17.08, size = 498, normalized size = 1.25

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(\frac{2\left(-16a^4+8a^2b^2+3b^4\right)\sin(c+dx)}{5b^4\left(b^2-a^2\right)}+\frac{2a^4\sin(c+dx)}{b^3\left(b^2-a^2\right)(a\cos(c+dx)+b)}-\frac{6a\tan(c+dx)}{5b^3}+\frac{2\tan(c+dx)\sec(c+dx)}{5b^2}\right.}{d(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(4*a^2 + b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*(2*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-4*a^2 - a*b
 + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + (4*a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(5*b^4*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b
*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-16*a^4
 + 8*a^2*b^2 + 3*b^4)*Sin[c + d*x])/(5*b^4*(-a^2 + b^2)) + (2*a^4*Sin[c + d
*x])/(b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])) - (6*a*Tan[c + d*x])/(5*b^3) +
 (2*Sec[c + d*x]*Tan[c + d*x])/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^5}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 2.04, size = 2480, normalized size = 6.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/5/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(8*cos(d*x+c)^2*a^4*b+8*c
os(d*x+c)^3*a^2*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^5-6*cos(d*x+c)^2*a^2*b^3-2*cos(d*x+c)*a
^3*b^2-16*cos(d*x+c)^3*a^4*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
```

```
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^5-6*cos(d*x+c)^3*a^3*b^2-5*co
s(d*x+c)^3*a*b^4-b^5+a^2*b^3+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^5+8*cos(d*x+c)^4*a^4*b-3*cos(
d*x+c)^4*a^2*b^3+2*cos(d*x+c)*a*b^4+8*cos(d*x+c)^4*a^3*b^2+3*cos(d*x+c)^4*a
*b^4-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^2*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^5-8*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^
2*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^2*a*b^4-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4*b-4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2+8*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a^2*b^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a*b^4+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^4*b-8*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3*b^
2-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^3*a^4*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3*b^2-2*cos(d*x+c)^2*b^5-16*cos(d*x+
c)^4*a^5+16*cos(d*x+c)^3*a^5+3*cos(d*x+c)^3*b^5-8*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^3-3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^3*a*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^4+16*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4*b
-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^3*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^2/(a-b)/(a+b
)/b^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2a\left(8a^2-5b^2\right)\cot(c+dx)\sqrt{\frac{b(\ldots}{}}}{}$$

```
[Out] 2/3*a*(8*a^2-5*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4*a+b)*cot(d*x+c)*EllipticF((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)-2*a^2*sec(d*x+c)*ta
n(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sec(d*x+
c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A] time = 0.50, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3845, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2a\left(8a^2-5b^2\right)\cot(c+dx)\sqrt{\frac{b(\ldots}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2*(2*a + b)*(4*
a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
```

```
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(4a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,d}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3b^2\left(a^2-b^2\right)d} -$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3b^2\left(a^2-b^2\right)d} +$$

$$= \frac{2a\left(8a^2-5b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{3b^4\sqrt{a+b}\,d}$$

Mathematica [A] time = 15.90, size = 470, normalized size = 1.45

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2a\left(5b^2-8a^2\right)\sin(c+dx)}{3b^3\left(b^2-a^2\right)}-\frac{2a^3\sin(c+dx)}{b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)}+\frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^3 + 2*a^2*b - 5*a*b
^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
```

```
 + b)] + a*(8*a^2 - 5*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*a*(-8
*a^2 + 5*b^2)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) - (2*a^3*Sin[c + d*x])/(b^
2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b
*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.66, size = 1792, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-4*cos(d*x+c)^2*a^2*b^2
-sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*b^4+b^4-a^2*b^2-8*cos(d*x+c)^2*a^3*b+5*cos(d*x+c)^2*a*b^3+4*cos(
d*x+c)*a^3*b+8*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^4-cos(d*x+c)^3*a*b^3-4*cos(d*x+c)*a*b^3+4*cos(
d*x+c)^3*a^3*b-8*cos(d*x+c)^3*a^4+8*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+5*cos(d*x+c)^3*a^2*b^2-8*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^3*b-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^3*b-5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b^2-5*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
```

```
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*cos(d*x+c)^2*a^4-sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^4-8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*cos(d*x+c)*a^3*b-2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+5*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3+8*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*
a^3*b-5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*cos(d*x+c)*a^2*b^2-5*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-cos(d*x+c)^2*b^4)/(b+a*cos(d*x+
c))/sin(d*x+c)/cos(d*x+c)/(a-b)/(a+b)/b^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(2a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+c}}{\sqrt{a+b}}\right.\right.}{b^3d\sqrt{a+b}}$$

```
[Out] -2*(2*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(
1/2)/b^3/d/(a+b)^(1/2)-2*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+se
c(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3839, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(2a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+c}}{\sqrt{a+b}}\right.\right.}{b^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) - (2*(2*a + b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b^2*Sqrt[a + b]*d) - (2*a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{ab}{2}-\frac{1}{2}\left(2a^2-b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b(a+b)} + \frac{\left(2a^2-b^2\right)\int \frac{\sec(}{}}{b\left(a^2\right.}$$

$$= -\frac{2\left(2a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{b^3\sqrt{a+b}\,d}$$

Mathematica [A] time = 14.33, size = 440, normalized size = 1.71

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(\frac{2\left(b^2-2a^2\right)\sin(c+dx)}{b^2\left(b^2-a^2\right)}+\frac{2a^2\sin(c+dx)}{b\left(b^2-a^2\right)(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{3/2}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-2*a^2 + b^2)*Sin[c + d*x])/(b^
2*(-a^2 + b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x])
)))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(
3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(2*a^3 + 2*a^2*b - a*b^2 - b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ 2*b*(-2*a^2 - a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] + (2*a^2 - b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)
/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^3}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.51, size = 1451, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-2*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a*b^2+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*sin(d*x+c)-2*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b+b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-b^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+2*cos(d*x+c)^2*a^3-cos(d*x+
c)^2*a^2*b-cos(d*x+c)^2*a*b^2-2*a^3*cos(d*x+c)+2*cos(d*x+c)*a^2*b+cos(d*x+c
)*a*b^2-cos(d*x+c)*b^3-a^2*b+b^3)/(b+a*cos(d*x+c))/sin(d*x+c)/b^2/(a+b)/(a-
b)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\cdots$$

```
[Out] 2*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/
(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b/d/(a+b)^(1/2)+2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3836, 4005, 3832, 4004}

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d)
 + (2*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} - \frac{a\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}\,d} + \frac{2\,\mathrm{co}}{}$$

Mathematica [A] time = 9.43, size = 249, normalized size = 1.05

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(a(a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)-4b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]*(4*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*b*(a + b)*Cos[(c +
d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] +
 a*(a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(b*(a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^2}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.21, size = 837, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b+sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^2-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b+a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b
-a^2*cos(d*x+c)+a*b*cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)/b/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(c+dx\right)}}{\left(a+b\sec{\left(c+dx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2\text{co}}{}$$

```
[Out] -2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+
b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/b/d/(a+b)^(1/2)-2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3833, 21, 3829, 3832, 4004}

$$-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) -
(2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
```

```
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} + \frac{b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,d}{a^2-b^2}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}\,d} + -\ 2$$

Mathematica [A] time = 10.31, size = 244, normalized size = 1.03

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left((a-b)\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)-4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*(a + b)*Cos[(c + d*
x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + (
a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\ \sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.26, size = 817, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b*sin(d*x+c)+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c)+b*co
s(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c + d\,x) \left(a + \frac{b}{\cos (c+d\,x)}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{a^2d}$$

```
[Out] 2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b
)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(c+}{\sqrt{a}}}{a\left(\right.}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} + \frac{}{a\left(\right.}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} - \frac{2\,\mathrm{c}}{}$$

Mathematica [C] time = 18.73, size = 1249, normalized size = 3.60

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
```

```
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 25.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

maple [B] time = 1.33, size = 1209, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
```

```
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*c
os(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+2*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+a*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)
-a*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x
+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(
d*x+c)-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*a^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-a*b*cos(d*x+c)+c
os(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=396

$$\frac{3b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}+\frac{b\left(a^2-3b^2\right)\tan(c+}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec}}$$

```
[Out] (a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a^2/b/d/(a+b)^(1/2)+(a+3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+3*b*cot(d*x+c)*EllipticPi((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(
d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+sin(d*x+c)/a/d/(
a+b*sec(d*x+c))^(1/2)+b*(a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x
+c))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3846, 4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\right.}{a^2bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((a + 3*b)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a^2*Sqrt[a + b]*d) + (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a
, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d)
 + Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(a^2 - 3*b^2)*Tan[c + d
*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{-\frac{3b}{2}+\frac{1}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{a}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{4}b\left(a^2-b^2\right)-\frac{1}{2}ab^2\sec(c+ \cdots}{\sqrt{a+b \cdots}}}{a^2\left(a \cdots\right.}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{4}b\left(a^2-b^2\right)+\left(-\frac{ab^2}{2}-\frac{1}{4}b\left(\cdots\right.\right.}{\sqrt{a+b\sec(\cdots}}}{a^2\left(a^2- \cdots\right.}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}} \cdots}{a^2b\sqrt{a+b}\,d}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}} \cdots}{a^2b\sqrt{a+b}\,d}$$

Mathematica [B] time = 15.62, size = 1069, normalized size = 2.70

$$\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2\sin(c+dx)b^3}{a^2\left(a^2-b^2\right)(b+a\cos(c+dx))}-\frac{2\sin(c+dx)b^2}{a^2\left(b^2-a^2\right)}\right)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\frac{\cdots}{1-\tan \cdots}} \cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b^2*Sin[c + d*x])/(a^2*(-a^2 +
b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a
 + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*Tan[(c + d*x)/2] + a^2*b
*Tan[(c + d*x)/2] - 3*a*b^2*Tan[(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2] - 2*a
^3*Tan[(c + d*x)/2]^3 + 6*a*b^2*Tan[(c + d*x)/2]^3 + a^3*Tan[(c + d*x)/2]^5
 - a^2*b*Tan[(c + d*x)/2]^5 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Tan[(c + d
*x)/2]^5 - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*
b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)
/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*T
an[(c + d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(
a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*
x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 24.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.39, size = 1662, normalized size = 4.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)^2*a^2*b+sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3+6*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*cos(d*x+c)*b^3-3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-a*b^2*cos(d*x+c)^3-3*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+3*cos
(d*x+c)^2*a*b^2-2*cos(d*x+c)*a*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+2*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^2-cos(d*x+c)*a^2*b-6*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b+6*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)+a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b-3*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
```

```
)/(a+b))^(1/2))*a+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+cos(d*x+c)^3*a^3-3*cos(d*x+c)^2*
b^3-cos(d*x+c)^2*a^3+3*cos(d*x+c)*b^3)/(b+a*cos(d*x+c))/sin(d*x+c)/a^2/(a+b
)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=470

$$-\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b}\left(4a^2+15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b}}{\right.\right.}}{4a^4d}$$

```
[Out] -1/4*(7*a^2-15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a^3/d/(a+b)^(1/2)+1/4*(2*a^2-5*a*b-15*b^2)*cot(d*x+c)*EllipticF((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-1/4*(4*a^2+15*
b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a^4/d-5/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)+1/2*cos
(d*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)-1/4*b^2*(7*a^2-15*b^2)*tan(d*
x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3846, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b^2\left(7a^2-15b^2\right)\tan(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\right.\right.}{4a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((7*a^2 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) + ((2*a^2 - 5*a*b -
15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)
*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(4*a^4*d) - (5*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a +
 b*Sec[c + d*x]]) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d
*x]]) - (b^2*(7*a^2 - 15*b^2)*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b
*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{5b}{2}+a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{4}\left(-4a^2-15b^2\right)-\frac{3}{2}ab\sec(c+dx}{(a+b\sec(c+dx)}}{2a^2}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2\left(7a^2-15b^2\right)\tan(c}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\sec}}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2\left(7a^2-15b^2\right)\tan(c}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\sec}}$$

$$= -\frac{\left(7a^2-15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{4a^3\sqrt{a+b}\,d}$$

$$= -\frac{\left(7a^2-15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{4a^3\sqrt{a+b}\,d}$$

Mathematica [C] time = 14.66, size = 1745, normalized size = 3.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^3*Sin[c + d*x])/(a^3*(-a^2 + b
^2)) + (2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2] - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^3*S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] + 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*b^
3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 7*a^3*b*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]^5 + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
+ 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (22*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*E
llipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a
^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
(30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
```

```
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + I*b*(7*a^3 - 7*a^2*b - 15*a*b^2 + 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (2*I)*(2*a^4 - a^3*b + 9*a^2*b^2 + 5*a*b^3 - 15*b^4
)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^3*Sqrt[(-a + b)
/(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2
)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
 d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 31.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.44, size = 2298, normalized size = 4.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-5*cos(d*x+c)^2*a^2*b^2
-2*cos(d*x+c)^4*a^2*b^2+8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4-30*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^4+7*cos(d*x+
c)^2*a^3*b-15*cos(d*x+c)^2*a*b^3+7*cos(d*x+c)*a^2*b^2-2*cos(d*x+c)*a^3*b+5*
cos(d*x+c)^3*a*b^3+10*cos(d*x+c)*a*b^3-5*cos(d*x+c)^3*a^3*b-4*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+15*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+8*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*
x+c)-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b^4*sin(d*x+c)-2*cos(d*x+c)^2*a^4-15*cos(d*x+c)*b^4+2*EllipticF((-1+cos(d*x
```

```
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-4*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-10*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)
-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin
(d*x+c)-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*a^2*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^3*a*sin(d*x+c)+22*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+15*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+2*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^3*b-4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2-10*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-7*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^3*b-7*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*co
s(d*x+c)*a^2*b^2+15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*cos(d*x+c)*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a+22*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^4
*a^4+15*cos(d*x+c)^2*b^4)/(b+a*cos(d*x+c))/sin(d*x+c)/a^3/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=427

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^3d\left(a^2-b^2\right)}+\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\cot}{}$$

```
[Out] 8/3*a*(4*a^4-7*a^2*b^2+2*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/(a-b)/b^5/(a+b)^(3/2)/d+2/3*(16*a^4+12*a^3*b-16*a^2*b^2
-9*a*b^3-b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(
a+b*sec(d*x+c))^(3/2)+4/3*a^3*(3*a^2-5*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a
+b*sec(d*x+c))^(1/2)+2/3*(2*a^2-b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/
(a^2-b^2)/d
```

Rubi [A] time = 0.93, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4090, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec}}{3b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)
*d) + (2*(16*a^4 + 12*a^3*b - 16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^4*(a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt
[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}\left(2a^2-b^2\right)\sec^2}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4}{}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2(}{}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2(}{}$$

$$= \frac{8a\left(4a^4-7a^2b^2+2b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3(a-b)b^5(a+b)^{3/2}d}$$

Mathematica [A] time = 19.09, size = 578, normalized size = 1.35

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{3b^4\left(b^2-a^2\right)^2} - \frac{2a^3\sin(c+dx)}{3b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2} - \frac{2\left(11a^3b^2\sin(c+dx)-7a^5\sin(c+d\right.}{3b^3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right.}{d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(4*a*(4*a^5 + 4*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4 + 2*b^5)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(
-16*a^5 - 4*a^4*b + 28*a^3*b^2 + 7*a^2*b^3 - 8*a*b^4 + b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c +
 d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-8*a*(4*a^4 - 7*a^
2*b^2 + 2*b^4)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2)^2) - (2*a^3*Sin[c + d*x])/
(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-7*a^5*Sin[c + d*x] + 11*
a^3*b^2*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (2*Tan
[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^5}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 2.04, size = 4176, normalized size = 9.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*4^(1/2)*(-8*cos(d*x+c)^4*a^6*b+13*cos(d*x+c)^4*a^4*b^3+8*cos(d*x+c)^4
*a^3*b^4-cos(d*x+c)^4*a^2*b^5+32*cos(d*x+c)^3*a^6*b+18*cos(d*x+c)^3*a^5*b^2
-56*cos(d*x+c)^3*a^4*b^3+8*cos(d*x+c)^3*a^3*b^4+16*cos(d*x+c)^3*a^2*b^5-2*c
os(d*x+c)^3*a*b^6-24*cos(d*x+c)^2*a^6*b+16*cos(d*x+c)^2*a^5*b^2+42*cos(d*x+
c)^2*a^4*b^3-28*cos(d*x+c)^2*a^3*b^4-13*cos(d*x+c)^2*a^2*b^5+8*cos(d*x+c)^2
*a*b^6-6*cos(d*x+c)*a^5*b^2+12*cos(d*x+c)*a^3*b^4-16*cos(d*x+c)^3*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^7-28*cos
(d*x+c)^4*a^5*b^2-cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^7-6*cos(d*x+c)*a*b^6-16*cos(d*x+c)^2*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^7-cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^7-2*b^5*a^2+a^4*b^3-7*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4+28*cos(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^3+28*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^3*b^4-8*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5-8*cos(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^6-16*cos(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6*b-
16*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*a^5*b^2+8*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^5-cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^6-16*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^6*b+28*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a^5*b^2+28*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
```

```
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^3-8*cos(d*x+c)^3*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^4-
8*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^5+16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6*b+20*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b^2-24*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^4*b^3-35*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^5+7*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
6-32*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a^6*b+12*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b^2+56*cos(d*x+c)^2*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^3+20*cos
(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^3*b^4-16*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^5-8*cos(d*x+c)^2*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^6+16*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
5*b^2+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^4*b^3-28*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^4-7*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^5+8*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^6+16*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^6*b+4*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b^2-28*cos(d*x+c)^3*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^3
+16*cos(d*x+c)^4*a^7-16*cos(d*x+c)^3*a^7-cos(d*x+c)^2*b^7+b^7)*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/cos(d*x+c)/(a-b)^2/(a
+b)^2/b^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(5/2), x)
```

3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(8a^4-15a^2b^2+3b^4\right)\cot(c+dx)}{}$$

```
[Out] -2/3*(8*a^4-15*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*(8*a^3+6*a^2*b-9*a*b^2-3*b^
3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b
^3/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c
))^(3/2)-8/3*a^2*(a^2-2*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^
(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3845, 4080, 4005, 3832, 4004}

$$-\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2b+8a^3-9ab^2-3b^3\right)\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^4*(a + b)^(3/2)
*d) - (2*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
 b*Sec[c + d*x])^(3/2)) - (8*a^2*(a^2 - 2*b^2)*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(4a^2-3b^2\right)\sec^2(c+d\ldots}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \ldots}{\ldots}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}} - \frac{\left(8a^3 \ldots\right.}{\ldots}$$

$$= -\frac{2\left(8a^4-15a^2b^2+3b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\ldots}}{3(a-b)b^4(a+b)^{3/2}d}$$

Mathematica [B] time = 23.53, size = 3345, normalized size = 9.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sin
[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (2*a^2*Sin[c + d*x])/(3*b*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (8*(-(a^4*Sin[c + d*x]) + 2*a^2*b^2*Sin[c + d*x])
)/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/
2)) - (2*(b + a*Cos[c + d*x])^2*((5*a^2)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c +
```

d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4)/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - b^2/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*b*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((Cos[(c + d*x)/2]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[(c + d*x)/2]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (8*a^4 - 15*a^2*b^2 + 3*b^4)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +

```
Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 -
((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((8*a^5 + 8*a^4*b - 15*a^3*b^2 -
15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1
- ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*
b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*
(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b^5)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 + 2
*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2 + 3*b^4)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c +
d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Ta
n[c + d*x]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.78, size = 3674, normalized size = 10.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*4^(1/2)*(8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^6-6*b^4*a^2+3*b^6+8*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6+3*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*b^6-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*b^6+8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*a^5*b+8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
```

)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3-15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5-8*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3+15*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4+6*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5+3*a^4*b^2+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^5+16*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^5*b-30*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^3*b^3+6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*a*b^5-8*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b-2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2+15*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3+6*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5-7*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-12*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4-8*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b-10*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2+13*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3-8*cos(d*x+c)^3*a^3*b^3-6*cos(d*x+c)^2*a^2*b^4+6*cos(d*x+c)*a*b^5+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6-3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6+4*cos(d*x+c)^3*a^5*b-16*cos(d*x+c)^2*a^5*b-10*cos(d*x+c)^2*a^4*b^2+12*cos(d*x+c)*a^5*b-8*cos(d*x+c)*a^4*b^2-22*cos(d*x+c)*a^3*b^3+30*cos(d*x+c)^2*a^3*b^3-6*cos(d*x+c)^2*a*b^5+15*cos(d*x+c)^3*a^4*b^2-3*cos(d*x+c)^3*a^2*b^4+15*cos(d*x+c)*a^2*b^4+21*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4+3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*

```
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*
b^5+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a^2*b^4+8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^5*b-15*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-15*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*si
n(d*x+c)*a^3*b^3-3*cos(d*x+c)*b^6-8*cos(d*x+c)^3*a^6+8*cos(d*x+c)^2*a^6)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+
b)^2/b^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)
```

3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=337

$$-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1}{}}}{3}$$

```
[Out] 4/3*a*(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d+2/3*(2*a^2+3*a*b-3*b^2)*cot(d*x+c)*Elliptic
F((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2/3*a
^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*tan(d*
x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3839, 4003, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1}{}}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^
2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*a^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*(a^2 - 3
*b^2)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
```

```
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3ab}{2}-\frac{1}{2}\left(2a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \cdots}{\cdots}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2a^2+\cdots\right.}{\cdots}$$

$$= \frac{4a\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+\cdots}{a-b}}\cdots}{3(a-b)b^3(a+b)^{3/2}d}$$

Mathematica [A] time = 16.09, size = 503, normalized size = 1.49

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(\frac{4a\left(3b^2-a^2\right)\sin(c+dx)}{3b^2\left(b^2-a^2\right)^2}-\frac{2a\sin(c+dx)}{3\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}-\frac{2\left(5ab^2\sin(c+dx)-a^3\sin(c+dx)\right)}{3b\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{5/2}}+\frac{4\sec^{\frac{5}{2}}(c\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*a*(-a^2 + 3*b^2)*Sin[c + d*x])/(
3*b^2*(-a^2 + b^2)^2) - (2*a*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d
*x])^2) - (2*(-(a^3*Sin[c + d*x]) + 5*a*b^2*Sin[c + d*x]))/(3*b*(-a^2 + b^2
)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*Cos[
c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(
```

```
a^3 + a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2 -
3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(3*(-(a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(
5/2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{3}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{3}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.33, size = 2733, normalized size = 8.11

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*4^(1/2)*(4*cos(d*x+c)^2*a^4*b+5*cos(d*x+c)^3*a^2*b^3-3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^5-12*cos(d*x+
c)^2*a^2*b^3+2*cos(d*x+c)*a^3*b^2-cos(d*x+c)^3*a^4*b+4*cos(d*x+c)^2*a^3*b^2
-3*cos(d*x+c)*a^4*b-6*cos(d*x+c)^3*a^3*b^2+6*cos(d*x+c)^2*a*b^4+7*cos(d*x+c
)*a^2*b^3-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
sin(d*x+c)*a^3*b^2-2*cos(d*x+c)^2*a^5+EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2-7*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3
-9*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*a*b^4+4*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2+6*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4-4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+
```

```
12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a^2*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^5-6*cos(d*x+c)*a*b^4+6*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3+6*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4-3
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
sin(d*x+c)*b^5-2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*sin(d*x+c)*a^5+2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2-EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3-6*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4-2*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^4*b+2*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^
4*b+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4*b-(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2-6*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*a^2*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*a*b^4+2*cos(d*x+c)^3*a^5-2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4*b+6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^2*a^3*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos
(d*x+c))^2/(a-b)^2/(a+b)^2/b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)
```

3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{3b^2d(a-b)\cdots}$$

```
[Out] 2/3*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*
(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d+2/3*a*tan(d*x+c)/(a^2
-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+
b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3836, 4003, 4005, 3832, 4004}

$$\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{3b^2d(a-b)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a - 3*
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*a*Tan[c + d*x])/(3*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Tan[c + d*x])/(3*(
a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
```

```
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{\sec(}{}}{}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(a-3b}{3}$$

$$= \frac{2\left(a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [A] time = 13.94, size = 486, normalized size = 1.53

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3b\left(b^2-a^2\right)^2}+\frac{2b\sin(c+dx)}{3\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{4\left(a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{5/2}} + 2\sec^{\frac{5}{2}}($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*
b*(-a^2 + b^2)^2) + (2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])
^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*Co
s[c + d*x])))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*S
ec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*b + 3
*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
```

```
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*Sqr
t[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^2}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.26, size = 2411, normalized size = 7.61

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*4^(1/2)*(-4*cos(d*x+c)^2*a^2*b^2+3*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+2*cos(d*x+c)^2*a^3*b
+6*cos(d*x+c)^2*a*b^3+cos(d*x+c)*a^2*b^2-sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4-2*cos(d*x+c)^3*a*b
^3-4*cos(d*x+c)*a*b^3-2*cos(d*x+c)^3*a^3*b+cos(d*x+c)^3*a^4-sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+3
*cos(d*x+c)^3*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*b^4*sin(d*x+c)+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+4*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+3*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^3-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*b-3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-3*sin(d*x+c)*cos(d*x+c)
```

```
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-cos
(d*x+c)^2*a^4+3*cos(d*x+c)*b^4+3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*a^2*sin(d*x+c)-3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*a*sin(d*x+c)-3*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*b^4*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*a^3*b+5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2*b^2+7*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b^3-2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
*a^3*b-4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*cos(d*x+c)*a^2*b^2-6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*cos(d*x+c)*b^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-3*cos(d*x+c)^2*b^4)*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2/b
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{8ab\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{3bd(a-}$$

```
[Out] -8/3*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-
b)/b/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d-2/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)-8/3*a*b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3833, 4003, 4005, 3832, 4004}

$$-\frac{8ab\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{3bd(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*(3*a - b)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*(a - b)*b*(a + b)^(3/2)*d) - (2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (8*a*b*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
```

```
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\sec(c+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(3a-b)\cdots}{3(a\cdots}$$

$$= -\frac{8a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d} + \frac{2(3\cdots}{\cdots}$$

Mathematica [A] time = 8.83, size = 360, normalized size = 1.18

$$-\frac{2\sec^3(c+dx)(a\cos(c+dx)+b)\left(b^2\left(b^2-a^2\right)\sin(c+dx)-b\left(b^2-5a^2\right)\sin(c+dx)(a\cos(c+dx)+b)+2a\cos\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b^2*(-a^2 + b^2)*Sin[c + d*x] - b*
(-5*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] - 4*a^2*(b + a*Cos[c + d*x
])^2*Sin[c + d*x] + 2*a*Cos[(c + d*x)/2]^2*(b + a*Cos[c + d*x])*(4*a*(a + b
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
 (3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + 2*a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])))/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.22, size = 1781, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*4^(1/2)*(3*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^3+8*cos(d*x+c)^2*a^2*b-4*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-4*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^3+3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3+sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+b^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-4*cos(d*x+c)^2*a*b^2+4*cos(d
*x+c)*a*b^2-5*a^2*cos(d*x+c)^3*b-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-4*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+7*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b+5*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^2-3*cos(d*x+c)*a^2*b-4*a^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-4*b^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+4*(co
```

```
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c
)-4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b+4*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+4*cos(d*x+c)^3*
a^3+cos(d*x+c)^3*b^3-4*cos(d*x+c)^2*a^3-cos(d*x+c)*b^3)*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c + d\,x)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c + dx)}{(a + b \sec (c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}+\frac{2b^2\left(7a^2-3b^2\right)\mathrm{t}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+}}$$

```
[Out] 2/3*(7*a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2/3*(6*a^2-a*b-3*b^2)*cot(d*x+c)*EllipticF(
(a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2*cot(d
*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/a^3/d+2/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*b^2
*(7*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2-ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\mathrm{s}}{}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (2*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*(6*a^
2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*Tan[c + d*x])/(3*a*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Tan[c + d*x])/(3
*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)+\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2\left(7a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{3a^2(a-b)(a+b)^{3/2}d}$$

$$= \frac{2\left(7a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 15.59, size = 1798, normalized size = 4.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b*(-7*a^2 + 3*b^2)*Sin[c + d*x])
/(3*a^2*(-a^2 + b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) - (8*(-2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(3*a^2*(a
^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b
+ a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*b*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2] - 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 7*a^3*b*Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*b^4*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (6*I)*a^4*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] + (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
6*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
```

```
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - I*b*(7*a^3 - 7*a^2*b - 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + I*(3*a^4 + 6*a^3*b - 13*a^2*b^2 - 2*a*b^3 +
6*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a^2*Sqrt[(-a
 + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*(-1 + Tan[(c + d*
x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + T
an[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 26.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c
)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

maple [B] time = 1.24, size = 3888, normalized size = 8.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*4^(1/2)*(-7*cos(d*x+c)^2*a^4*b-3*cos(d*x+c)^3*a^2*b^3-6*cos(d*x+c)^2*
sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*a^5-4*cos(d*x+c)^2*a^2*b^3-6*cos(d*x+c)*a^3*b^2+7*cos(d*x+c)^3*a^4*b+14*
cos(d*x+c)^2*a^3*b^2-8*cos(d*x+c)^3*a^3*b^2+4*cos(d*x+c)^3*a*b^4-6*cos(d*x+
c)^2*a*b^4+7*cos(d*x+c)*a^2*b^3-7*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2-6*sin(d*x+c)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+3*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+3*cos(d*x+c)^2*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5-6*
cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*a^5-6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
```

```
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+3*cos(d*x+c)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5-6*sin
(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^4*b+7*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*sin(d*x+c)*a^3*b^2-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3-2*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4-14*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
a^3*b^2+6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*a*b^4-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-3*cos(d*x+c
)*b^5+12*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a^3*b^2-6*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4-6*cos(d*x+c)*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^
4*b+12*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a^3*b^2+12*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3-6*cos(d*x+c)*sin(d*x+
c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4
+2*cos(d*x+c)*a*b^4-7*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*a^2*b^3+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4+6*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3*b^2+EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2*b^3-2*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b^4+12*sin(d*x+c)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3+3*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a
^4*b+9*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*a^4*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^4+6*(cos(d*x
```

```
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*a^4*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^2*a^3*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^3+3*cos(d*x+c)^2*b^5-7*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos
(d*x+c)^2*a^4*b-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/
sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/2), x)
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=510

$$\frac{5b\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^4d}+\frac{b\left(3a^2-5b^2\right)\text{ta}}{3a^2d\left(a^2-b^2\right)(a+b}$$

```
[Out] sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)+1/3*(3*a^4-26*a^2*b^2+15*b^4)*cot(d*x
+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/b/(a+b)
^(3/2)/d+1/3*(3*a^3+21*a^2*b-5*a*b^2-15*b^3)*cot(d*x+c)*EllipticF((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+5*b*cot(d*x+c)*E
llipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(
a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a
^4/d+1/3*b*(3*a^2-5*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+
1/3*b*(3*a^4-26*a^2*b^2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3846, 4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(-26a^2b^2+3a^4+15b^4\right)\tan(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{\left(21a^2b+3a^3-5ab^2-15b^3\right)\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*a^4 - 26*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)
*d) + ((3*a^3 + 21*a^2*b - 5*a*b^2 - 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*(
a + b)^(3/2)*d) + (5*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + Sin[
c + d*x]/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2 - 5*b^2)*Tan[c + d*x]
)/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2
 + 15*b^4)*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{-\frac{5b}{2}+\frac{3}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{a}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{15}{4}b\left(a^2-b^2\right)-}{}}{}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b\right.}{3a^3\left(a^2-b^2\right)^2}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b\right.}{3a^3\left(a^2-b^2\right)^2}$$

$$= \frac{\left(3a^4-26a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$= \frac{\left(3a^4-26a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^3(a-b)b(a+b)^{3/2}d}$$

Mathematica [B] time = 17.67, size = 1481, normalized size = 2.90

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-4*b^2*(-5*a^2 + 3*b^2)*Sin[c + d*
x])/(3*a^3*(-a^2 + b^2)^2) + (2*b^4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) + (2*(-11*a^2*b^3*Sin[c + d*x] + 7*b^5*Sin[c + d*x]))/(3*
a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) -
((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2
)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan
[(c + d*x)/2]^2)]*(3*a^5*Tan[(c + d*x)/2] + 3*a^4*b*Tan[(c + d*x)/2] - 26*a
^3*b^2*Tan[(c + d*x)/2] - 26*a^2*b^3*Tan[(c + d*x)/2] + 15*a*b^4*Tan[(c + d
*x)/2] + 15*b^5*Tan[(c + d*x)/2] - 6*a^5*Tan[(c + d*x)/2]^3 + 52*a^3*b^2*Ta
n[(c + d*x)/2]^3 - 30*a*b^4*Tan[(c + d*x)/2]^3 + 3*a^5*Tan[(c + d*x)/2]^5 -
 3*a^4*b*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*Ta
n[(c + d*x)/2]^5 + 15*a*b^4*Tan[(c + d*x)/2]^5 - 15*b^5*Tan[(c + d*x)/2]^5
- 30*a^4*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*b*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
```

```
)] + (3*a^5 + 3*a^4*b - 26*a^3*b^2 - 26*a^2*b^3 + 15*a*b^4 + 15*b^5)*Ellipt
icE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + 2*a*b*(6*a^3 + 9*a^2*b - 2*a*b^2 - 5*b^3)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*Sqrt[1 + T
an[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2
)))
```

fricas [F] time = 32.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.85, size = 4580, normalized size = 8.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/6/d*4^(1/2)*(3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^6+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^6+15*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6+3*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*b+3
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^4*b^2-26*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^3*b^3-26*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a^2*b^4+15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5+12*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2+18*EllipticF((-1+cos(d*x+c))/
```

```
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3-4*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4-10*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5+15
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^
2*sin(d*x+c)*a*b^5+6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^5*b-52*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^3*b^3+30*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*sin(d*x+c)*a*b^
5+12*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a^5*b+18*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-4*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3-10*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^2*b^4-23*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-11*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4+12*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5*
b+30*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*a^4*b^2+14*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3-6*cos(d*x+c)^4*a^4*b^2+3*cos(d*
x+c)^4*a^2*b^4-34*cos(d*x+c)^3*a^3*b^3+20*cos(d*x+c)^3*a*b^5-14*cos(d*x+c)^
2*a^2*b^4+10*cos(d*x+c)*a*b^5-30*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b+60*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^3*b^3-30*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a*b^5-30*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^5*b-30*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4
*b^2+60*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a^3*b^3+60*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+15*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^6+6*cos(d*x+c)^3
*a^5*b-6*cos(d*x+c)^2*a^5*b-17*cos(d*x+c)^2*a^4*b^2-3*cos(d*x+c)*a^4*b^2-18
*cos(d*x+c)*a^3*b^3+52*cos(d*x+c)^2*a^3*b^3-30*cos(d*x+c)^2*a*b^5+26*cos(d*
x+c)^3*a^4*b^2-15*cos(d*x+c)^3*a^2*b^4+26*cos(d*x+c)*a^2*b^4-30*(cos(d*x+c)
```

```
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^6-30*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*b^6-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^4*b^2+60*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+3*cos(d*x+c)^4*a^6-14*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*a^2*b^4-10*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*a*b^5+15*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^4+3*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^5*b-26*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a^4*b^2-26*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3-15*cos(d*x+c)*b^6-3*cos(d*x+c)^3
*a^6-30*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^5+15*cos(d*x+c)^2*b^6)*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)^2/(a+b)^2/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=562

$$-\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}}-\frac{\sqrt{a+b}\left(4a^2+35b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{4a^5d}$$

```
[Out] -7/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(3/2)+1/2*cos(d*x+c)*sin(d*x+c)/a/
d/(a+b*sec(d*x+c))^(3/2)-1/12*(33*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)*Ellip
ticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+
c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d+1/
12*(a+3*b)*(6*a^3-45*a^2*b+35*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1
/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-1/4*(4*a^2+35*b^2)*cot(d*x
+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a^5/d-1/12*b^2*(27*a^2-35*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+
c))^(3/2)-1/12*b^2*(33*a^4-170*a^2*b^2+105*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/
d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3846, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b^2\left(-170a^2b^2+33a^4+105b^4\right)\tan(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{b^2\left(27a^2-35b^2\right)\tan(c+dx)}{12a^3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{(a+3b)\left(-45a^2b+6a^3+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*a^4*(a - b)*(a + b)^(3
/2)*d) + ((a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(12*a^4*(a -
b)*(a + b)^(3/2)*d) - (Sqrt[a + b]*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(4*a^5*d) - (7*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (C
os[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(27*a^2
 - 35*b^2)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
- (b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2
*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{7b}{2}+a\sec(c+dx)+\frac{5}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{4}\left(-4a^2-35b^2\right)-\frac{5}{2}ab\sec(\ldots}{(a+b\sec(\ldots}}{2\ldots}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\ldots}{12a^3\left(a^2-b^2\right)d(a+\ldots}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\ldots}{12a^3\left(a^2-b^2\right)d(a+\ldots}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\ldots}{12a^3\left(a^2-b^2\right)d(a+\ldots}$$

$$= -\frac{\left(33a^4-170a^2b^2+105b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+\ldots}{a+b}}\ldots}{12a^4(a-b)(a+b)^{3/2}d}$$

$$= -\frac{\left(33a^4-170a^2b^2+105b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+\ldots}{a+b}}\ldots}{12a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 16.38, size = 2285, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^3*(-13*a^2 + 9*b^2)*Sin[c + d*
x])/(3*a^4*(-a^2 + b^2)^2) - (2*b^5*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) - (4*(-7*a^2*b^4*Sin[c + d*x] + 5*b^6*Sin[c + d*x]))/(3*a
^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(4*a^3)))/(d*(a +
 b*Sec[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2]^2)]*(33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 33*a^4*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 170*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] - 170*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*
a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*b^6*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 66*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 +
 340*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 210*a*b^5*Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]^5 - 33*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 170*a^
3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 170*a^2*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 + 105*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]^5 - 105*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (24*I)*a^6*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-
((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
 b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
```

```
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (24*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-33*a^5 + 33*a^4*b + 170*a^3*b
^2 - 170*a^2*b^3 - 105*a*b^4 + 105*b^5)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*(6*a^6 - 3*a^5*b + 57*a^4*b^2 + 54*a^3*b^3 - 184*a^
2*b^4 - 35*a*b^5 + 105*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(12*a^4*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(
5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c +
d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b \sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{2}}{b^{3} \sec\left(dx+c\right)^{3}+3\,ab^{2} \sec\left(dx+c\right)^{2}+3\,a^{2}b \sec\left(dx+c\right)+a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{2}}{\left(b \sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.84, size = 5638, normalized size = 10.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=535

$$-\frac{2\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^4d}+\frac{2b^2\left(13a^2-5b^2\right)\mathrm{ta}}{15a^2d\left(a^2-b^2\right)^2(a+b\,\mathrm{s}}$$

```
[Out] 2/15*(58*a^4-41*a^2*b^2+15*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2/15*(45*a^4-13*a^3*b-36*a^
2*b^2+5*a*b^3+15*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/5*b^2*tan(d*x+c)
/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(5/2)+2/15*b^2*(13*a^2-5*b^2)*tan(d*x+c)/a^
2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(3/2)+2/15*b^2*(58*a^4-41*a^2*b^2+15*b^4)*
tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(-41a^2b^2+58a^4+15b^4\right)\tan(c+dx)}{15a^3d\left(a^2-b^2\right)^3\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{15a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^{3/2}}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*a^3*(a - b)^2*(a + b)^(
5/2)*d) - (2*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b^3 + 15*b^4)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*a^3*(a - b)^2*(a + b)^(5/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a^4*d) + (2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(5/2)) + (2*b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Tan[c +
d*x])/(15*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2b^2\tan(c+dx)}{5a(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}(a^2-b^2)+\frac{5}{2}ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{5a(a^2-b^2)}$$

$$= \frac{2b^2\tan(c+dx)}{5a(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} + \frac{4\int}{}$$

$$= \frac{2b^2\tan(c+dx)}{5a(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2}{15}$$

$$= \frac{2b^2\tan(c+dx)}{5a(a^2-b^2)\,d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2(a^2-b^2)^2\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2}{15}$$

$$= \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

$$= \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

Mathematica [C] time = 16.53, size = 2346, normalized size = 4.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((2*b*(58*a^4 - 41*a^2*b^2 + 15*b^4)
*Sin[c + d*x])/(15*a^3*(-a^2 + b^2)^3) + (2*b^4*Sin[c + d*x])/(5*a^3*(a^2 -
 b^2)*(b + a*Cos[c + d*x])^3) + (2*(-19*a^2*b^3*Sin[c + d*x] + 11*b^5*Sin[c
 + d*x]))/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2*(74*a^4*b^2*Si
n[c + d*x] - 65*a^2*b^4*Sin[c + d*x] + 23*b^6*Sin[c + d*x]))/(15*a^3*(a^2 -
 b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(7/2)) + (2*(b + a*
Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*b*Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2] + 58*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
] - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 41*a^2*b^4*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c +
 d*x)/2] + 15*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 116*a^5*b*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 82*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^3 - 30*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*
a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 58*a^4*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]^5 + 41*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*a*b^5*S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^6*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (30*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (90*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*
a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
```

```
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*a^6*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (90*I)*a^4*b^2*EllipticPi[-((
a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*a^2*b^4*Ellipt
icPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-58*
a^5 + 58*a^4*b + 41*a^3*b^2 - 41*a^2*b^3 - 15*a*b^4 + 15*b^5)*EllipticE[I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(15*a^6 + 45*a^5*b - 103*a^4*b^2
- 23*a^3*b^3 + 86*a^2*b^4 + 10*a*b^5 - 30*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)]))/(15*a^3*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^3*d*(a
+ b*Sec[c + d*x])^(7/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2
]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c
 + d*x)/2]^2)))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{b^4\sec\left(dx+c\right)^4+4\,ab^3\sec\left(dx+c\right)^3+6\,a^2b^2\sec\left(dx+c\right)^2+4\,a^3b\sec\left(dx+c\right)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c
)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

maple [B] time = 1.57, size = 7838, normalized size = 14.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-7/2), x)
```

3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6b}{}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^{\frac{5}{2}}(c+dx)\,dx + b\int \sec^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{\sec(}$$
$$= \frac{6b\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\sec^{\frac{5}{2}}(c+}{}$$
$$= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6b\sqrt{\sec(c+dx)}\sin(c+}{5d}$$
$$= -\frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{}$$

Mathematica [A] time = 0.36, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(10a\sin(2(c+dx)) + 20a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) + 21b\sin(c+dx) + 9b\sin(3(c+dx)) - 36b\,c\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*a*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*b*Sin[c + d*x] + 10*a
*Sin[2*(c + d*x)] + 9*b*Sin[3*(c + d*x)]))/(30*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^3 + a\sec(dx+c)^2\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c) + a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 9.10, size = 502, normalized size = 3.32

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\right.}}{3\sqrt{-2\left(\sin^4\right.}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\sqrt{c}}{}$$

```
[Out] 2/3*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^{\frac{3}{2}}(c+dx)\,dx + b\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - a\int \frac{}{\sqrt{\mathrm{se}}}$$
$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \left(a\sqrt{\cos(}\right.$$
$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}\,F\left(\right.}{}$$

Mathematica [A] time = 0.27, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)(3a\cos(c+dx)+b) - 6a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2b\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*a*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(b + 3*a*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(dx+c\right)^{2} + a\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right) + a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [B] time = 8.11, size = 397, normalized size = 3.23

$$\frac{2\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*b*sin(1/2*d*x+1/2*c)^2+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*sin(1/2*d*x+
1/2*c)^2-12*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

3.581 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E(}{d}$$

```
[Out] 2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\,dx = a\int \sqrt{\sec(c+dx)}\,dx + b\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(a\sqrt{\cos(c+dx)}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\sec(c+dx)}\,\sin(c}{d}$$
$$= -\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\Big(\frac{1}{2}}{}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)-b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]))/d
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [A] time = 3.89, size = 148, normalized size = 1.53

$$-\frac{2\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,a+\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-2*b*sin(1/2*d*x+1
```

```
/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+b\int \sqrt{\sec(c+dx)}\,dx\\ &= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx+\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\\ &= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [A] time = 3.22, size = 152, normalized size = 2.03

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\cdots\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{d}$$

```
[Out] 2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c$$
$$= \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\right.$$
$$= \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}$$

Mathematica [A] time = 0.16, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(a\left(\sin(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
[Out] (Sqrt[Sec[c + d*x]]*(6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.15, size = 228, normalized size = 2.26

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-3*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3d}$$

```
[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}b\int \sqrt{\sec(c+$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c$$
$$= \frac{6a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

Mathematica [A] time = 0.40, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(3a\cos(c+dx)+5b)+18a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b\sqrt{\cos(c+dx)}\,F\left(\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(18*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*b + 3*a*Cos[c + d*x])*
Sin[2*(c + d*x)]))/(15*d)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

maple [A] time = 3.79, size = 262, normalized size = 2.06

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(24a+20b)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*b-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6b\sqrt{\mathrm{c}}}{}$$

```
[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/
21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6b\sqrt{\mathrm{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + b\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(3b)\int \frac{1}{\sqrt{\sec(c+d}}\\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,dx +\\
&= \frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} +\\
&= \frac{6b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{21d}\end{aligned}$$

Mathematica [A] time = 0.62, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(15a\cos(2(c+dx))+65a+42b\cos(c+dx))+100a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a + 42*b*Cos[c + d*
x] + 15*a*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.62, size = 290, normalized size = 1.92

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-360a-168b)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a-63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{2(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+d}{5}$$

```
[Out] 2/21*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*s
in(d*x+c)/d+2/7*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec(d
*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*
(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+d}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(7*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab$$

$$= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}$$

$$= \frac{12ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}$$

$$= -\frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{}}{}$$

Mathematica [A] time = 0.99, size = 139, normalized size = 0.70

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(20\left(7a^2+5b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(5\left(7a^2+5b^2\right)\cos(2(c+dx))+35a^2\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
+ 20*(7*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(35*a
^2 + 55*b^2 + 273*a*b*Cos[c + d*x] + 5*(7*a^2 + 5*b^2)*Cos[2*(c + d*x)] + 6
3*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec\left(dx+c\right)^4+2\,ab\sec\left(dx+c\right)^3+a^2\sec\left(dx+c\right)^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

maple [B] time = 10.77, size = 689, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*b/(8*sin(1/2*d*x+1/2*c)^6-12*
sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2 \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=175

$$\frac{2\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c}{}$$

```
[Out] 4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/5*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(5*a^2+3*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(5*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)\int$$

$$= \frac{2\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

$$= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2\left(5a^2+3b^2\right)\sqrt{\sec(c}}{5d}$$

$$= -\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + 4ab\sqrt{\cos}$$

Mathematica [A] time = 1.40, size = 126, normalized size = 0.72

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12\left(5a^2+3b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(3\left(5a^2+3b^2\right)\cos(2(c+dx))+15\left(a^2\right.\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a^2 + 3*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a
^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(5*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*d)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec\left(dx+c\right)^3+2\,ab\sec\left(dx+c\right)^2+a^2\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

maple [B] time = 10.23, size = 660, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2 \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.588 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{d}$$

```
[Out] 2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*a^2+b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\,\left(a^2+b^2\sec^2(c+dx)\right)\,d$$

$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - (2ab$$

$$= \frac{4ab\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \Big(2ab$$

$$= -\frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(3a^2+b^2\right)\sqrt{c}}{}$$

Mathematica [A] time = 0.36, size = 93, normalized size = 0.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\left(\left(3a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + b\sin(c+dx)(6a\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ (3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + b*(b + 6*a*C
os[c + d*x])*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec\left(dx+c\right)^2 + 2\,ab\sec\left(dx+c\right) + a^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

maple [B] time = 8.64, size = 514, normalized size = 3.81

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(6\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b+12*a*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2}{}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = (2ab)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(2ab\sqrt{\cos(c+dx)}\,\sqrt{\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + (($$
$$= \frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{d}$$

Mathematica [A] time = 0.20, size = 82, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + b*(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]
)))/d
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

maple [A] time = 4.45, size = 202, normalized size = 1.87

$$-\frac{2\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,ab-\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2+EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] 2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2639, 4045, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-a^2-3b^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(\left(-a^2-3\right.\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{3d}$$

Mathematica [A] time = 0.21, size = 87, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + a^2\sin(2(c+dx)) + 12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*Sin[2*(c
 + d*x)]))/(3*d)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2 + 2\,ab\sec(dx+c) + a^2}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.73, size = 283, normalized size = 2.53

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{Ell}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2+3*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-2*a^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+d}}{}$$

```
[Out] 2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2ab)\int\sqrt{\sec(c+dx)}\,dx - \frac{1}{5}\left(-3a^2-5b^2\right)$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{3}$$

Mathematica [A] time = 0.48, size = 100, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(6\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(2(c+dx))(3a\cos(c+dx)+10b)+20ab\sqrt{\cos}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*b +
3*a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

maple [A] time = 4.09, size = 321, normalized size = 2.28

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(24a^2+40ab\right)\left(\sin^4\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^2+40*a*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-6*a^2-20*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*a*b-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sec\left(c+dx\right)\right)^2}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)
```

3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c}{5d\sec^{\frac{3}{2}}(c}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+2/21*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c}{5d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a^2 + 7*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}dx = (2ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(6ab)\int \frac{1}{\sqrt{\sec(c+dx)}}dx - \frac{1}{7}\left(-5a^2-7b\right.$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} - \frac{1}{21}\left(-5a^2-7b^2\right.$$
$$= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c}{5d\sec^{\frac{3}{2}}(c}$$
$$= \frac{12ab\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\,F}{21}$$

Mathematica [A] time = 0.81, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(15a^2\cos(2(c+dx))+65a^2+84ab\cos(c+dx)+70b^2\right)+20\left(5a^2+7b^2\right)\sqrt{\cos(\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 20*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a^2
+ 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

fricas [F] time = 1.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(7/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

maple [A] time = 4.14, size = 362, normalized size = 2.07

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360a^2-336ab\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2+35*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2-126*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2), x)
```

3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=234

$$\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx}}{2}$$

```
[Out] 2/21*b*(21*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a*b^2*sec(d*x+c)^
(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+2
/5*a*(5*a^2+9*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(5*a^2+9*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*a^2 + 9*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b^2
*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\Big($$

$$= \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\Big($$

$$= \frac{2b\left(21a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{32ab^2\sec^{\frac{5}{2}}(c+dx)\sin(c}{35d}$$

$$= \frac{2a\left(5a^2+9b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b\left(21a^2+5b^2\right)\sec^{\frac{3}{2}}(c+}{21d}$$

$$= \frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\left(5\right.}{}$$

$$= -\frac{2a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(2\right.}{}$$

Mathematica [A] time = 3.74, size = 177, normalized size = 0.76

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(40b\left(21a^2+5b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-168a\left(5a^2+9b^2\right)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
 + d*x)/2, 2] + 40*b*(21*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x
)/2, 2] + 2*(210*a^2*b + 110*b^3 + 63*a*(5*a^2 + 13*b^2)*Cos[c + d*x] + 10*
(21*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 105*a^3*Cos[3*(c + d*x)] + 189*a*b^2*
Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \sec(dx+c)^4 + 3\,ab^2 \sec(dx+c)^3 + 3\,a^2 b \sec(dx+c)^2 + a^3 \sec(dx+c)\right)\sqrt{\sec(dx+c)}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c
)^2 + a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

maple [B] time = 12.80, size = 847, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a^2*b*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*b^2*a/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

3.594 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$\frac{6b\left(5a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)\cdots}{}$$

```
[Out] 8/5*a*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(3/2)*(a+b*sec(d
*x+c))*sin(d*x+c)/d+6/5*b*(5*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b*(
5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(a^2+b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{6b\left(5a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*b*(5*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
```

```
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx = \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= -\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(a\right.}{}$$

Mathematica [A] time = 1.65, size = 134, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(5a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)\left(3\left(5a^2+b^2\right.\right.}{}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*b*(5*a^2 + b^2)*EllipticE[(c +
 d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (b*(5*(3*a^2 + b^
2) + 10*a*b*Cos[c + d*x] + 3*(5*a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(2*Cos[c + d*x]^(5/2))))/(5*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3)*sqrt(sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^3 \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

maple [B] time = 11.12, size = 738, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*b^2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^3 \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^3\,\sqrt{\frac{1}{\cos \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=158

$$\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d}$$

```
[Out] 16/3*a*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c
)*sec(d*x+c)^(1/2)/d+2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a\left(3a^2-b^2\right)+\frac{1}{2}b\left(9a^2+\right.}{\sqrt{}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a\left(3a^2-b^2\right)+4ab^2\sec^2}{\sqrt{\sec(c+dx)}}$$

$$= \frac{16ab^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx}{3d}$$

$$= \frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{16ab^2\sqrt{\sec(c+dx)}}{3d}$$

$$= \frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(}}{}$$

Mathematica [A] time = 0.54, size = 106, normalized size = 0.67

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(6a\left(a^2-3b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2\left(9a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\,\mathrm{s}\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(6*a*(a^2 - 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*
x)/2, 2] + b*(2*(9*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]
+ 2*b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

maple [B] time = 8.74, size = 631, normalized size = 3.99

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(18\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3*sin(1
/2*d*x+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+18*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2-36*a*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*b^3+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+18
*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*b^3*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b(3a^2-b^2}{}$$

```
[Out] 2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/3*b*(a^2-3*b^2)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3841, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b(3a^2-b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4a^2b+\frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx)-\frac{1}{2}b\left(\right.}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4a^2b-\frac{1}{2}b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx +$$

$$= -\frac{2b\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + ($$

$$= \frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{2b\left(a^2-3b^2\right)\sqrt{\sec(c}}{3d}$$

$$= \frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\left(a^2+9b^2\right)\sqrt{\cos(}}{}$$

Mathematica [A] time = 0.63, size = 108, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(a^3\cos(c+dx)+3b^3\right)+2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-6b\left(b^2-3a\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*b*(-3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + 2*(3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

maple [A] time = 3.98, size = 303, normalized size = 1.83

$$-\frac{2\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+9*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-2*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{5d}$$

```
[Out] 2/5*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*b*sin(d*x+c)
/d/sec(d*x+c)^(1/2)+6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{3}{2}a\left(a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}b\,(}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{1}{2}b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx +$$

$$= \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \left(b\left(a^2+b^2\right)\right)\int \sqrt{\sec(c+}$$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + $$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(a^2+b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 0.50, size = 106, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(10b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
a^2*(5*b + a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

maple [A] time = 3.86, size = 376, normalized size = 2.41

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(8a^3+20a^2b\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(8*a^3+20*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-2*a^3-10*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-3*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2), x)
```

3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)\sqrt{\cos(c}}{}$$

```
[Out] 32/35*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))*sin(d*x+
c)/d/sec(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2
/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5
*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}a\left(5a^2+21b^2\right)\sec(c+dx)+\frac{1}{2}}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(5a^2+21b^2\right)\sqrt{c}}{}$$

Mathematica [A] time = 1.07, size = 132, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(a\sin(2(c+dx))\left(15a^2\cos(2(c+dx))+65a^2+126ab\cos(c+dx)+210b^2\right)+20a\left(5a^2+21b^2\right)\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 20*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + a*(65*a^2 + 210*b^2 + 126*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)
])*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.76, size = 421, normalized size = 2.12

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360a^3-504a^2b\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(7/2), x)
```

3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

```
[Out] 40/63*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*a*(7*a^2+27*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2
/21*b*(15*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*a*(7*a^2+27*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(15*a^2+7*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.24, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a^2*b*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*a*(7*a^2 + 27*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*b*(15*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{10a^2b+\frac{1}{2}a\left(7a^2+27b^2\right)\sec(c+dx)+}{\sec^{\frac{7}{2}}(c+dx}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{10a^2b+\frac{1}{2}b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(a+b\sec(c+dx))\sin(c+}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+$$

$$= \frac{2a\left(7a^2+27b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a\left(7a^2+27b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2b\left(15a^2+7b^2\right)\surd}{}$$

Mathematica [A] time = 1.44, size = 159, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(120b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+168a\left(7a^2+27b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 120*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
```

```
x)/2, 2] + (7*a*(43*a^2 + 108*b^2)*Cos[c + d*x] + 5*(234*a^2*b + 84*b^3 + 5
4*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(126
0*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

maple [A] time = 3.53, size = 470, normalized size = 2.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2240a^3+2160a^2b\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+105*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=287

$$\frac{14b^2(7a^2+b^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{8ab(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx}}{}$$

```
[Out] 8/21*a*b*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+14/45*b^2*(7*a^2+b^2)*
sec(d*x+c)^(5/2)*sin(d*x+c)/d+44/63*a*b^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9
*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+2/15*(15*a^4+54*a^2*b
^2+7*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/15*(15*a^4+54*a^2*b^2+7*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(7*a^2+5*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3842, 4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{14b^2(7a^2+b^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{8ab(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(54a^2b^2+15a^4+7b^4)\sin}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^4 + 54*a^2*
b^2 + 7*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (8*a*b*(7*a^2 + 5*b^
2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (14*b^2*(7*a^2 + b^2)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(45*d) + (44*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d) + (2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
9*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int \sec^{\frac{3}{2}}(c+dx)$$

$$= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{9d}$$

$$= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{9d}$$

$$= \frac{8ab\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{14b^2\left(7a^2+b^2\right)\sec^{\frac{5}{2}}(c+}{45d}$$

$$= \frac{2\left(15a^4+54a^2b^2+7b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{8ab\left(7a^2+5b^2\right)}{}$$

$$= \frac{8ab\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2\left(15a\right.}{}$$

$$= -\frac{2\left(15a^4+54a^2b^2+7b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{15d}$$

Mathematica [A] time = 2.43, size = 256, normalized size = 0.89

$$-\frac{2(a+b\sec(c+dx))^4\left(-315a^4\sin(c+dx)-420a^3b\tan(c+dx)-1134a^2b^2\sin(c+dx)-60ab\left(7a^2+5b^2\right)\sqrt{\cos}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^4*(21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - 60*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2] - 315*a^4*Sin[c + d*x] - 1134*a^2*b^2*Sin[c + d*x
] - 147*b^4*Sin[c + d*x] - 420*a^3*b*Tan[c + d*x] - 300*a*b^3*Tan[c + d*x]
- 378*a^2*b^2*Sec[c + d*x]*Tan[c + d*x] - 49*b^4*Sec[c + d*x]*Tan[c + d*x]
- 180*a*b^3*Sec[c + d*x]^2*Tan[c + d*x] - 35*b^4*Sec[c + d*x]^3*Tan[c + d*x
]))/(315*d*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4\sec(dx+c)^5+4\,ab^3\sec(dx+c)^4+6\,a^2b^2\sec(dx+c)^3+4\,a^3b\sec(dx+c)^2+a^4\sec(dx+c)\right)\sqrt{\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^5 + 4*a*b^3*sec(d*x + c)^4 + 6*a^2*b^2*sec(d*x +
 c)^3 + 4*a^3*b*sec(d*x + c)^2 + a^4*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

maple [B] time = 15.93, size = 1174, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*a*b^3*(-1/56*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1
/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-12/5*a
^2*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^4*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+8*a^3*b*(-1/6*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)
^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d
*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^4\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4,x)
[Out] Timed out
```

3.601 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=247

$$\frac{2b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{8ab\left(5a^2+3b^2\right)\sqrt{\cos(c+\dots}}{}$$

```
[Out] 2/21*b^2*(39*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+36/35*a*b^3*sec(d*x+c
)^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)
/d+8/5*a*b*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-8/5*a*b*(5*a^2+3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*a^4+42*a^2*b^2+5*
b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.37, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3842, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(42a^2b^2+21a^4+5b^4\right)\sqrt{\dots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-8*a*b*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*(5*a^2 + 3*b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b^2*(39*a^2 + 5*b^2)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(21*d) + (36*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(35*d) + (2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^4\,dx = \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{\sec(c+dx}$$

$$= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx)}{7d}$$

$$= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx)}{7d}$$

$$= \frac{8ab\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b^2\left(39a^2+5b^2\right)\sec^{\frac{3}{2}}(}{21d}$$

$$= \frac{8ab\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b^2\left(39a^2+5b^2\right)\sec^{\frac{3}{2}}(}{21d}$$

$$= -\frac{8ab\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(}{}$$

Mathematica [A] time = 1.72, size = 168, normalized size = 0.68

$$\frac{2\sec^{\frac{7}{2}}(c+dx)\left(-84ab\left(5a^2+3b^2\right)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\left(84a\left(5a^2+3b^2\right)\cos^3(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 5*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Cos[c + d*x]^(7/2)*Ellip
ticF[(c + d*x)/2, 2] + b*(15*b^3 + 5*b*(42*a^2 + 5*b^2)*Cos[c + d*x]^2 + 84
*a*(5*a^2 + 3*b^2)*Cos[c + d*x]^3)*Sin[c + d*x] + 42*a*b^3*Sin[2*(c + d*x)]
))/(105*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4 \sec\left(dx+c\right)^4 + 4\,ab^3 \sec\left(dx+c\right)^3 + 6\,a^2b^2 \sec\left(dx+c\right)^2 + 4\,a^3b \sec\left(dx+c\right) + a^4\right)\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^4\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

maple [B] time = 12.66, size = 925, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^4
*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)-8/5*a*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*a^3*b*(-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/s
in(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+12*a^2*b^2*(-1/6*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*
d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
```

```
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^4\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4,x)
[Out] Timed out
```

3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5a^4-$$

```
[Out] 28/15*a*b^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*(29*a^2+3*b^2)*sin(d*x+c)
*sec(d*x+c)^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)*sec(d*x+c)^(1/2)/
d+2/5*(5*a^4-30*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d+8/3*a*b*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3842, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(-30a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*(29*a^2 + 3*b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (28*a*b^3*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
 !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(\frac{1}{2}a\left(\right.\right.}{}$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+dx}{5d}$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2b^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+dx}{5d}$$

$$= \frac{2b^2\left(29a^2+3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2}{}$$

$$= \frac{8ab\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2b^2\left(29a^2+3b^2\right)\sqrt{}}{}$$

$$= \frac{2\left(5a^4-30a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab\left(3a^2+\right.}{}$$

Mathematica [A] time = 2.38, size = 146, normalized size = 0.70

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(b\left(80a\left(3a^2+b^2\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\sin(c+dx)\left(9\left(10a^2+b^2\right)\cos(2(c+dx))+\right.\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Cos[c + d*x]^(5/2)*Ell
ipticE[(c + d*x)/2, 2] + b*(80*a*(3*a^2 + b^2)*Cos[c + d*x]^(5/2)*EllipticF
[(c + d*x)/2, 2] + 2*b*(15*(6*a^2 + b^2) + 40*a*b*Cos[c + d*x] + 9*(10*a^2
+ b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])))/(30*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \sec (dx+c)^4 + 4\,ab^3 \sec (dx+c)^3 + 6\,a^2b^2 \sec (dx+c)^2 + 4\,a^3 b \sec (dx+c) + a^4}{\sqrt{\sec (dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec (dx+c) + a)^4}{\sqrt{\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

maple [B] time = 10.30, size = 907, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5*b^4/(8*sin(1/2
*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x
+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^
2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)+12*a^2*b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1)+8*a*b^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec (dx+c) + a)^4}{\sqrt{\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)} dx$

Optimal. Leaf size=208

$$-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{s}}{d}$$

```
[Out] -2/3*b^2*(a^2-b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^2*(a+b*sec(d*x+c))^2
*sin(d*x+c)/d/sec(d*x+c)^(1/2)-4/3*a*b*(a^2-6*b^2)*sin(d*x+c)*sec(d*x+c)^(1
/2)/d+8*a*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(a
^4+18*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3841, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2(18a^2b^2+a^4+b^4)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (8*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(3*d) - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(5a^2b+\frac{1}{2}a\left(a^2+9\right.\right.}{\sqrt{\mathrm{se}}}$$

$$= -\frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{4}{9}$$

$$= -\frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{4}{9}$$

$$= -\frac{4ab\left(a^2-6b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

$$= \frac{2\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{4ab\left(a^2-6b^2\right.}{}$$

$$= \frac{8ab\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(a^4+18a^2b^2+b^4\right)}{}$$

Mathematica [A] time = 1.24, size = 130, normalized size = 0.62

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(a^4\cos(2(c+dx))+a^4+24ab^3\cos(c+dx)+2b^4\right)}{\cos^{\frac{3}{2}}(c+dx)} + 24ab\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\left(a\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(24*a*b*(a^2 - b^2)*EllipticE[(c + d
*x)/2, 2] + 2*(a^4 + 18*a^2*b^2 + b^4)*EllipticF[(c + d*x)/2, 2] + ((a^4 +
```

```
2*b^4 + 24*a*b^3*Cos[c + d*x] + a^4*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c +
 d*x]^(3/2)))/(3*d)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

maple [B] time = 5.02, size = 777, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*a*(a^3+6*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^4+12*a*b^3+b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+18*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3*b+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)*
sin(1/2*d*x+1/2*c)^2+a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+18*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+12*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(3/2), x)
```

3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}-\frac{2b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

```
[Out] 2/5*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+28/15*a^3*b*sin(d*
x+c)/d/sec(d*x+c)^(1/2)-2/5*b^2*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2
/5*(3*a^4+30*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+
8/3*a*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.37, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4074, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(30a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (28*a^3*b*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(5*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(7a^2b+\frac{3}{2}a\left(a^2+5\right.\right.}{\sec}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2\left(3a^2+29b\right.}{}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2\left(3a^2+29b\right.}{}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a^2(a+b\sec(c+}{5d\sec^{\frac{3}{2}}}$$

$$= \frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} -$$

$$= \frac{2\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab\left(a^2+\right.}{}$$

Mathematica [A] time = 0.67, size = 138, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(80ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(3a^4\cos(2(c+dx))+3a^4+40a^3\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 80*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(3*a^4 + 30*b^4 + 40*a^3*b*Cos[c + d*x] + 3*a^4*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(5/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)

maple [B] time = 4.59, size = 619, normalized size = 2.99

$$-\frac{2\left(-24\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)

[Out] -2/15*(-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(3*a+10*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*a^4+20*a^3*b+15*b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+20*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+60*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-9*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-90*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(5/2), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

```
[Out] 36/35*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))^2*sin(d*
x+c)/d/sec(d*x+c)^(5/2)+2/21*a^2*(5*a^2+39*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/
2)+8/5*a*b*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/2
1*(5*a^4+42*a^2*b^2+21*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(42a^2b^2+5a^4+21b^4\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{8ab\left(3a^2+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (8*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (36*a^3*b*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2)) + (2*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(9a^2b+\frac{1}{2}a\left(5a^2+\right.\right.}{\text{se}}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2\left(5a^2+39b\right.}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2\left(5a^2+39b\right.}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^4+42a^2b^2+\right.}{}$$

Mathematica [A] time = 0.94, size = 142, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(a^2\sin(2(c+dx))\left(15a^2\cos(2(c+dx))+65a^2+168ab\cos(c+dx)+420b^2\right)+336ab\left(3a^2+5b^2\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(336*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 20*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + a^2*(65*a^2 + 420*b^2 + 168*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

maple [A] time = 3.73, size = 476, normalized size = 2.26

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(240a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360a^4-672a^3b\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^4-672*a^3*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^4+672*a^3*b+840*a^2*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-80*a^4-168*a^3*b-420*a^2*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*a^2*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+105*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
3*b-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(7/2), x)
```

3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{21d}$$

```
[Out] 44/63*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+14/45*a^2*(a^2+7*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)
+8/21*a*b*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*a^4+54*a^2*b^
2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(5*a^2+7*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.40, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3841, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (44*a^3*b*Sin[c + d*x
])/(63*d*Sec[c + d*x]^(5/2)) + (14*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Se
c[c + d*x]^(3/2)) + (8*a*b*(5*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(11a^2b+\frac{1}{2}a\left(7a^2+\right.\right.}{\text{s}}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2\left(a^2+7b^2\right.}{}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2\left(a^2+7b^2\right.}{}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{2\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8ab\left(5a^2\right.}{}$$

Mathematica [A] time = 1.69, size = 168, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(480ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+168\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 480*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + a*(7*a*(43*a^2 + 216*b^2)*Cos[c + d*x] + 5*(312*a^2*
b + 336*b^3 + 72*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c
 + d*x)]))/(1260*d)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4}{\sec(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

maple [A] time = 4.10, size = 529, normalized size = 2.16

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2240a^4+2880a^3b\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^4+2880*a^3*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^4-4320*a^3*b-3024*a^2*b^2)*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^4+3360*a^3*b+3024*a^2*b^2+1680*a*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^4-960*a^3*b-756*a^2*b^2-840*a
*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*a^3*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+420*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1
134*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=289

$$\frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{15d}$$

```
[Out] 52/99*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/77*a^2*(9*a^2+59*b^2)*sin(d*x+c
)/d/sec(d*x+c)^(5/2)+8/45*a*b*(7*a^2+9*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2
/11*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/231*(45*a^4+330*
a^2*b^2+77*b^4)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+8/15*a*b*(7*a^2+9*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/231*(45*a^4+330*a^2*b^2+77*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.46, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3841, 4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(330a^2b^2+45a^4+77b^4\right)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2\left(330a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (8*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (52*a^3*b*Sin[c +
d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*a^2*(9*a^2 + 59*b^2)*Sin[c + d*x])/(77
*d*Sec[c + d*x]^(5/2)) + (8*a*b*(7*a^2 + 9*b^2)*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sin[c + d*x])/(231*d*Sqrt
[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(13a^2b+\frac{3}{2}a\left(3a^2+1\right.\right.}{\text{se}}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2\left(9a^2+59b^2\right)}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2\left(9a^2+59b^2\right)}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \cdots$$

$$= \frac{8ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \cdots$$

$$= \frac{8ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2\left(45a^4+330a^2b^2+\cdots\right.}{}$$

Mathematica [A] time = 2.11, size = 199, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(14784ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+240\left(45a^4+330a^2b^2+77b^4\right)\sqrt{\cos(c+dx)}\,\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(14784*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 240*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + (616*a*b*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*(159
3*a^4 + 10296*a^2*b^2 + 1848*b^4 + 72*(8*a^4 + 33*a^2*b^2)*Cos[2*(c + d*x)]
 + 616*a^3*b*Cos[3*(c + d*x)] + 63*a^4*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)])
)/(27720*d)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4}{\sec(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

maple [A] time = 4.10, size = 586, normalized size = 2.03

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(20160a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-50400a^4-49280a^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*a^4*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*a^4-49280*a^3*b)*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*a^4+98560*a^3*b+47520*a^2*b^2)*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*a^4-91168*a^3*b-71280*a^2*b^2-22
176*a*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*a^4+41888*a^3*b+5
5440*a^2*b^2+22176*a*b^3+4620*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-2790*a^4-7392*a^3*b-15840*a^2*b^2-5544*a*b^3-2310*b^4)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-6468*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-8316*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^3+675*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+1155*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{b^2d}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/b/d-2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/d+2
*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A] time = 0.51, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*b*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) - (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(b^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}b\sec(c+dx)-\frac{3}{2}a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{3b}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^2}{4}+ab\sec(c+dx)+\frac{1}{4}(3}{\sqrt{\sec(c+dx)}\,(a+b}}{3b^2}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^3}{4}+\frac{1}{4}a^2b\sec(c+dx)}{\sqrt{\sec(c+dx)}}d}{3a^2b^2}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b^2} + \int$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3bd}$$

Mathematica [A] time = 3.25, size = 165, normalized size = 0.88

$$-\frac{\cot(c+dx)\left(-2\left(3a^2+3ab+b^2\right)\sqrt{-\tan^2(c+dx)}\,F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+6a^2\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/3*(Cot[c + d*x]*(-(b^2*Sec[c + d*x]^(5/2)) + b^2*Cos[2*(c + d*x)]*Sec[c
+ d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] - 2*(3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[-Tan[c + d*x]^2] + 6*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

maple [A] time = 9.50, size = 450, normalized size = 2.39

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{b^2\left(a^2-ab\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^2*a^3/(a^2
-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+2/b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a/b^2*
(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{bd}$$

```
[Out] 2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+
b)/d
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
- (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
```

```
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{b}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} - ($$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

Mathematica [A] time = 5.23, size = 83, normalized size = 0.71

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+bE\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Se
c[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 4.22, size = 353, normalized size = 3.02

$$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{\frac{5}{2}}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x)), x)
```

3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a+b)/d
```

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3849, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx\\ &= \frac{2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.37, size = 63, normalized size = 1.29

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 3.23, size = 150, normalized size = 3.06

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d
```

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3848, 2803, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{a}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

Mathematica [A] time = 0.28, size = 47, normalized size = 0.51

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Ta
n[c + d*x]^2])/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

maple [A] time = 3.86, size = 187, normalized size = 2.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{a(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=135

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sqrt{\cos(c+}{}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A] time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sqrt{\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{a^2} + \frac{\left(b^2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} + \frac{\left(\sqrt{\cos(c+d\right.}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [A] time = 7.20, size = 176, normalized size = 1.30

$$\frac{\cot(c+dx)\left(-2b\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+a\sec^{\frac{7}{2}}(c+dx)-a\sec^{\frac{3}{2}}(c+dx)+a\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*(-(a*Sec[c + d*x]^(3/2)) - a*Cos[2*(c + d*x)]*Sec[c + d*x]^(3
/2) + a*Sec[c + d*x]^(7/2) + a*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*EllipticPi[-
(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)
```

fricas [F] time = 67.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)^2+a\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [A] time = 3.83, size = 226, normalized size = 1.67

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\cdots}\right.\right.}{a^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=172

$$-\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\left(a^2+\right.}{}$$

```
[Out] 2/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^3/d-2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(
a+b)/d
```

Rubi [A] time = 0.37, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^3*d) - (2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{3a} \\
&= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^3} - \frac{b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a^3} \\
&= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{3a^3} - \\
&= -\frac{2b^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d} + \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+d}} \\
&= -\frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\left(a^2+3b^2\right)\sqrt{\cos(c-}}{}
\end{aligned}$$

Mathematica [A] time = 7.13, size = 194, normalized size = 1.13

$$-\frac{\cot(c+dx)\left(-a^2\sqrt{\sec(c+dx)}+a^2\cos(3(c+dx))\sec^{\frac{3}{2}}(c+dx)-12b^2\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

[Out] -1/6*(Cot[c + d*x]*(-(a^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) - 6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a^2*Cos[3*(c + d*x)]*Sec[c + d*x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 4*a*(a - 3*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 12*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)

fricas [F] time = 69.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)^{3}+a\sec(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] integral(sqrt(sec(d*x + c))/(b*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

maple [B] time = 4.48, size = 516, normalized size = 3.00

$$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\left(4a^{3}-4a^{2}b\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^{3}+2a^{2}b\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a^3-4*a^2*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3+2*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+3*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=342

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d\left(a^2-b^2\right)}+\frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3b^2d\left(a^2-b^2\right)}$$

```
[Out] 1/3*(5*a^2-2*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c
)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))-a*(5*a^2-4*b^2)*sin(d*x+c
)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+1/3*(5*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+a^2*(5*a^2-7*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

Rubi [A] time = 0.94, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d\left(a^2-b^2\right)}-\frac{a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx}}{b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) + (a^2*(5*a^2 -
 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - 2\int \frac{\sqrt{\sec}}{}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d}$$

$$= \frac{a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} - \frac{a\left(5a^2-4b^2\right)}{b^3}$$

$$= \frac{a\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 6.87, size = 653, normalized size = 1.91

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{a\left(4b^2-5a^2\right)\sin(c+dx)}{b^3\left(b^2-a^2\right)} - \frac{a^3\sin(c+dx)}{b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)} + \frac{2\tan(c+dx)}{3b^2}\right)}{d} + \frac{2\left(40a^3b-28ab^3\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+}}{a\left(1-\cos^2(c+dx)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*(45*a^4 - 44*a^2*b^2 - 4*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + (2*(40*a^3*b - 28*a*b^3)*Cos[c + d*x]^2*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + ((15*a^4 - 12*a^2*b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4
*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*E
llipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a
*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)
))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((a*(-5*a^2 + 4*b^2)*
Sin[c + d*x])/(b^3*(-a^2 + b^2))) - (a^3*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(b
 + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 14.65, size = 1002, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a^3/b^3/(a^2
-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+2/b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/b^3*
a*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
+2/b^2*a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=279

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d\right.}{bd\left(a^2-b^2\right)}$$

```
[Out] -a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+(3*a^2-2*b^
2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(3*a^2-2*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-a*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A] time = 0.64, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+d\right.}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^2*(a^2 - b^2)*d)) - (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a -
 b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \frac{\frac{1}{4}a\left(3a^2\right.}{}}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \frac{\frac{1}{4}a^2\left(3a\right.}{}}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{a\int\sqrt{\sec(}}{2b\left(a^2\right.}$$

$$= -\frac{a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} + \frac{\left(3a^2-2b^2\right)\sqrt{\sec}}{b^2\left(a\right.}$$

$$= -\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{b\left(a^2-\right.}$$

Mathematica [A] time = 6.36, size = 351, normalized size = 1.26

$$\frac{\frac{2b\sin(c+dx)\left(-3a^3+2b\left(b^2-a^2\right)\sec(c+dx)+2ab^2\right)}{\left(b^2-a^2\right)\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+b)} + \frac{\cot(c+dx)\left(6a^3\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2b\left(3a^2-2b^2\right)\sqrt{-\tan^2(c+dx)}\,E\left(\sin\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*(-3*a^3 + 2*a*b^2 + 2*b*(-a^2 + b^2)*Sec[c + d*x])*Sin[c + d*x])/((-a
^2 + b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-3*a^2*
b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x)]*
Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3*a^2
 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] -
 2*(3*a^3 + 3*a^2*b - 4*a*b^2 - 2*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 11.06, size = 868, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^2/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))+2/b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1)-2/b*a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)
-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E}{bd\left(a^2-b^2\right)}$$

```
[Out] -a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+a*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

Rubi [A] time = 0.40, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2
 - b^2)*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (a^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^3}{2}-\frac{1}{2}a^2b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2b\left(a^2-b^2\right)} + \frac{\left(a^2-3b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((a*Sin[c + d*x])/(b*(-a^2 + b^2)) + (a*Sin[c + d*x])/(
(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((2*(3*a^2 - 4*b^2)*Cos[c + d*x]^2*
(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
 d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos[c + d*x]^2*
EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + (Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^
2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a),
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
 - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*
x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*b*(a + b
)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 7.16, size = 608, normalized size = 2.84

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}{b\left(a^{2}-b^{2}\right)\left(2a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\cdots\right.\right.}}{(a+b)b\sqrt{-2\left(\sin\cdots\right.}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(a^2-b^2
```

```
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b)*a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**2, x)
```

3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}\right.}{d\left(a^2-b^2\right)}$$

```
[Out] a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))-(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a/d/(a^2-b^2)+(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.36, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 -
 b^2)*d)) - (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(a^2+b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2\left(a^2-b^2\right)} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{2a\left(a^2-b^2\right)} + \frac{\left(\left(a^2+b^2\right)\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} + \frac{a\sqrt{\sec(c+dx)}\,\sin(c}{\left(a^2-b^2\right)d(a+b\sec(c}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d} - \frac{b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}}{a\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.73, size = 628, normalized size = 3.02

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2\left(\frac{b\sin(c+dx)}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}-\frac{\sin(c+dx)}{b^2-a^2}\right)}{d(a+b\sec(c+dx))^2} + \frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(\frac{\sin(c+dx)\cos(2(c}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(-(Sin[c + d*x]/(-a^2 + b^2)) +
(b*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*
x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*a*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[
c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos[c + d*x]^2*Ellip
ticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 -
 Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)
) + (Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 -
 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] -
4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b*(b + a*Cos[c + d*x])*(1 - Cos
[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(4*(-a + b)*(a + b)
*d*(a + b*Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 8.32, size = 707, normalized size = 3.40

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{\left(a^{2}-ab\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-2/a*b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
```

```
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{ad\,(}$$

```
[Out] -b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))+b*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d/(a^2-b^2)+(2*a^2-b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.37, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\,\sqrt{\sec}}{ad\,(}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2
 - b^2)*d) + ((2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)
*(a + b)^2*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*S
ec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{ab}{2}-\left(a^2-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)\int \frac{\sec^{\frac{3}{2}}(c+d}{a+b\sec(c+}}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} + \frac{\left(2a^2-b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)}$$

$$= -\frac{b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} - \frac{b\sqrt{\sec(c+dx)}\,\text{si}}{\left(a^2-b^2\right)d(a+b\,\text{s}}$$

$$= \frac{b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 5.14, size = 251, normalized size = 1.11

$$\cos(2(c+dx))\csc(c+dx)\sqrt{\sec(c+dx)}\left(-\left(3a^2-b^2\right)\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+b)\Pi\left(-\frac{b}{a};\text{si}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Sec[c + d*x]]*(a*(a - b)*(b + a*Cos[c +
 d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-T
an[c + d*x]^2] - (3*a^2 - b^2)*(b + a*Cos[c + d*x])*EllipticPi[-(b/a), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*b*
(-(b*Tan[c + d*x]^2) + (b + a*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a^2*(a - b)*(a + b)
*d*(b + a*Cos[c + d*x])*(-2 + Sec[c + d*x]^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 8.39, size = 788, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*b/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2*b^2/a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\left(4a^2-3b^2\right)\sqrt{\cos(}}{}$$

```
[Out] b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))+(2*a^2-3*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(4*a^2-3*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b^2*(5*a^2
-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(
a+b)^2/d
```

Rubi [A] time = 0.44, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{3b^2}{2}+ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{a\left(-a^2+\frac{3b^2}{2}\right)-\left(-a^2b+b\left(-a^2+\frac{3b^2}{2}\right)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{a^3\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)} - \frac{\left(b\left(4a\right.\right.}{}$$

$$= \frac{b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} + \frac{b^2}{a\left(a\right.}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} - \frac{b\left(4a^2-3b\right.}{}$$

Mathematica [B] time = 6.77, size = 603, normalized size = 2.47

$$\frac{2\left(b^2-2a^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(a+b\sec(c+dx))\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)} + \frac{\left(3b^2-2a^2\right)\sin(c+dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-((b^2*Sin[c + d*x])/(a^2*(-a^2 + b^2))) - (b^3*Sin[c
+ d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((2*(-2*a^2 + b^2)*Cos
[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a),
 ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos
[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + ((-2*a^2 + 3*b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*
(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a
^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[
c + d*x]^2)))/(4*a*(-a + b)*(a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

maple [B] time = 10.32, size = 809, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
```

```
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{2}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=304

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\left(2a^4+16a^2b^2-15b^4\right)\sqrt{\cos(c+}}{3a^4d\,(a}$$

```
[Out] 1/3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+b^2*sin(d*x+c
)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2)-b*(4*a^2-5*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-1
5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d-b^3*(7*
a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b
)/(a+b)^2/d
```

Rubi [A] time = 0.68, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3847, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\left(16a^2b^2+2a^4-15b^4\right)\sqrt{\cos(c+}}{3a^4d\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -((b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d
) - (b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)
*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{5b^2}{2}+ab\sec(c+dx)-}{\sec^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{s}}}{a\left(a^2-b\right.}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(}$$

$$= -\frac{b^3\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^4(a-b)(a+b)^2d} +$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4+\right.}{}$$

Mathematica [B] time = 6.85, size = 634, normalized size = 2.09

$$\frac{\frac{2\left(4a^3+8ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(a+b\sec(c+dx))\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)}{a\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)} + \frac{2\left(5b^3-8a^2b\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((2*(-8*a^2*b + 5*b^3)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c +
d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - C
os[c + d*x]^2)) + (2*(4*a^3 + 8*a*b^2)*Cos[c + d*x]^2*EllipticPi[-(b/a), Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]
*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^2*b
+ 15*b^3)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x
]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*a^2*(a -
b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((b^3*Sin[c + d*x])/(a^3*(-a^2 + b^2))
+ (b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[2*(c + d*
x)]/(3*a^2)))/d
```

fricas [F] time = 116.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^
2*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 11.49, size = 1064, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*(a+b)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*b^3/a^3/(a^2-a*b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(1/((a + b*sec(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(}}{4b^2d\left(a^2-b^2\right.}$$

```
[Out] -1/2*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*a
^2*(5*a^2-11*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*
x+c))+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/(a^2-b^
2)^2/d-1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(
a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

Rubi [A] time = 0.96, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(-29a^2b^2+15a^4+8b^4\right)\sin(c+dx}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - (a*(5*a^2 - 11*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2
)^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3
*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^
3*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int}{}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} + \frac{(15a}{}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a\left(5a^2-11b^2\right.}{}$$

Mathematica [A] time = 6.88, size = 721, normalized size = 1.86

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)}{2b\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{11a^2b^2\sin(c+dx)-5a^4\sin(c+dx)}{4b^2\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(b^2-a^2\right)^2}\right)}{d} - \frac{2\left(45a^5-95a^3b^2+56ab^4\right)\sin}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
-1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(40*a^4*b - 80*a^2*b^3 + 16*b^5)*C
os[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + ((15*a^5 - 29*a^3*b^2 + 8*a*b^4)*Cos[2*(c + d*x)]*(a
 + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*
(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a),
```

```
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c
+ d*x]]*(((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2
) + (a^2*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (-5*a^4*
Sin[c + d*x] + 11*a^2*b^2*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(b + a*Cos[c
+ d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 16.75, size = 2014, normalized size = 5.19

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^3/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-2/b*a*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^
2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)
/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
```

```
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),
2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/
(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/b^3*(-(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2*a/b^2*(a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2
)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=315

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{4bd\left(a^2-b^2\right.}$$

```
[Out] -1/2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/4*a
^2*(a^2-3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))+3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-
b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^
2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A] time = 0.70, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{4bd\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2
*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (a^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (3*a^2*(a^2
 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Se
c[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(3a^2-4b^2\right)\sec^2(c+d}{(a+b\sec(c+dx))^2}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= \frac{3\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} - \frac{a^2\,\text{se}}{2b\left(a^2-\right.}$$

$$= \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^2-7b^2\right)\sqrt{\cos(c+}}{}$$

Mathematica [B] time = 6.82, size = 692, normalized size = 2.20

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3a\left(3b^2-a^2\right)\sin(c+dx)}{4b^2\left(b^2-a^2\right)^2} - \frac{a\sin(c+dx)}{2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2} + \frac{a^3\sin(c+dx)-7ab^2\sin(c+dx)}{4b\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d} + \frac{2\left(8a^3b-32ab^3\right)\sin(c+dx)\cos^2(c+dx}{a(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*(9*a^4 - 19*a^2*b^2 + 16*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^3*b - 32*a*b^3)*Cos[c + d*x]^2*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - S
ec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
+ ((3*a^4 - 9*a^2*b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*
b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*Elli
pticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/
(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((3*a*(-a^2 + 3*b^2)*S
in[c + d*x])/(4*b^2*(-a^2 + b^2)^2) - (a*Sin[c + d*x])/(2*(-a^2 + b^2)*(b +
 a*Cos[c + d*x])^2) + (a^3*Sin[c + d*x] - 7*a*b^2*Sin[c + d*x])/(4*b*(-a^2
+ b^2)^2*(b + a*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)

maple [B] time = 7.98, size = 1203, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3, x)

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=313

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2}$$

```
[Out] -1/2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*a
*(a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+1
/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d
+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d
+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

Rubi [A] time = 0.68, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^
2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) - (a^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2 - 7*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{1}{4}a^2 \cdots}{\cdots}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{1}{4}a^3 \cdots}{\cdots}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(3\left(a \cdots\right.\right.}{\cdots}$$

$$= \frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} - \frac{a^2\sqrt{\sec(c \cdots}}{2b\left(a^2-b^2\right) \cdots}$$

$$= \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\,F \cdots}{4a\left(a^2 \cdots\right.}$$

Mathematica [B] time = 6.81, size = 728, normalized size = 2.33

$$\frac{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{\left(a^2+5b^2\right)\sin(c+dx)}{4b\left(b^2-a^2\right)^2} + \frac{b\sin(c+dx)}{2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2} + \frac{3\left(a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{4\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^3} + \frac{\sec^3(c+d \cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(3*a^3 - 9*a*b^2)*Cos[c + d*x]^2
*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqr
t[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
+ d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*b + 16*b^
3)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + ((a^3 + 5*a*b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c +
 d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
 - Sec[c + d*x]^2))))/(16*(a - b)^2*b*(a + b)^2*d*(a + b*Sec[c + d*x])^3) +
 ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*(-1/4*((a^2 + 5*b^2)*Sin[c + d*
x])/(b*(-a^2 + b^2)^2) + (b*Sin[c + d*x])/(2*(-a^2 + b^2)*(b + a*Cos[c + d*
x])^2) + (3*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(4*(-a^2 + b^2)^2*(b + a
*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 13.50, size = 1760, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2)))+2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
```

```
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**3, x)
```

3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{4a^2d\left(a^2-b^2\right)^2}$$

```
[Out] 1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/4*(a^2+b
^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))-1/4*(5*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d-1/4*b*(7*a
^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d+1/4*
(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^2/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 0.65, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{4a^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10
*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (a*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{4}a(5}{}}{}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{4}a^2(}{}}{}$$

$$= \frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\left(b\left(7a\right.\right.}{}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d} + \frac{a\sqrt{\sec}}{2\left(a^2-b\right.}$$

$$= -\frac{\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} - \frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+}}{4}$$

Mathematica [B] time = 6.77, size = 719, normalized size = 2.35

$$\frac{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(\frac{\left(5a^2+b^2\right)\sin(c+dx)}{4a\left(a^2-b^2\right)^2}+\frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b^3\sin(c+dx)-7a^2b\sin(c+dx)}{4a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^3} - \frac{\sec^3(c+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-a^2 - 5*b^2)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[
Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (48*b*Cos[c + d*
x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x]
)*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c +
 d*x]^2)) + ((5*a^2 + b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b +
4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*
EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b +
a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2
))))/((a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])
^3*Sec[c + d*x]^(7/2)*(((5*a^2 + b^2)*Sin[c + d*x])/(4*a*(a^2 - b^2)^2) + (
b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-7*a^2*b*Sin[
c + d*x] + b^3*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*
(a + b*Sec[c + d*x])^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)

maple [B] time = 13.65, size = 1858, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^2/a^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-4*b/a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2

```
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=323

$$-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}}{4a^2d(a^2-b^2)^2}$$

```
[Out] -1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*b*(7*
a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+3/4*b
*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d
+1/4*(8*a^4-5*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 0.67, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{(-5a^2b^2+8a^4+3b^4)\sqrt{\cos(c+dx)}\sqrt{s}}{4a^3d(a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*
d) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) - (
b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{3}{4}b(3a\cdots}{\cdots}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{3}{4}ab(3\cdots}{\cdots}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{(3b\,(3a\cdots}{\cdots}$$

$$= -\frac{3b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3d} - \frac{b\sqrt{\sec\cdots}}{2\left(a^2-b\cdots\right.}$$

$$= \frac{3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+3b^4\right)\sqrt{c\cdots}}{\cdots}$$

Mathematica [B] time = 6.86, size = 744, normalized size = 2.30

$$\frac{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(\frac{3b\left(b^2-3a^2\right)\sin(c+dx)}{4a^2\left(b^2-a^2\right)^2}+\frac{11a^2b^2\sin(c+dx)-5b^4\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2a^2\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}\right)}{d(a+b\sec(c+dx))^3}+\frac{\sec^3(c\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-5*a^2*b - b^3)*Cos[c + d*x]^2*
(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
 d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^3 + 8*a*b^2
)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*
Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x]
)*(1 - Cos[c + d*x]^2)) + ((9*a^2*b - 3*b^3)*Cos[2*(c + d*x)]*(a + b*Sec[c
+ d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*E
llipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(
2 - Sec[c + d*x]^2))))/(16*a*(a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^3)
+ ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*((3*b*(-3*a^2 + b^2)*Sin[c + d
*x])/(4*a^2*(-a^2 + b^2)^2) - (b^3*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*(b + a*
Cos[c + d*x])^2) + (11*a^2*b^2*Sin[c + d*x] - 5*b^4*Sin[c + d*x])/(4*a^2*(a
^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 13.74, size = 1936, normalized size = 5.99

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/a^2
*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2))-2/a^3*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/
2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+
b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b
)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+6*b^2/a^3*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3,x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{\left(c+dx \right)}}}{\left(a+b\sec{\left(c+dx \right)}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=342

$$\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c}}{4a^4d\left(\right.}$$

```
[Out] 1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b^
2*(11*a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x
+c))+1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b^4
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b)^2/(a+b)
^3/d
```

Rubi [A] time = 0.75, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3847, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)\sqrt{\cos(}}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a -
 b)^2*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{5b^2}{2}+2ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c\cdot}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\text{s}}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\text{s}}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\text{s}}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c}$$

$$= \frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{4a^4(a-b)^2(a+b)^3d}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 6.91, size = 707, normalized size = 2.07

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{b^2\left(7b^2-13a^2\right)\sin(c+dx)}{4a^3\left(b^2-a^2\right)^2}+\frac{b^4\sin(c+dx)}{2a^3\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{3\left(3b^5\sin(c+dx)-5a^2b^3\sin(c+dx)\right)}{4a^3\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right)}{d}+\frac{2\left(8ab^3-32a^3b\right)\sin(c+d}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((2*(8*a^4 - 7*a^2*b^2 + 5*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Se
c[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
 ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2
*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*
(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d
*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*(b^2*(-
13*a^2 + 7*b^2)*Sin[c + d*x])/(a^3*(-a^2 + b^2)^2) + (b^4*Sin[c + d*x])/(2*
a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (3*(-5*a^2*b^3*Sin[c + d*x] + 3*b
^5*Sin[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/d
```

fricas [F] time = 135.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^4+3\,ab^2\sec(dx+c)^3+3\,a^2b\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec \left(dx+c\right)+a\right)^{3} \sqrt{\sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

maple [B] time = 15.08, size = 1957, normalized size = 5.72

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-12*b^2/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-8/a^4*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+

```
1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^3\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=406

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}-\frac{b\left(24a^4-65a^2\right.}{}$$

```
[Out] 1/12*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2
)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/4*
b^2*(13*a^2-7*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/sec(d*x+c)
^(1/2)-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+128*a^4*b^2-223*a^2*b^4+105*b^6)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a^
4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^5/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 1.02, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3847, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{\left(-61a^2b^2+8a\right.}{12a^3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 -
223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c
 + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x])/(
2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(13*a^2
 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{7b^2}{2}+2ab\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec}}{2a\left(a^2-\right.}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2\left(13a^2-\right.}{4a^2\left(a^2-b^2\right)^2d\sqrt{\sec}}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\,\mathrm{s}}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\,\mathrm{s}}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\,\mathrm{s}}$$

$$= -\frac{b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{4a^5(a-b)^2(a+b)^3d}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 6.97, size = 731, normalized size = 1.80

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(2(c+dx))}{3a^3}-\frac{b^5\sin(c+dx)}{2a^4\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b^3\left(11b^2-17a^2\right)\sin(c+dx)}{4a^4\left(b^2-a^2\right)^2}+\frac{19a^2b^4\sin(c+dx)-13b^6\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right)}{d}+\frac{2\left(16a^5+112\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((2*(-56*a^4*b + 73*a^2*b^3 - 35*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt
[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*
(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c
 + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^5 + 112*a^3*b^2 - 56*a*b^4)*Cos[c
 + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c +
 d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)) + ((-72*a^4*b + 195*a^2*b^3 - 105*b^5)*Cos[2*(c + d*x)]*(a
 + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*
(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a),
```

```
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec
[c + d*x]]*((b^3*(-17*a^2 + 11*b^2)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^2) -
(b^5*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (19*a^2*b^4
*Sin[c + d*x] - 13*b^6*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*
x])) + Sin[2*(c + d*x)]/(3*a^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 16.28, size = 2216, normalized size = 5.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+20*b^3/a^4/(a^2-a*b)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))-2/a^5*b^5*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^
2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^
2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/
b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
```

```
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a
^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1
/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b
)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{3}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
[Out] Integral(1/((a + b*sec(c + d*x))**3*sec(c + d*x)**(3/2)), x)
```

3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)
^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.65, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3855, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3855

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*
Csc[e + f*x])^(n - 1))/(f*(2*n - 1)), x] + Dist[d^2/(2*n - 1), Int[((d*Csc[
e + f*x])^(n - 2)*Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
```

```
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\int \frac{-a+a\sec^2(}{\sqrt{\sec(c+dx)}\sqrt{a+}}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+}}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{s}\right.}{2\sqrt{a+}}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+}}{}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\right.}{d\sqrt{a+b}}$$

Mathematica [C] time = 5.64, size = 321, normalized size = 1.35

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}} - \frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{b-a}{a+b}\right)+a\Pi\right.\right.}{ab\sqrt{\frac{1}{a-b}}\sqrt{a}}\right.}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*a*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*Sqrt[-((a*(-1 + Cos[c
 + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b
*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a -
b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*Tan[c + d*x]))/(4*d*Sqrt[Sec[c + d
*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c)+a}\, \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [C] time = 1.82, size = 789, normalized size = 3.33

$$-\frac{\left(2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\ \mathrm{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \frac{a+b}{a-b}, \frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)\,a - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a-((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a+(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b
))^(1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(
1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(dx+c)+a}\, \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\, \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec (c + dx)} \sec^{\frac{3}{2}} (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\left(a\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{b+a\cos\right.}{}$$
$$= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right.}{}$$
$$= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(\right.}{d\sqrt{a}}$$

Mathematica [A] time = 2.31, size = 96, normalized size = 0.70

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(aF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{d(a+b)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [C] time = 1.49, size = 283, normalized size = 2.05

$$\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)b+2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(dx+}{\sin(dx}\right.\right.}{d(-1+\cos(dx+c))(b-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
[Out] 2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)^2*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d*x+c))/
((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+
b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3856, 2655, 2653}

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$= \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 1.00

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.75, size = 925, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((
```

```
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*b+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+
c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b+((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+}}{3ad\sqrt{\sec(c+}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d/sec(d*x+c)^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3857, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+}}{3ad\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a} + \frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,d}{3a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\left(a^2-b^2\right)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{}}}{3a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{3a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a-}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{s}}$$

Mathematica [A] time = 0.63, size = 156, normalized size = 0.81

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a\sin(c+dx)(a\cos(c+dx)+b)+b(a+b)\sqrt{\frac{a\cos}{}}\right.}{3ad\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*S
in[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [B] time = 1.84, size = 1021, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+cos(d*x+c)*sin(
d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2
)-b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2
*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] -4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d/sec(d*x+c)^(3/2)+2/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/s
ec(d*x+c)^(1/2)+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (
2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\int \frac{b+3a\sec(c+dx)+2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-9a^2}{\sqrt{\sec(c}}}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-\right.\right.}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-\right.\right.}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-\right.\right.}{}$$

$$= -\frac{4b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9-\frac{2b^2}{a^2}\right)E\left(\frac{1}{2}(c+\right.}{15d\sqrt{\frac{b+a\,\mathrm{c}}{}}}$$

Mathematica [A] time = 0.95, size = 203, normalized size = 0.83

$$\frac{\sqrt{a+b\sec(c+dx)}\left(2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2+8ab\cos(c+dx)+2b^2\right)+8b\left(b^2-a^2\right)\sqrt{\frac{a\cos(c+}{a+}}\right.}{30a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*b*(-a^2
+ b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(3*a^2 + 2*b^2 + 8*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)

maple [B] time = 1.82, size = 1736, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)

[Out] -2/15/d*(-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3+2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-9*a^2*b*((a-b)/(a+b))^(1/2)-a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+4*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+2*b^3*((a-b)/(a+b))^(1/2)+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^2/((a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec{(c + dx)}}}{\sec^{\frac{5}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=305

$$\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-1\right.}{}$$

```
[Out] 2/105*(25*a^4-17*a^2*b^2-8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/7*sin(d*x+c)
*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*b*sin(d*x+c)*(a+b*sec(d*x+c
))^(1/2)/a/d/sec(d*x+c)^(3/2)+2/105*(25*a^2-4*b^2)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*b*(19*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))
^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x
+c)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2\left(-17a^2b^2+25a^4-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{105a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d*Sqrt[a + b*Se
c[c + d*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*
x]^(3/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105
*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\int \frac{b+5a\sec(c+dx)+4b\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}(-25a^2+4}{\text{se}}}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a^2-4b}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a^2-4b}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a^2-4b}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a^2-4b}{}$$

$$= \frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}} + \frac{2b(19a^2+}{10}$$

Mathematica [A] time = 1.34, size = 237, normalized size = 0.78

$$\frac{\sqrt{a+b\sec(c+dx)}\left(8(25a^4-17a^2b^2-8b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)(15a^3\cos(3(c+dx)\right.}{420a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(8*b*(19*a^3 + 19*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(25*
a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(136*a^2*b - 16*b^3 + a*(145*a^2 - 4*b^2)*Cos
[c + d*x] + 36*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c + d*
x]))/(420*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 2.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)

maple [B] time = 1.90, size = 2050, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)

[Out] -2/105/d*(-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+25*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-25*a^3*b*((a-b)/(a+b))^(1/2)-19*a^2*b^2*((a-b)/(a+b))^(1/2)+4*a*b^3*((a-b)/(a+b))^(1/2)-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+26*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-19*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+18*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+25*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*b^4*((a-b)/(a+b))^(1/2)-19*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+19*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-19*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-19*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-8*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+19*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-19*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+$$

```
[Out] 7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c
)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1
/2)+1/2*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-5/4*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)+5/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/d
```

Rubi [A] time = 0.99, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3866, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (5*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(4*d) + (b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{5a\sqrt{}}{}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b}{}}}{}$$

Mathematica [C] time = 6.60, size = 549, normalized size = 1.84

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{5}{4}a\tan(c+dx)+\frac{1}{2}b\tan(c+dx)\sec(c+dx)\right)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}+\frac{(a+b\sec(c+dx))^{3/2}\left(-\frac{2(-a^2-8b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{\sqrt{a\cos(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]

[Out] ((a + b*Sec[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - (2*(-a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((10*I)*a^2*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((a + b*Sec[c + d*x])^(3/2)*((5*a*Tan[c + d*x])/4 + (b*Sec[c + d*x]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

maple [C] time = 1.74, size = 1744, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)

[Out] 1/4/d*(5*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-5*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c

```
os(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*cos(d*x+c)^3*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*b^2-6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^2+5*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-5*cos(d*x+c)^2*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*cos(d*x+
c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*b^2-6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^2-5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-2*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-
5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b
^2+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(b+a*cos(d*x+c))/sin(d*x+c
)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.635 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b\sqrt{a}}{}$$

```
[Out] (2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d
*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-b*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)+b*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/d
```

Rubi [A] time = 0.73, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3866, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (b*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{-\frac{ab}{2}+a^2\sec(c}{\sqrt{\sec(c+d}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(3ab)\int \frac{\sec}{\sqrt{a+b}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}b\int \frac{\sqrt{a+b\sec(}}{\sqrt{\sec(c+}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\left(\left(2a^2+b^2\right)\sqrt{b+}\right.}{}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{\sec(c+}}{}$$

$$= \frac{\left(2a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b}{}}}{}$$

Mathematica [C] time = 8.38, size = 394, normalized size = 1.58

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{8a^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^2} + \frac{4b\tan(c+dx)}{a\cos(c+dx)+b} + \frac{10ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^2} - \frac{2i\csc(c+dx)\sqrt{-}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((8*a^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + (10*a*b*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b +
 a*Cos[c + d*x])^2 - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b
*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)
/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*(b + a*Cos[c + d*
x])^(3/2)) + (4*b*Tan[c + d*x])/(b + a*Cos[c + d*x])))/(4*d*Sec[c + d*x]^(3
/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

maple [C] time = 1.60, size = 1207, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
[Out] -1/d*(6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+2*cos(d*x+c)^2*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^2-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*b^2+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+2*cos(d*x+c)*sin(
d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+cos(d*x+c)
*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^2+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-b^2*((a-b)/(a+b))^(1/2))*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin
(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2b^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b}}{d\sqrt{s}}$$

```
[Out] 2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^
(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3868, 3856, 2655, 2653, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2b^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b}}{d\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$= (ab)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{\left(a\sqrt{a+b\sec(c+d\right.}}{\sqrt{b+a\cos(}}$$

$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b+a\cos(c+dx)}\right.}{\sqrt{a+}}$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\left(ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\frac{b}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{d\sqrt{a+b\sec(c}}$$

Mathematica [A] time = 2.55, size = 129, normalized size = 0.62

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}(a+b\sec(c+dx))^{3/2}\left(a(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b\left(aF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(a + b)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)] + b*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]))*(a + b*Sec[c + d*x])^(3/2))/(d*(b + a*Cos[c +
 d*x])^2*Sec[c + d*x]^(3/2))
```

fricas [F] time = 5.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

maple [C] time = 1.84, size = 1367, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d*(-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*b^2+cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+2*cos(d*x+c
)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*b^2-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+
c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a^2-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b-a*b*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c-}}{3d\sqrt{\sec(c+}}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d/sec(d*x+c)^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3864, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c-}}{3d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\int \frac{-4ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}\left(-a^2+b^2\right.$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{}}$$

Mathematica [A] time = 0.75, size = 156, normalized size = 0.83

$$\frac{(a+b\sec(c+dx))^{3/2}\left(2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)(a\cos(c+dx)+b)+8b(a+b)\right.}{3d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d
*x])*Sin[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

fricas [F] time = 2.18, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

maple [B] time = 1.89, size = 1219, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+3*cos(d*x+c)*
sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*b^2+4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-4*cos(d*x+c)*sin(d*x+c)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(
d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(
d*x+c)+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)-4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
```

```
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2)-4*b^2*((a-b)
/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+
c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] 2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/d/sec(d*x+c)^(3/2)+4/5*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*
x+c)^(1/2)+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2
)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*b*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\int \frac{-6ab-\left(3a^2+5b^2\right)\sec(c+dx)-2ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{\frac{3}{2}a}{\sqrt{s}}}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2\right.\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2\right.\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2\right.\right.}{}$$

$$= \frac{2b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)E\left(\frac{1}{2}(c\right.}{5ad\sqrt{\frac{b+a}{}}}$$

Mathematica [A] time = 1.23, size = 197, normalized size = 0.82

$$\frac{(a+b\sec(c+dx))^{3/2}\left(2a\sin(c+dx)\left(a^2\cos(2(c+dx))+a^2+6ab\cos(c+dx)+4b^2\right)+4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+}{a+b}}\right.}{10ad\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 4*b*(a^2 - b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(a^2 + 4*b^2 + 6*a*b*Cos[c + d*x] + a^2*Cos[2*(c + d*x)])*Sin[c + d
*x]))/(10*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

maple [B] time = 1.80, size = 1707, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5/d*(cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-3*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+4*
cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+3*cos(d*x+c)*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^3-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-cos(d*x+
c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*b^3+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-3*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-3*a^2*
b*((a-b)/(a+b))^(1/2)-2*a*b^2*((a-b)/(a+b))^(1/2)-b^3*((a-b)/(a+b))^(1/2))*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d
*x+c)/(b+a*cos(d*x+c))/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/2), x)
```

3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-3\right.}{}$$

```
[Out] 2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/7*a*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+16/35*b*sin(d*x+c)*(a+b*sec(d*
x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*a^2+3*b^2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^
(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+
c)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{105a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c +
 d*x]^(5/2)) + (16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d
*x]^(3/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(10
5*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{1}{7}\int \frac{-8ab-\left(5a^2+7b^2\right)\sec(c+dx)-4ab\sec}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}a(25}{}}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2\right.}{}$$

$$= \frac{2\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(41-\right.}{1}$$

Mathematica [A] time = 1.97, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{3/2}\left(8\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(15a^3\cos(3(c+d\right.\right.}{420}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(16*b*(41*a^3 + 41*a^2*b - 3*a*b^2 - 3*b^3)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(
25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(178*a^2*b + 12*b^3 + a*(145*a^2 + 108*b^2
)*Cos[c + d*x] + 78*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c
 + d*x]))/(420*a^2*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

maple [B] time = 1.97, size = 2050, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*(6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+25*cos(d*x+c)*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-25*a^3*b
*((a-b)/(a+b))^(1/2)-82*a^2*b^2*((a-b)/(a+b))^(1/2)-3*a*b^3*((a-b)/(a+b))^(
1/2)+27*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^3*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-82*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^3*b+55*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+6*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a*b^3+39*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-6*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+
25*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+6*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*b^4*((a-b)/(a+b))^(1/2)-82*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+51*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+82*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-82*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)-6*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-82*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b+51*cos(d*x+
c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a^2*b^2+6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+82*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3*b-82*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+
c)/(b+a*cos(d*x+c))/a^2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=369

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}+\frac{b(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c\right.}{24d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b
*sec(d*x+c))^(1/2)+13/12*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(
1/2)/d+1/3*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/24*(3
3*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin(d*x+c)*sec(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.35, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}+\frac{b(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c\right.}{24d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (13*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d
) + (b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{}$$

$$=\frac{13ab\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d}+\frac{b^2\sec^{\frac{5}{2}}(c+dx)}{}$$

$$=\frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+\frac{13ab}{}$$

$$=\frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+\frac{13ab}{}$$

$$=\frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+\frac{13ab}{}$$

$$=\frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d}+\frac{13ab}{}$$

$$=\frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{8d\sqrt{a+b\sec(c+dx)}}+\frac{(3}{}$$

$$=\frac{b\left(59a^2+16b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}}+\frac{5a}{}$$

Mathematica [C] time = 6.69, size = 602, normalized size = 1.63

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2\sin(c+dx)+16b^2\sin(c+dx)\right)+\frac{13}{12}ab\tan(c+dx)\sec(c+dx)+\frac{1}{3}b^2\text{t}\right.}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/96*(a*(a + b*Sec[c + d*x])^(5/2)*((-104*a*b*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2
*(3*a^2 - 104*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(33*a^2 + 16*b^2)*S
qrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*
(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)
^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b,
 I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])
)*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 -
a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b +
a*Cos[c + d*x])^2)))))/(d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(33*a^2*Sin[c + d*x] + 16*b^2*Sin
[c + d*x]))/24 + (13*a*b*Sec[c + d*x]*Tan[c + d*x])/12 + (b^2*Sec[c + d*x]^
2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

maple [C] time = 1.81, size = 2295, normalized size = 6.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/24/d*(30*cos(d*x+c)^4*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+26*((a-b)/(a+b))^(1/2
)*cos(d*x+c)^4*a^2*b+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a*b^2+18*((a-b)/(a
+b))^(1/2)*cos(d*x+c)^3*a*b^2-59*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b+33*
cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-33*cos(d*x+c)^4*sin(d*x+c)*EllipticE((
```

```
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3+33*cos(
d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-
33*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3
*b^3-8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^3+16*cos(d*x+c)^4*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+
18*cos(d*x+c)^4*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3+30*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3
-33*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*a^3+16*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+18*cos(d*x+c
)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3-8*b^3*((a-b)/(a+b))^(1/2)+120*cos(d*x+c)^3*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*a*b^2+33*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-16*cos(d*x+c)^3*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*a*b^2+26*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*cos(d*x+c)^3*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+
120*cos(d*x+c)^4*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+33*cos(d*x+c)^4*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a^2*b-16*cos(d*x+c)^4*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+26*cos(d*x+c)^4*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*
cos(d*x+c)^4*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d
*x+c))^(3/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos \left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.641 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=314

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+
b*sec(d*x+c))^(1/2)+1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d-9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+9/4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.07, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*Sec[c + d*x]^(3/2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{a\left(8a^2+11b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{b(1}{}$$

Mathematica [C] time = 6.60, size = 560, normalized size = 1.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{9}{4}ab\tan(c+dx)+\frac{1}{2}b^2\tan(c+dx)\sec(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2} + \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2\left(16a^3+4ab^2\right)\sqrt{\frac{a\cos}{}}}{\sqrt{a\cos}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(16*a^3 + 4*a*b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]]
+ (2*(21*a^2*b + 8*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((18*I)*a^2*Sqrt[(a -
 a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x
)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c
+ d*x])/(Sqrt[(a - b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*
Sec[c + d*x])^(5/2)*((9*a*b*Tan[c + d*x])/4 + (b^2*Sec[c + d*x]*Tan[c + d*x
])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

maple [C] time = 1.54, size = 1982, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/4/d*(9*cos(d*x+c)^3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-9*cos(d*x+c)^3*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-8*co
s(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2*cos(d*x+c)^3*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b^2+4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3-30*((b+a*cos(d*x+c))/(1+cos(d*
```

```
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*
sin(d*x+c)*a^2*b-8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3+9*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
*sin(d*x+c)*a^2*b-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2-8*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+4*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*b^3-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-8*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^2*b-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b^2+9*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a^2*b-9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*b^3+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+2*b^3*((a-b)/(a+b
))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=263

$$\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\,\mathrm{si}}{}$$

```
[Out] b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+
(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d
*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b^2*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.78, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3842, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{\frac{1}{2}a\left(2a^2-b^2\right)+3a^2b\sec(c+}{\sqrt{\sec(c+dx)}\,\sqrt{a+}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}\left(5ab^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{\left(b\left(4a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right.}{2\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec}}{d}$$

$$= \frac{b\left(4a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(\right.}{d\sqrt{a+}}$$

Mathematica [C] time = 6.62, size = 538, normalized size = 2.05

$$\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^{5/2}}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2} + \frac{a(a+b\sec(c+dx))^{5/2}\left(\frac{2\left(2a^2+9b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{a\cos(c+dx)+b}} + \frac{2i\left(2a^2-b^2\right)\sin(c+d}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])^2*Sec
[c + d*x]^(3/2)) + (a*(a + b*Sec[c + d*x])^(5/2)*((24*a*b*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
 d*x]] + (2*(2*a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(2*a^2 - b
^2)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
```

```
b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a
^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*
(b + a*Cos[c + d*x])^2))))/(4*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2))
```

fricas [F] time = 4.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

maple [C] time = 1.79, size = 1949, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^2*b-Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2*sin(d*x+c)*a*b^2+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*b^3-2*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c
)*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+10*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+2*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-2*cos
(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*a^2*b-cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
```

```
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+cos(d*x+c)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
b^3-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^2+10*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+2*((a-b)/(a+b
))^(1/2)*cos(d*x+c)^3*a^3-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+2*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^2*a^2*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^3-b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a
+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=262

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^3\sqrt{\sec(c+}}{}$$

```
[Out] 2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2
)+2/3*a^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+14/3*a*b*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3841, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^3\sqrt{\sec(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx)+\frac{3}{2}b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2}+\frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(7ab)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx + \frac{1}{3}\left(a\left(a\right.\right.$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(a\left(a^2+2b^2\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+\right.}{3\sqrt{a+b\sec(c+dx}}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{d}$$

Mathematica [C] time = 6.51, size = 409, normalized size = 1.56

$$(a+b\sec(c+dx))^{5/2}\left(\frac{2a\left(a^2+9b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^3} + \frac{b\left(7a^2+6b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^3} + \frac{2a^2\sin(c+dx)}{(a\cos(c+dx)+b)^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*a*(a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (b*
(7*a^2 + 6*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((7*I)*Sqrt[-((a*(-1 + Cos[c +
d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a
 + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a
 + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a
- b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-
1)]*(b + a*Cos[c + d*x])^(5/2)) + (2*a^2*Sin[c + d*x])/(b + a*Cos[c + d*x])
^2))/(3*d*Sec[c + d*x]^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

maple [C] time = 1.79, size = 1661, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
[Out] -2/3/d*(6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+cos(d*x+c)*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
-7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*s
in(d*x+c)*b^3+7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-7*cos(d*x+c)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+(
(a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+7*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
```

```
-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)+8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a*b^2-a^2*b*((a-b)/(a+b))^(1/2)-7*a*b^2*((a-b)/(a+b)
)^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3
/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{16b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d/sec(d*x+c)^(3/2)+22/15*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/
sec(d*x+c)^(1/2)+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*
x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{16b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (22*a*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2}+\frac{3}{2}a\left(a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}b\left(\right.}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{4\int}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{1}{15}\left(-\right.$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{\left(8b\left(a\right.\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{\left(8b\left(a\right.\right.}{}$$

$$= \frac{16b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+23b^2\right)E\left(\frac{1}{2}(\right.}{15d\sqrt{\frac{b+a\,c}{}}}$$

Mathematica [A] time = 1.80, size = 200, normalized size = 0.84

$$\frac{(a+b\sec(c+dx))^{5/2}\left(2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2+28ab\cos(c+dx)+22b^2\right)+32b\left(a^2-b^2\right)\sqrt{\frac{a\cos}{}}\right.}{30d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(4*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 32*b*(
a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)] + 2*a*(3*a^2 + 22*b^2 + 28*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x
)])*Sin[c + d*x]))/(30*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

maple [B] time = 1.92, size = 1931, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+23*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+17*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-23*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-23*cos(d*x+c)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-9*a^
2*b*((a-b)/(a+b))^(1/2)-11*a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a^3+23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(
1/2)*a^3+14*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+34*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a*b^2-5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-23*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a*b^2+15*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^3-9*cos(d*x+c)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a^2*b+23*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+17*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-23*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^2-23*b^3*((a-b)/(a+b))^(1/2)+9*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-23*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)+15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c)
)^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b
)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin(c$$

```
[Out] 2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/7*a^2*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+6/7*a*b*sin(d*x+c)*(a+b*sec(d*x+c
))^(1/2)/d/sec(d*x+c)^(3/2)+2/21*(5*a^2+9*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^
(1/2)/d/sec(d*x+c)^(1/2)+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(
a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\right.}{21ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(21*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^
(5/2)) + (6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3
/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt
[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2}+\frac{1}{2}a\left(5a^2+21b^2\right)\sec(c+dx)+}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} - \frac{4\int}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5}{}$$

$$= \frac{2\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}} + \frac{2b\left(29a^2\right.}{}$$

Mathematica [A] time = 2.48, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(8\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(3a^3\cos(3(c+dx\right.\right.}{84a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(8*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(5
*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(44*a^2*b + 36*b^3 + a*(29*a^2 + 72*b^2)*Cos[
c + d*x] + 24*a^2*b*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)])*Sin[c + d*x]
))/(84*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

maple [B] time = 1.97, size = 2050, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/21/d*(-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-5*a^3*b*(
(a-b)/(a+b))^(1/2)-29*a^2*b^2*((a-b)/(a+b))^(1/2)-9*a*b^3*((a-b)/(a+b))^(1/
2)+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a^3*b+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-29*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-3*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a*b^3+12*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^4+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+2*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-5*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+5*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-3*b^4*((a-b)/(a+b))^(1/2)-29*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+27*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+29*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-29*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-29*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b+27*cos(d*x+c)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a^2*b^2-3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+29*cos(d*x+c)*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
*b-29*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
```

```
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b
+a*cos(d*x+c))/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sqrt{\sec(c+dx)}}+\frac{2a^2\sin(c+dx}{9d\,\mathrm{se}}$$

```
[Out] 4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/9*a^2*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+38/63*a*b*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/315*(49*a^2+75*b^2)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/315*b*(163*a^2+5*b^2)*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/315*(147*a^4+279*a^2*b^2-10*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sqrt{\sec(c+dx)}}+\frac{4b\left(-62a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(9*d*Sec[c + d*x]^(7/2)) + (38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2}+\frac{1}{2}a\left(7a^2+27b^2\right)\sec(c+dx)+\frac{3}{2}\cdots}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec\cdots}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} - \frac{4\int \cdots}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a\cdots\right.}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a\cdots\right.}{\cdots}$$

$$= \frac{4b\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(147a^4\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.89, size = 286, normalized size = 0.79

$$\frac{(a+b\sec(c+dx))^{5/2}\left(32b\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(35a^4\cos(4(c+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^
2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)] + 32*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(301*a^4
+ 1984*a^2*b^2 + 40*b^4 + 4*a*b*(619*a^2 + 160*b^2)*Cos[c + d*x] + 8*(42*a^
4 + 85*a^2*b^2)*Cos[2*(c + d*x)] + 260*a^3*b*Cos[3*(c + d*x)] + 35*a^4*Cos[
4*(c + d*x)])*Sin[c + d*x]))/(2520*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x
]^(5/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

maple [B] time = 2.09, size = 2788, normalized size = 7.68

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*(35*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5+14*cos(d*x+c)^4*((a-b)/(a
+b))^(1/2)*a^5+98*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-147*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^5-10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+147*cos(d*x+c)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*a^5+10*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^5-147*a^4*b*((a-b)/(a+b))^(1/2)-163*
a^3*b^2*((a-b)/(a+b))^(1/2)-279*a^2*b^3*((a-b)/(a+b))^(1/2)-5*a*b^4*((a-b)/
(a+b))^(1/2)+130*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b+170*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a^3*b^2+82*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-279*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+199*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*
b^3+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+80*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*a^2*b^3+272*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-5*cos(d*x+c)^2*((
a-b)/(a+b))^(1/2)*a*b^4-65*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-147*cos(d*x
+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^5-147*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+279*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-279*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-10*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+261*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-279*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+
155*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)+10*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+147*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*((b+a*cos(d*x+c))/(1
```

```
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+10*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)-147*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)+10*b^5*((a-b)/(a+b))^(1/2)-279*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3*b^2+155*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+10*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-
147*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a^4*b+279*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^2-279*cos(d
*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a^2*b^3-10*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4+261*cos(d*x+c)*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c
))^(9/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
[Out] Timed out
```

3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4b^2d}-$$

```
[Out] -1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)
^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^
(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c
))^(1/2)+1/2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d+3/4*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c))/(a
+b))^(1/2)/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+
c))^(1/2)/b^2/d
```

Rubi [A] time = 0.89, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3860, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4b^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d) + (Sec[c + d*x]^(3/2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(a+2b\sec(c+dx)-3a\sec^2(c+dx\right.}{\sqrt{a+b\sec(c+dx)}}}{4b}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{2bd}$$

$$= \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\sec(c+dx)}\,\sqrt{a}}{}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} + \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\right.}{4d\sqrt{a+b\,\mathrm{se}}}$$

Mathematica [C] time = 6.44, size = 397, normalized size = 1.27

$$\sqrt{\sec(c+dx)}\left(b\left(9a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+4ab^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2b\tan(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + b*(9*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] + ((3*I)*Sqrt[-((a*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos
[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipt
icPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)])))/Sqrt[(a - b)^(-1)] + 2*b*(2*b - 3*a*Cos[c + d*x])*(b + a*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/(8*b^3*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

maple [C] time = 1.71, size = 1755, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(6*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*cos(d*
x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*b^2-3*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+c)^3*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
```

```
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b-6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^2+6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a*b+4*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b-6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^2+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-2*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+
3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b
^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a
*cos(d*x+c))/((a-b)/(a+b))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}+\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
d/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+
c)^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A] time = 0.62, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3860, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}+\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(b*d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\int \frac{-a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{2b} - \frac{a\int -$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx - \frac{\int \frac{\sqrt{a+}}{\sqrt{s}}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+a}}{bd}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 9.43, size = 329, normalized size = 1.34

$$\frac{\sqrt{\sec(c+dx)}\left(4\tan(c+dx)(a\cos(c+dx)+b) - 6a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right) - \frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)] - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*
x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sq
rt[(a - b)^(-1)]*b) + 4*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(4*b*d*Sqrt[a +
 b*Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

maple [C] time = 1.75, size = 996, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a+2*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b-2*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2/sin(d*x+c)/(b
+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3859, 2807, 2805}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 1.00

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

maple [C] time = 1.69, size = 216, normalized size = 3.18

$$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\operatorname{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-2\operatorname{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\sqrt{\frac{b+a\cos}{\cos}}}{d\left(b+a\cos(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{a-b}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-2*EllipticPi((-1+cos
```

```
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(b+a
*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}\\
&= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}\\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 67, normalized size = 1.00

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.60, size = 171, normalized size = 2.55

$$\frac{2\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\cos(dx+c)\left(\sin^2(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticF}\left(\frac{(-1+\cos(dx+c)}{\sin(dx+c}\right.}{d(-1+\cos(dx+c))(b+a\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*sin
(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))/(-1+cos(d*x+c))/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec}}{a\sqrt{b+}}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+}}{a\sqrt{\frac{b+a\,c}{a}}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{ad\sqrt{\frac{b+a\cos(c+d}{a+b}}}$$

Mathematica [A] time = 2.96, size = 96, normalized size = 0.68

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(a
*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^2+a\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 1.83, size = 736, normalized size = 5.18

$$-\frac{2\left(-\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a+\sqrt{\frac{\cdots}{(1+\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*b-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)
/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/s
in(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+ \cdots}{3a \cdots}$$

```
[Out] 2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/a/d/sec(d*x+c)^(1/2)-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3863, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+ \cdots}{3a \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a^2} + \frac{1}{3}\left(1+\frac{2b^2}{a^2}\right.$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(\right.\right.}{3\sqrt{a+b\sec(c}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx\right.\right.}{3\sqrt{a+b\sec(c+}$$

$$= \frac{2\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} - \frac{4bE\left(\frac{1}{2}(c+\right.}{3a^2d\sqrt{\frac{b-}{}}}$$

Mathematica [A] time = 0.70, size = 147, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2a\sin(c+dx)(a\cos(c+dx)+b)-4b(a+b)\sqrt{\frac{a}{}}\right.}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d*x])*
Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^3+a\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 1.98, size = 1024, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*cos(d*x+c)*
sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a*b+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x
```

```
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+2*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-
b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))/a^2/((a-b
)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\,\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\sec(c+dx)}}-\frac{2b(7a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b\ldots}{15\ldots}$$

```
[Out] -2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(
1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*sec(
d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)-8/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)
/a^2/d/sec(d*x+c)^(1/2)+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3863, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b(7a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\ldots\right)}{15a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (
8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{\int \frac{4b-3a\sec(c+dx)-2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}} + \frac{2(9}{}$$

Mathematica [A] time = 0.85, size = 193, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2-2ab\cos(c+dx)-8b^2\right)-4b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+}{a+b}}\right.}{30a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - 4*b*(7*a^2 + 8*b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(3*a^2 - 8*b^2 - 2*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b\sec(dx+c)^4+a\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^4 + a*
sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 2.11, size = 1736, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-9*a^2*b*
((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d*x+c)*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a
^3-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a*b^2+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+2*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-8*b^3*((a-b)/(a+b)
)^(1/2)+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(
d*x+c))/a^3/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec\left(dx + c\right) + a} \sec\left(dx + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\,\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/2)), x)
```

3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d(a^2-b^2)}-\frac{(3a^2-b^2)\sqrt{a+b\sec}}{b^2d(a^2-b^2)\sqrt{\sec}}$$

```
[Out] -2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d
/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)-(3*a^2-b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d
```

Rubi [A] time = 1.00, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3845, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d(a^2-b^2)}-\frac{(3a^2-b^2)\sqrt{a+b\sec}}{b^2d(a^2-b^2)\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\text{si}}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\text{si}}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\text{si}}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,\text{si}}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{b^2d\sqrt{a+b\sec(c+d}}$$

Mathematica [C] time = 4.85, size = 478, normalized size = 1.39

$$\sec^{\frac{3}{2}}(c+dx)\left(\frac{4\tan(c+dx)(a\cos(c+dx)+b)\left(\left(ab^2-3a^3\right)\cos(c+dx)-a^2b+b^3\right)}{b^4-a^2b^2}-\frac{a(a\cos(c+dx)+b)^{3/2}\left(\frac{2\left(9a^2-7b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{a\cos(c+dx)+b}}+\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-((a*(b + a*Cos[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[
c + d*x]] + (2*(9*a^2 - 7*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(3*a^2
- b^2)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))
/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipti
cPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
b)/(a + b)])))/(a^2*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))) + (4*(b
+ a*Cos[c + d*x])*(-(a^2*b) + b^3 + (-3*a^3 + a*b^2)*Cos[c + d*x])*Tan[c +
d*x])/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [C] time = 1.86, size = 1501, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2+4*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-3*cos(d*x
+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
```

```
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2-6*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/
(a+b))^(1/2))*a^2-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b+6*cos(d*x+c)*
sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a^2+4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-3*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+cos(
d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*b^2-6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-6*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a*b+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-3*((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1/2
))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(7/2)/(b
+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\Big(}{bd\sqrt{a+b\sec(c+d}}$$

```
[Out] -2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c
),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/b/d/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c)
)^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3845, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\Big(}{bd\sqrt{a+b\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b^2)*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b\left(a^2-b^2\right)} + \frac{\left(\sqrt{b+a\cos(c+dx)}\right.}{b\sqrt{}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c}{a+b}}}}{b\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.52, size = 557, normalized size = 2.70

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}{bd\left(b^2-a^2\right)(a+b\sec(c+dx))^{3/2}} + \frac{\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^{3/2}\left(\frac{2\left(3a^2-2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}\right.}{\sqrt{a\cos(c+dx)+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(-a^2 + b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^
(3/2)*((4*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Co
s[c + d*x]] + ((2*I)*a^2*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos
[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt
[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipt
icF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b
)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Co
s[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
 a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(2*(a - b)*b*(a + b)*d*(a +
 b*Sec[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [C] time = 1.57, size = 1144, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-2*cos(d*x+c)*sin(d*x+c)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*b+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-2*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*sin(d*x
+c)-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-a*((a-b)/
(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c
))^(5/2)/(b+a*cos(d*x+c))/sin(d*x+c)/b/((a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3844, 21, 3856, 2655, 2653}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 -
 b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
```

```
&& IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}\\
&= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a^2-b^2}\\
&= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}}\\
&= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}}\\
&= -\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 103, normalized size = 0.82

$$-\frac{2\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left((a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-a\sin(c+dx)\right)}{d(a-b)(a+b)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a + b)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - a*Sin[c + d*x]))/((a -
 b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.92, size = 501, normalized size = 3.98

$$-\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}} - \mathrm{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)+EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)*((a-b)/(a+b))^(1/2)-((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(b+a*cos(d*x+c))/sin(d*x+c)/((a
-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(\right.}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(
a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)
/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3843, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(\right.}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,d}{a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c}}$$

Mathematica [A] time = 0.62, size = 156, normalized size = 0.78

$$\frac{2\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-ab\sin(c+dx)+b(a+b)\sqrt{\frac{a\cos(c+dx}{a+b}}\right.}{ad(a-b)(a+b)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(b*(a + b)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - a*b*Sin[c
 + d*x]))/(a*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.73, size = 510, normalized size = 2.55

$$2\left(-\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-\sqrt{\frac{b+}{(1+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*
x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)*(1/
cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)/a/((a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{a^2d\sqrt{a+b\,\mathrm{se}}}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-4*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
a^2/d/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)
^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3847, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{a^2d\sqrt{a+b\,\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2} + \frac{\left(a^2-2b\right.}{}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{a^2\sqrt{a+b\sec(c+}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int}{a^2\sqrt{a+b\sec(c+d}}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}\right.}{a^2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.72, size = 165, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(b\left(ab\sin(c+dx)-2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)+\left(a^3+a^2b-2ab^2-2b^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{a^2d(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + b*(-2*(a^2 - b^2)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*
b*Sin[c + d*x])))/(a^2*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^3+2ab\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 1.79, size = 999, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*(cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-cos(d*x+c)*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*s
```

```
in(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-2*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*b^2+a*b*((a-b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))^(1/2))*(
(b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/si
n(d*x+c)/(a+b)/((a-b)/(a+b))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2\left(a^2+8b^2\right)\sqrt{\sec}}{3}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*
(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d
*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)-2/3*b*(5*a^2-8*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2\left(a^2+8b^2\right)\sqrt{\sec}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+2b^2+\frac{1}{2}ab\sec(}{\sec^{\frac{3}{2}}(c+dx)\sqrt{}}}{a(a^2}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{a+}}{3a^2(a^2-b}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{a+}}{3a^2(a^2-b}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{a+}}{3a^2(a^2-b}$$

$$= \frac{2b^2\sin(c+dx)}{a(a^2-b^2)\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-4b^2)\sqrt{a+}}{3a^2(a^2-b}$$

$$= \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}} - \frac{2b(5a}{3}$$

Mathematica [A] time = 0.96, size = 203, normalized size = 0.70

$$\frac{2\sqrt{\sec(c+dx)}\left(a\sin(c+dx)\left(a(a^2-b^2)\cos(c+dx)+b(a^2-4b^2)\right)+(a^4+7a^2b^2-8b^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(\right.\right.}{3a^3d(a-b)(a+b)\sqrt{a+b\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^4 + 7*a^2*
b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
)/(a + b)] + a*(b*(a^2 - 4*b^2) + a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])
)/(3*a^3*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 2.14, size = 1315, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-5*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+cos(d*x+
c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*cos(d*x+c)*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b
^2+((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
2*b-5*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-4*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-4*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+4*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-a^2*b*((a-b)/(
a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^(1/2)+8*b^3*((a-b)/(a+b))^(1/2))*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b
+a*cos(d*x+c))/a^3/((a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(dx + c\right) + a\right)^{\frac{3}{2}} \sec\left(dx + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sec{(c + dx)})^{\frac{3}{2}}\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), x)
```

3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}-\frac{8b\left(a^2+4b^2\right)\sqrt{\sec(c}{5a^4d}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-8/5*
b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/a^4/d/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(3/2)-2/5*b*(3*a^2-8*b^2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/5*(3*a^4+8
*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2
-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\left(3a^2-8b^2\right)\sin(c+}{5a^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a^4*d*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^
(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2*b*(3*a^2 - 8*b
^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+3b^2+\frac{1}{2}ab\sec(c+dx}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b}}}{a\left(a^2-b\right.}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{se}}}{5a^2\left(a^2-b^2\right)a}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{se}}}{5a^2\left(a^2-b^2\right)a}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{se}}}{5a^2\left(a^2-b^2\right)a}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{se}}}{5a^2\left(a^2-b^2\right)a}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\,\mathrm{se}}}{5a^2\left(a^2-b^2\right)a}$$

$$= -\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5a^4d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4\right.}{}$$

Mathematica [A] time = 1.46, size = 250, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(2a\sin(c+dx)\left(a^4-4ab\left(a^2-b^2\right)\cos(c+dx)-7a^2b^2+\left(a^4-a^2b^2\right)\cos(2(c+dx))+16b^4\right)-16b\right.}{10a^4d(a-}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(3*a^5 + 3*a^4*b + 8*a^3*b^2 + 8*a^2*b^3 - 16*a*b^4
- 16*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(
a + b)] - 16*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(a^4 - 7*a^2*b^2 + 16*b^4 - 4*
a*b*(a^2 - b^2)*Cos[c + d*x] + (a^4 - a^2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*
x]))/(10*a^4*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^5+2\,ab\sec(dx+c)^4+a^2\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^5 +
2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 1.89, size = 1861, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
[Out] -2/5/d*(-16*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4+3*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a^4+((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4+2*((a-b)/(a+b))^(1/
2)*cos(d*x+c)^2*a^4-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b+8*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^2*a^2*b^2-3*a^3*b*((a-b)/(a+b))^(1/2)-8*a*b^3*((a-b)/(a+
b))^(1/2)-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*a^3*b+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+2*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+cos(d*x+c)
^4*((a-b)/(a+b))^(1/2)*a^3*b+16*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-3*a^4*((
a-b)/(a+b))^(1/2)*cos(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*b^4*((a-
b)/(a+b))^(1/2)-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-12*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*cos(d*x+c)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b-12*co
s(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*a^2*b^2-16*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+8*cos(d*x+c)*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*
x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^4/((a-b)/(a+b))^(1/2)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{\left(5a^2-3b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}{3b^2d\left(a^2-b^2\right)\sqrt{a}}$$

```
[Out] -2/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(5*a^2-9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))^(1/2)+1/3*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c
))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-5*a
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/b^3/d/(a+b*sec(d*x+c))^(1/2)-1/3*(15*a^4-26*a^2*b^2+3*b^4)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)+1/3*(15*a^4-26*a^2*b^2+3*b^4)*sin(d*x+c)*sec(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d
```

Rubi [A] time = 1.41, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3845, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{\left(-26a^2b^2+15a^4+3b^4\right)\sin(c}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((5*a^2 - 3*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
d*x]]) - (5*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^3*d*Sqrt[a + b*Sec[c + d*x]]) - ((15
*a^4 - 26*a^2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((15*a^4
 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(5a^2-3b^2\right)\sec^2(c}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3 d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3b\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{\left(5a^2-3b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\right.}{b^3 d\sqrt{a+}}$$

Mathematica [C] time = 6.82, size = 677, normalized size = 1.48

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{2a^3\sin(c+dx)}{3b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}-\frac{4\left(5a^3b^2\sin(c+dx)-3a^5\sin(c+dx)\right)}{3b^3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{b^3}\right)}{d(a+b\sec(c+dx))^{5/2}} - \frac{a\sec^{\frac{5}{2}}(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/12*(a*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*((2*(20*a^3*b - 36*a
*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(45*a^4 - 86*a^2*b^2 + 33*b^4)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
 + a*Cos[c + d*x]] + ((2*I)*(15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[(a - a*Cos[c
 + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b
*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/
(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]
^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^
```

```
2))))/((a - b)^2*b^3*(a + b)^2*d*(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[
c + d*x])^3*Sec[c + d*x]^(5/2)*((-2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(
b + a*Cos[c + d*x])^2) - (4*(-3*a^5*Sin[c + d*x] + 5*a^3*b^2*Sin[c + d*x]))
/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + Tan[c + d*x]/b^3))/(d*(a + b
*Sec[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)^{\frac{9}{2}}}{(b \sec (dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [C] time = 1.77, size = 4591, normalized size = 10.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d*(15*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+3*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^5+30*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5-30*cos(d*x+c)^3*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^5+30*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5+15*cos(d*x+c)^2*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^5+3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5-30*cos(d*x+c)^2*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+15*
cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^5+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^5-15*cos(d*x+c)^3*((a
-b)/(a+b))^(1/2)*a^5+3*a^3*b^2*((a-b)/(a+b))^(1/2)+3*a^2*b^3*((a-b)/(a+b))^
(1/2)-3*a*b^4*((a-b)/(a+b))^(1/2)+21*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b
^2-15*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b+23*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^2*b^3-5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+5*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^3*b^2-29*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3-3*cos(d*x+c)
```

*((a-b)/(a+b))^(1/2)*a*b^4+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3-29*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4+20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3-30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4-3*b^5*((a-b)/(a+b))^(1/2)+30*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b-30*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^2-30*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3-26*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+3*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-20*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+36*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+18*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+60*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b-60*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3-30*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4+15*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-26*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-26*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-50*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+16*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+54*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+18*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b+30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-

```
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3*b^2-20*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+36*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+
18*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^4+15*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b-26*cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a^2*b^3-30*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b)*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/(b+a*cos(d*x+c))^
2/b^3/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F(}{3bd(a^2-b^2)\sqrt{a+b\sec(}}$$

```
[Out] -2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*sec
(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*s
ec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*
x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/
2)
```

Rubi [A] time = 1.10, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3845, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F(}{3bd(a^2-b^2)\sqrt{a+b\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}\left(a^2-b^2\right)\sec^2(}{(a+b\sec(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3b\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{b^2d\sqrt{a+b\sec}}$$

Mathematica [C] time = 5.93, size = 487, normalized size = 1.32

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{2a^2b\left(7b^2-3a^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{\left(a^2-b^2\right)^2}+\frac{2a^2b^2\sin(c+dx)(a\cos(c+dx)+b)}{b^2-a^2}+\frac{4ab^2\left(a^2-3b^2\right)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{5/2}F\left(\frac{1}{2}(c+dx)|\frac{2}{a-}\right.}{(a-b)^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((4*a*b^2*(a^2 - 3*b^2)*((b + a*Cos[c + d*x])/(a + b))^
(5/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(a - b)^2 + (b*(9*a^4 - 19*a^2
*b^2 + 6*b^4)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(a - b)^2 + (I*((a - b)^(-1))^(3/2)*(3*a^2 - 7*b^2)*Sqrt
[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*(
b + a*Cos[c + d*x])^(5/2)*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sq
rt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Elli
pticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)])))/(a + b)^2 + (2*a^2*b^2*(b + a*Cos[c + d*x])*
Sin[c + d*x])/(-a^2 + b^2) + (2*a^2*b*(-3*a^2 + 7*b^2)*(b + a*Cos[c + d*x])
^2*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [C] time = 1.56, size = 3854, normalized size = 10.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-6*cos(d*x+c)*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*a^4+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-6*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-
3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4+12*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+
c)*a^3*b-12*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a*b^3-6*cos(d*x+c)^2*sin(d*
x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*a^2*b^2+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b-6*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^3-7*cos(d*x+c)^2*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^2-4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+9*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(
```

```
d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^3+6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2+4*a^3*
b*((a-b)/(a+b))^(1/2)+a^2*b^2*((a-b)/(a+b))^(1/2)-7*a*b^3*((a-b)/(a+b))^(1/
2)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^3*b-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+7*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a*b^3+3*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+3*cos(d*x+c)^2*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^4-6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-
6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+6*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+9*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)-7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-10*cos(d*x+c)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a^3*b+5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2+12*cos(d*x+c)*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+3*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b-7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-7*cos(d*x+c)*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))
^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F(}{3d\left(a^2-b^2\right)\sqrt{a+b\sec(}}$$

```
[Out] -2/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2
/3*a*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(
d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/s
ec(d*x+c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3845, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F(}{3d\left(a^2-b^2\right)\sqrt{a+b\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*(a^2 - b^2)^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 1.15, size = 169, normalized size = 0.61

$$-\frac{2\sec^{\frac{3}{2}}(c+dx)\left(a\sin(c+dx)\left(-a^2+4ab\cos(c+dx)+5b^2\right)-(a-b)(a+b)^2\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{3d(a-b)^2(a+b)^2(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^(3/2)*(-4*b*(a + b)^2*((b + a*Cos[c + d*x])/(a + b))^(3/2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(a + b)^2*((b + a*Cos[c +
d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-a^2 + 5*b^
2 + 4*a*b*Cos[c + d*x])*Sin[c + d*x]))/(3*(a - b)^2*(a + b)^2*d*(a + b*Sec[
c + d*x])^(3/2))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.70, size = 1343, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-3*cos(d*x
+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*a*b+4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+4*cos(d*x+c)*sin(d*x+c)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^
2+cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-3*cos(d*x+c)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*b^2+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^2-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+4*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/
2)*a^2-a*b*((a-b)/(a+b))^(1/2)+4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))
/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*
x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*(a
^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/
3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3844, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{a}{2}-\frac{3}{2}b\sec(c+dx)+a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{\frac{1}{4}\cdots}{\sqrt{s\cdots}}}{\cdots}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{b\int \frac{\cdots}{\sqrt{a\cdots}}}{3a\cdots}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(b\sqrt{b\cdots}\right.}{\cdots}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(b\sqrt{\frac{b+\cdots}{\cdots}}\right.}{\cdots}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2+b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\cdots}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+d\cdots}{a+b}}}$$

Mathematica [A] time = 1.11, size = 178, normalized size = 0.63

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\sin(c+dx)(a\cos(c+dx)+b)\left(a\left(3a^2+b^2\right)\cos(c+dx)+2b\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2} - \frac{2(a+b)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{5/2}\left(\left(3a^2+b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b(a-b)F\left(\frac{1}{2}(c\cdots\right.\right.}{a(a-b)^2}\cdots\right)}{3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*a
^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*b*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*b*(a^2
+ b^2) + a*(3*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*d*(
a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.56, size = 1822, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-3*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)^2*sin(d*x+c)*a^3-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2+3*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3+2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*cos(d*x+c
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*a^3-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-cos(d*x+c)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-
cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*b^3+3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-((a-b)/(a+b))^(
1/2)*cos(d*x+c)^2*a^2*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b^3-2*a^2*b*((a-b)/(a+b))^(1/2)+a*b^2*((a-b)/(a+b))^(1/2)
-b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c)
)^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/a/((a-b)/(
a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(5/2), x)
```

3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{3a^2d\left(a^2-b^2\right)\sqrt{a+}}$$

```
[Out] -2/3*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2/3*b
*(5*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(
1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3843, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{3a^2d\left(a^2-b^2\right)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{b}{2}-\frac{3}{2}a\sec(c+dx)+b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(3a^2-b^2\right)E\left(\frac{1}{2}\right.}{3a^2\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 1.31, size = 196, normalized size = 0.65

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{ab\sin(c+dx)\left(\left(2ab^2-6a^3\right)\cos(c+dx)-5a^2b+b^3\right)}{\left(a^2-b^2\right)^2}+\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(\left(6a^2b-2b^3\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(\right.}{(a-b)^2}\right.}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((6*a^2*b - 2*b^3)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (3*a^3 -
3*a^2*b - 2*a*b^2 + 2*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^
2 + (a*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.52, size = 2070, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-3*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^3*b-2*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*a^2*b^2+5*a^2*b^2*((a-b)/(a+b))^(1/2)-a*b^3*((a-b)/(a+b))^(1/
2)-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b^3+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^2*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+2*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*b^4-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-2*b^4*((a-b)
/(a+b))^(1/2)+3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)+6*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3*b-2*cos(d*x+c)^2*sin(d*x+c
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*a*b^3-5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-2*cos(d*x+c)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+6*cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3*b+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*cos(d*x+c)*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx + c)}}{(b\sec(dx + c) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c + dx)}}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{3a^3d\left(a^2-b^2\right)\sqrt{a+b}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*
x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*
(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3847, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{3a^3d\left(a^2-b^2\right)\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c +
d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+2b^2+\frac{3}{2}ab\sec(c+dx)-b^2\sec^2(c}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\,\sin}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= -\frac{2b\left(9a^2-8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3\right.}{}$$

Mathematica [A] time = 1.38, size = 208, normalized size = 0.66

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{ab^2\sin(c+dx)\left(a\left(9a^2-5b^2\right)\cos(c+dx)+8a^2b-4b^3\right)}{\left(a^2-b^2\right)^2}+\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(\left(3a^4-15a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.\right.}{(a-b)}\right.}{3a^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
+ b*(-9*a^3 + 9*a^2*b + 8*a*b^2 - 8*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]))/(a - b)^2 + (a*b^2*(8*a^2*b - 4*b^3 + a*(9*a^2 - 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^4+3\,ab^2\sec(dx+c)^3+3\,a^2b\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 1.87, size = 3103, normalized size = 9.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-8*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^5+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5+3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*cos(d
*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^5+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^5+3*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^5-3*a^3*b^2*((a-b)/(a+b))^(1/2)-11*a^2*b^3*((a-b)/(a+b))^(1/2)
+4*a*b^4*((a-b)/(a+b))^(1/2)-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2+3*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^
2*b^3+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-12*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^3*b^2+18*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+8*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b^4-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+18*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-6
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-15*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*b^5*((a-b)/(a+b))^(
1/2)-15*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-9*c
os(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^4*b+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*cos(d*x+c)
```

```
^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b^3-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+14*cos(d*x+c)*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^3+8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+3*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b-15*
cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*a^3*b^2-15*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3+8*cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a*b^4-12*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)/(b+a*cos(d*x+c))^2/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/a^3/((a-b)/(a
+b))^(1/2)/(a+b)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), x)
```

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2\left(a^4+1\right.}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2
-b^2)^2/d/sec(d*x+c)^(1/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-13a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

```
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab \cdots}{\sec^{\frac{3}{2}}(c+\cdots}}{3a \cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4b^2 (\cdots}{3a^2\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4b^2 (\cdots}{3a^2\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4b^2 (\cdots}{3a^2\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4b^2 (\cdots}{3a^2\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4b^2 (\cdots}{3a^2\left(a^2-b^2\right)^2 d \cdots}$$

$$= \frac{2\left(a^4+16a^2b^2-16b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx \cdots}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 1.79, size = 257, normalized size = 0.66

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{a\sin(c+dx)\left(a^6+\left(a^3-ab^2\right)^2\cos(2(c+dx))-25a^2b^4+4ab\left(a^4-8a^2b^2+5b^4\right)\cos(c+dx)+16b^6\right)}{2\left(a^2-b^2\right)^2}+\frac{\left(\frac{a\cos(c+\cdots}{a+\cdots}\right.\cdots}{\cdots}\right)}{3a^4d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*(-4*(2*a^4*b - 7*a^2*b^3 + 4*b^5)*EllipticE[(c + d*x)/2, (2*a)/(a +
b)] + (a^5 - a^4*b + 16*a^3*b^2 - 16*a^2*b^3 - 16*a*b^4 + 16*b^5)*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + (a*(a^6 - 25*a^2*b^4 + 16*b^6 +
4*a*b*(a^4 - 8*a^2*b^2 + 5*b^4)*Cos[c + d*x] + (a^3 - a*b^2)^2*Cos[2*(c + d
*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a + b*Sec[c + d*x])^(5/2)
)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

maple [B] time = 2.04, size = 3615, normalized size = 9.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(a^4*b^2*((a-b)/(a+b))^(1/2)-7*a^3*b^3*((a-b)/(a+b))^(1/2)-20*a^2*b^4
*((a-b)/(a+b))^(1/2)+8*a*b^5*((a-b)/(a+b))^(1/2)+28*cos(d*x+c)*sin(d*x+c)*(
1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b^4+16*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+16*cos(d*x+c)^2*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5-9*co
s(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^5*b-16*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+12*cos(d*x+c
)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3*b^3+16*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+8*cos(d*x+c)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a^5*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b^2-28*cos(d*x+c)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b
^3+16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*b^6-cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+16*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a*b^5-6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^3-6*cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^4-7*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*
b+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b^2+34*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^3*b^3-8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4-24*cos(d*x+c)^2*((
```

```
a-b)/(a+b))^(1/2)*a*b^5+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b-14*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^4*b^2-22*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3+34*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4-cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5
*b+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b^2+cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a^3*b^3+6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b+6*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^4*b^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+8*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*(1/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-28*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^4*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^5*b*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-16*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3*(1/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+12
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)+16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^5*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+16*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6*(1/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-16*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*b^6-cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6+cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^6+16*b^6*((a-b)/(a+b))^(1/2)-28*cos(d*x+c)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2
*b^4+16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^5-10*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-25*cos(d
*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^4*b^2-4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+8*cos(d*x+c)^2*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^5*b-28*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^3)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))^2/
a^4/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=474

$$\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{4b\left(7a^4-4\right.}{}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)+8/
3*b^2*(3*a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)/(a+b*sec(
d*x+c))^(1/2)-2/15*b*(17*a^4+116*a^2*b^2-128*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)+2/15*(3*a^4-71*a^2*b^2+48*b^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)-4/15*b*(7*a^4-49*a^2*b^2+32*b^4)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+2/15*(9*a
^6+55*a^4*b^2-212*a^2*b^4+128*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+
c))^(1/2)/a^5/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-71a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*(17*a^4 + 116*a^2*b^2 - 128*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2
)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^
5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d
*x])^(3/2)) + (8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
ec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 71*a^2*b^2 + 48*b
^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d*Sec[c +
d*x]^(3/2)) - (4*b*(7*a^4 - 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+4b^2+\frac{3}{2}ab}{\sec^{\frac{5}{2}}(c+d}}{3a}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2b\left(17a^4+116a^2b^2-128b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec}}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.20, size = 292, normalized size = 0.62

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(a\left(\frac{10b^5\sin(c+dx)}{b^2-a^2}-\frac{10b^4\left(11b^2-15a^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^2}-28b\sin(c+dx)(a\cos(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((2*((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*b^6)*EllipticE[(c + d*x)/2,
```

```
(2*a)/(a + b)] + b*(-17*a^5 + 17*a^4*b - 116*a^3*b^2 + 116*a^2*b^3 + 128*a
*b^4 - 128*b^5)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + a*((10*
b^5*Sin[c + d*x])/(-a^2 + b^2) - (10*b^4*(-15*a^2 + 11*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*b*(b + a*Cos[c + d*x])^2*Sin[c + d*x
] + 3*a*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])))/(15*a^5*d*(a + b*Sec[c +
d*x])^(5/2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}}{b^3\sec\left(dx+c\right)^6+3\,ab^2\sec\left(dx+c\right)^5+3\,a^2b\sec\left(dx+c\right)^4+a^3\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^6 +
3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

maple [B] time = 2.07, size = 4586, normalized size = 9.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^7+6*cos(d*x+c)^3*((a-b)/(a+b)
)^(1/2)*a^7+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^7-9*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^7-9*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^7-128*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^7+
5*a^4*b^3*((a-b)/(a+b))^(1/2)-50*a^3*b^4*((a-b)/(a+b))^(1/2)-148*a^2*b^5*((
a-b)/(a+b))^(1/2)+64*a*b^6*((a-b)/(a+b))^(1/2)-9*a^5*b^2*((a-b)/(a+b))^(1/2
)+17*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^6*b-38*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^5*b^2+42*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b^3+254*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^3*b^4-64*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^5-192*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^6+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a
^6*b-3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^5*b^2-3*cos(d*x+c)^5*((a-b)/(a+b)
)^(1/2)*a^4*b^3-8*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6*b-8*cos(d*x+c)^4*((a
-b)/(a+b))^(1/2)*a^5*b^2+6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^6*b+42*cos(d*
x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b^2+42*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*
b^3-48*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^4-48*cos(d*x+c)^3*((a-b)/(a+b
))^(1/2)*a^2*b^5+8*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b^3+8*cos(d*x+c)^4*
((a-b)/(a+b))^(1/2)*a^3*b^4-18*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^6*b+16*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b^2-94*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^
3-164*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^4+260*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^5+128*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^6+128*EllipticE((-1+cos
```

(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^7*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+55
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)*a^5*b^2-212*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^4+128*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a*b^6-17*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^6*b-72*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
2*sin(d*x+c)*a^5*b^2-116*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^4*b^3+96*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*si
n(d*x+c)*a^3*b^4+128*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^2*b^5+9*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a^6*b+55*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2+55*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^3-2
12*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*a^3*b^4-212*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^5+128*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a*b^6-26*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^6*b-89*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*
x+c)*a^5*b^2-188*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^4*b^3-20*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^3
*b^4+224*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^2*b^5+128*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a*b^6+128*
b^7*((a-b)/(a+b))^(1/2)+9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^7+128*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*b^7-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^7+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)

```
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6*b*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+55*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-212*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-17*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b^2*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-72*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-116*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+96*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^5*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+128*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^6*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/s
in(d*x+c)/(b+a*cos(d*x+c))^2/a^5/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

```
[Out] -3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+3*
sec(d*x+c))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*5^(1/2)*(2+3*sec(d*x+c))^(1/2)/d/(3+2*cos(
d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2, 4/
5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$$
$$= -\frac{\left(3\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}\,dx}{2\sqrt{2+3\sec(c+dx)}} + \frac{\sqrt{2+3\sec}}{2\sqrt{3}}$$
$$= -\frac{3\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}} + \frac{\sqrt{5}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{3+2\cos(}}$$

Mathematica [A] time = 0.15, size = 78, normalized size = 0.64

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2+2\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 + 2*
sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.77, size = 405, normalized size = 3.32

$$-\frac{\left(2i\sin(dx+c)\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\cos(dx+c)\sqrt{5}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},\sqrt{5}\right)\sqrt{2}+i\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/10/d*(2*I*sin(d*x+c)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*5^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5
^(1/2)/sin(d*x+c),5^(1/2))*2^(1/2)+I*sin(d*x+c)*10^(1/2)*((3+2*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*5^(1/2)*EllipticE
(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*2^(1/2)+2*I*5^(1/2)*Elli
pticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/2)*((3+2*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+I
*5^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/
2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)+20*cos(d*x+c)^2+10*cos(d*x+c)-30)*((3+2*cos(d*x+c))/cos(d*x+c))
^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(3+2*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \sec(dx+c)+2}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \sec(c+dx)+2}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) + 2)*sqrt(sec(c + d*x))), x)
```

3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2)-
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2*I)*(-2+3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*Sec
[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
```

```
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$$
$$= \frac{\left(3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{2\sqrt{-2+3\sec(c+dx)}} - \frac{\sqrt{-2+3}}{2}$$
$$= \frac{3\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}} - \frac{E\left(\frac{1}{2}(c+\right.}{d\sqrt{3-}}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 0.62

$$-\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3F\left(\frac{1}{2}(c+dx)\middle|-4\right)\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c + d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[-2 + 3*Sec[c + d*x]]))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2-2\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 - 2*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 1.88, size = 374, normalized size = 3.43

$$\frac{\left(-2i\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sin(dx+c)-i\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(-2*I*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-I*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*
EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2
-10*cos(d*x+c)+6)*(-(-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2
)/sin(d*x+c)/(-3+2*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\,\sec\left(dx+c\right)-2}\,\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sec\left(c+dx\right)-2}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2*I)*(2-3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x
)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{-3+2\cos(c+dx)}\,dx}{2\sqrt{-3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{-3+2\cos(c+dx)}\right.}{2\sqrt{}}$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{3-2\cos(c+dx)}\,dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{3-2\cos(c+dx)}\right.}{2\sqrt{2-}}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{3\sqrt{3-2\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d\sqrt{2-3\sec(}}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.63

$$-\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3F\left(\frac{1}{2}(c+dx)\middle|-4\right)\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]]))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2-2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 -
2*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

maple [B] time = 1.85, size = 405, normalized size = 3.75

$$-\frac{\left(2i\sqrt{5}\ \cos\left(dx+c\right)\sin\left(dx+c\right)\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\ \sqrt{2}\ \sqrt{\frac{1}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-5i\sqrt{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/10/d*(2*I*5^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^
(1/2)/sin(d*x+c),1/5*5^(1/2))-5*I*5^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*(-3+2*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))+2*I*5^(1/2)*EllipticF(I*
(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*I*5^(1/2)*El
lipticE(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+20*c
os(d*x+c)^2-50*cos(d*x+c)+30)*((-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d
*x+c))^(1/2)/sin(d*x+c)/(-3+2*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\ \sec\left(dx+c\right)+2}\ \sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}\ \sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3\sec\left(c+dx\right)}\ \sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2/5*5^(1/2))*5^(1/2)*(-2-3*sec(d*x+c))^(1/2)/d/(3+2*cos(d*x+c))^(1/2)/se
c(d*x+c)^(1/2)-3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d
*5^(1/2)/(-2-3*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[3
 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)-\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$$

$$= -\frac{\sqrt{-2-3\sec(c+dx)}\int \sqrt{-3-2\cos(c+dx)}\,dx}{2\sqrt{-3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{-3-2\cos\right.}{}$$

$$= -\frac{\left(\sqrt{5}\sqrt{-2-3\sec(c+dx)}\right)\int \sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}\,dx}{2\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{3+2\right.}{}$$

$$= -\frac{\sqrt{5}\,E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{3\sqrt{3+2\cos(c+dx)}\,I}{\sqrt{5}\,d\sqrt{-2}}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.63

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\,\sec\left(dx+c\right)-2}\sqrt{\sec\left(dx+c\right)}}{3\,\sec\left(dx+c\right)^2+2\,\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 +
2*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.86, size = 390, normalized size = 3.17

$$-\frac{\left(2i\cos(dx+c)\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-5i\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
[Out] -1/10/d*(2*I*cos(d*x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),1/5*5^(1/2))-5*I*cos(d*x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)
*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),1/5*5^(1/2))+2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5
*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*
5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)-20*cos(d*x+c)^2-10*cos(d*x+c)+30)*(-(3+2*cos(d*x
+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(3+2*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
[Out] int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(1/(sqrt(-3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}\,E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

```
[Out] -4/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(3+
2*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(3+2*sec(d*x+c))^(1/2)/d/(2
+3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}\,E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)/2
, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \frac{1}{3}\int \frac{\sqrt{3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$$
$$= -\frac{\left(2\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}\,dx}{3\sqrt{3+2\sec(c+dx)}} + \frac{\sqrt{3+2\sec}}{3\sqrt{2}}$$
$$= -\frac{4\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}} + \frac{2\sqrt{5}\,E\left(\frac{1}{2}(c+d\right.}{3d\sqrt{2+3\cos}}$$

Mathematica [A] time = 0.16, size = 81, normalized size = 0.64

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)-2F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2+3\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 + 3*
sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.80, size = 409, normalized size = 3.22

$$\frac{\left(3\sin(dx+c)\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\sin(dx+c)}, i\sqrt{5}\right)\sqrt{5}\,\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{2} - \sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(3*sin(d*x+c)*cos(d*x+c)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d
*x+c),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*2^(1/2)-sin(d*x+c)*cos(d*x+c)*EllipticE(1/5*5^(1/2)
*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*5^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*2^(1/2)+3*5^(1/2)*EllipticF(1
/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*10^(1/2)*((2+3*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5^(1/2)*
EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*10^(1/2)*((2+3*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-30*cos(d*x+c)^2+10*cos(d*x+c)+20)*((2+3*cos(d*x+c))/cos(d*x+c))^(1/2)/(1
/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 \sec\left(dx + c\right) + 3} \sqrt{\sec\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}+3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 \sec{\left(c + dx \right)} + 3} \sqrt{\sec{\left(c + dx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) + 3)*sqrt(sec(c + d*x))), x)
```

3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{4\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),6^(1/2))*(3-2*sec(d*x+c))^(1/2)/d/(-2+3*cos(d*x+c))^(1/2)/sec(d*x+c)^
(1/2)+4/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*sec(d*
x+c))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{4\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{1}{3}\int \frac{\sqrt{3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{-2+3\cos(c+dx)}\,dx}{3\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{-2+3\cos(c+\right.}{}$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{4\sqrt{-2+3\cos(c+dx)}F\left(\frac{1}{2}\right.}{3d\sqrt{3-2\mathrm{se}}}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.64

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\left(4F\left(\frac{1}{2}(c+dx)\middle|6\right)+2E\left(\frac{1}{2}(c+dx)\middle|6\right)\right)}{3d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2-3\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 -
3*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.91, size = 381, normalized size = 3.37

$$\frac{2\left(3\sin(dx+c)\cos(dx+c)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{\sin(dx+c)}, \frac{i\sqrt{5}}{5}\right)\sqrt{5} - 5\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/15/d*(3*sin(d*x+c)*cos(d*x+c)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF(5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*I*5
^(1/2))*5^(1/2)-5*sin(d*x+c)*cos(d*x+c)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)
,1/5*I*5^(1/2))*5^(1/2)+3*5^(1/2)*EllipticF(5^(1/2)*(-1+cos(d*x+c))/sin(d*x
+c),1/5*I*5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)-5*5^(1/2)*EllipticE(5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)
,1/5*I*5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-15*cos(d*x+c)^2+25*cos(d*x+c)-10)*((-2+3*cos(d*x+c))/cos(
d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2 \sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=129

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] -4/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-3
+2*sec(d*x+c))^(1/2)+2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*El
lipticE(cos(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(-3+2*sec(d*x+c))^(1/2)/d/
(2-3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2654, 3858, 2662}

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}\,E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c + P
i + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]]*S
qrt[Sec[c + d*x]])
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{-3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$$
$$= \frac{\left(2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}\,dx}{3\sqrt{-3+2\sec(c+dx)}} - \frac{\sqrt{-3+2\,\text{se}}}{3\sqrt{2}}$$
$$= \frac{4\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}} - \frac{2\sqrt{5}\,E\left(\frac{1}{2}(c\right.}{3d\sqrt{2}}$$

Mathematica [A] time = 0.09, size = 72, normalized size = 0.56

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\left(4F\left(\frac{1}{2}(c+dx)\middle|6\right)+2E\left(\frac{1}{2}(c+dx)\middle|6\right)\right)}{3d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2-3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 - 3*
sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.78, size = 370, normalized size = 2.87

$$-\frac{2\left(3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\sqrt{5}\right)-i\sin(dx+c)\cos(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*I*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-
I*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+3*I*Ellipt
icF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((-2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))
/sin(d*x+c),I*5^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^2+5*cos(d*x+c)-2)*(-(-2+3*cos(d*x+c))
/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 \sec(dx+c) - 3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2\sec(c+dx) - 3}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1
/2*c),6^(1/2))*(-3-2*sec(d*x+c))^(1/2)/d/(-2-3*cos(d*x+c))^(1/2)/sec(d*x+c)
^(1/2)+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/
2*d*x+1/2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3862, 3856, 2654, 3858, 2662}

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
 - 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Ellipt
icF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{-3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right) - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$$

$$= -\frac{\sqrt{-3-2\sec(c+dx)}\int \sqrt{-2-3\cos(c+dx)}\,dx}{3\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{-2-3\cos\right.}{}$$

$$= -\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{4\sqrt{-2-3\cos(c+}}{3c}$$

Mathematica [A] time = 0.12, size = 81, normalized size = 0.70

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\right)}{3\sqrt{5}d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2+3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 +
3*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

maple [C] time = 1.88, size = 394, normalized size = 3.43

$$\left(-3i\cos(dx+c)\sin(dx+c)\sqrt{2}\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)+5i\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/15/d*(-3*I*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2
)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),1/5*I*5^(1/2))+5*I*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d
*x+c))/sin(d*x+c),1/5*I*5^(1/2))-3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,1/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),1/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+30*cos(d*x+c)^2-10*cos(d*x+c)-20)*(-(2+3
*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x
+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\sec(dx+c)-3}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2\sec(c+dx)-3}\,\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+3*s
ec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3858, 2661}

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{3+2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{2+3\sec(c+dx)}}\\ &= \frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{2+3\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.00

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

maple [C] time = 1.49, size = 142, normalized size = 2.33

$$-\frac{i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},\sqrt{5}\right)\sqrt{10}\,\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{3+2\cos(dx+c)}{\cos(dx+c)}}\,\cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{5d\left(2\left(\cos^{2}\left(dx+c\right)\right)+\cos\left(dx+c\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/
2)*(1/cos(d*x+c))^(1/2)*((3+2*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*sin(
d*x+c)^2*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)/(2*cos(d*x+c)^2+cos(d*x+c)-3)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) + 2), x)
```

3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3858, 2661}

$$\frac{2\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{-2+3\sec(c+dx)}}\\ &= \frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 1.00

$$\frac{2\sqrt{3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

maple [A] time = 1.58, size = 137, normalized size = 2.54

$$\frac{i\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{-\frac{-3+2\cos(dx+c)}{\cos(dx+c)}}\,\cos\left(dx+c\right)\left(\sin^{2}\left(dx+c\right)\right)\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{-\frac{2(}{}}}{d\left(2\left(\cos^{2}\left(dx+c\right)\right)-5\cos\left(dx+c\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
[Out] I/d*(1/cos(d*x+c))^(1/2)*(-(-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*s
in(d*x+c)^2*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(2*cos(d*x+c)
^2-5*cos(d*x+c)+3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) - 2), x)
```

3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{-3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}\\
&= \frac{\left(\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}\\
&= \frac{2\sqrt{3-2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c) - 2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

maple [A] time = 1.53, size = 144, normalized size = 2.67

$$-\frac{i\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\cos(dx+c)\left(\sin^2(dx+c)\right)\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{}}{5d\left(2\left(\cos^2(dx+c)\right)-5\cos(dx+c)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)*sin(d*x+c)^2*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(
d*x+c),1/5*5^(1/2))*(1/cos(d*x+c))^(1/2)*((-3+2*cos(d*x+c))/cos(d*x+c))^(1/
2)/(2*cos(d*x+c)^2-5*cos(d*x+c)+3)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{\left(c + dx \right)}}}{\sqrt{2 - 3\sec{\left(c + dx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2-3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2 - 3*sec(c + d*x)), x)
```

3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-2-3*
sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{-3-2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3-2\cos(c+dx)}}\,dx}{\sqrt{-2-3\sec(c+dx)}}\\
&= \frac{\left(\sqrt{3+2\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}}\,dx}{\sqrt{5}\,\sqrt{-2-3\sec(c+dx)}}\\
&= \frac{2\sqrt{3+2\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-2-3\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{2\sqrt{2\cos(c+dx)+3}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}\,d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-3\sec\left(dx+c\right)-2}\,\sqrt{\sec\left(dx+c\right)}}{3\sec\left(dx+c\right)+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c) + 2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-3\sec\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

maple [C] time = 1.65, size = 139, normalized size = 2.28

$$\frac{i\sqrt{2}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{-\frac{3+2\cos(dx+c)}{\cos(dx+c)}}\;\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, \frac{\sqrt{5}}{5}\right)\left(\sin^2\left(dx+c\right)\right)}{5d\left(2\left(\cos^2\left(dx+c\right)\right)+\cos\left(dx+c\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*(1/cos(d*x+c))^(1/2)*(-(3+2*cos(d*x+c))/cos(d*x+c))^(1/2)*El
lipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*sin(d*x+c)^2*cos(d*x+c)/(
2*cos(d*x+c)^2+cos(d*x+c)-3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-3\sec\left(dx+c\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{-3\sec{(c+dx)}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2-3*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-3*sec(c + d*x) - 2), x)
```

3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{3\cos(c+dx)+2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

```
[Out] 2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(3+2*
sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3858, 2661}

$$\frac{2\sqrt{3\cos(c+dx)+2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{2+3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{3+2\sec(c+dx)}}\\ &= \frac{2\sqrt{2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{3+2\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.00

$$\frac{2\sqrt{3\cos(c+dx)+2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

maple [C] time = 1.46, size = 145, normalized size = 2.38

$$\frac{\cos\left(dx+c\right)\operatorname{EllipticF}\left(\frac{\sqrt{5}\left(-1+\cos\left(dx+c\right)\right)}{5\sin\left(dx+c\right)}, i\sqrt{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\,\sqrt{2}\,\sqrt{\frac{1}{1+\cos\left(dx+c\right)}}\,\left(\sin^{2}\left(dx+c\right)\right)\sqrt{\frac{1}{\cos\left(dx+c\right)}}\,\sqrt{2\cdots}}{5d\left(3\left(\cos^{2}\left(dx+c\right)\right)-\cos\left(dx+c\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*cos(d*x+c)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2)
)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*((2+3*cos(d*x+c))/cos(d*x+c))^(1
/2)/(3*cos(d*x+c)^2-cos(d*x+c)-2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{2}{\cos\left(c+d\,x\right)}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) + 3), x)
```

3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*sec(d*x+c))^(1
/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3858, 2661}

$$\frac{2\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{-2+3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{3-2\sec(c+dx)}}\\ &= \frac{2\sqrt{-2+3\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.00

$$\frac{2\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\sec\left(dx+c\right)+3}\sqrt{\sec\left(dx+c\right)}}{2\sec\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c) - 3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\sec\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

maple [C] time = 1.56, size = 138, normalized size = 2.56

$$\frac{2\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\left(\sin^2\left(dx+c\right)\right)\cos\left(dx+c\right)\text{EllipticF}\left(\frac{\sqrt{5}\left(-1+\cos(dx+c\right.}{\sin(dx+c)}\right.}{5d\left(3\left(\cos^2\left(dx+c\right)\right)-5\cos\left(dx+c\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*(1/cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*((-2+3*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*cos(d*x
+c)*EllipticF(5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))/(3*cos(d*x+
c)^2-5*cos(d*x+c)+2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\sec\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec (c + dx)}}{\sqrt{3 - 2\sec (c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3-2*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3 - 2*sec(c + d*x)), x)
```

3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}$$

```
[Out] -2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+
1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-3+
2*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3858, 2662}

$$\frac{2\sqrt{2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}\,dx}{\sqrt{-3+2\sec(c+dx)}}\\ &= \frac{2\sqrt{2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}\,d\sqrt{-3+2\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.87

$$\frac{2\sqrt{3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-3 + 2*Sec[c + d*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

maple [C] time = 1.61, size = 136, normalized size = 2.19

$$\frac{2i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\sqrt{5}\right)\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{-\frac{-2+3\cos(dx+c)}{\cos(dx+c)}}}{d\left(3\left(\cos^2\left(dx+c\right)\right)-5\cos\left(dx+c\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 2*I/d*(1/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*cos(d*x+c)*sin(d*x+c)^2*(1/c
os(d*x+c))^(1/2)*(-(-2+3*cos(d*x+c))/cos(d*x+c))^(1/2)/(3*cos(d*x+c)^2-5*co
s(d*x+c)+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) - 3), x)
```

3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2\sqrt{-3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

```
[Out] -2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/
2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*sec(d*x+c))^
(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3858, 2662}

$$\frac{2\sqrt{-3\cos(c+dx)-2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{-2-3\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2-3\cos(c+dx)}}\,dx}{\sqrt{-3-2\sec(c+dx)}}\\ &= \frac{2\sqrt{-2-3\cos(c+dx)}\,F\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.11

$$\frac{2\sqrt{3\cos(c+dx)+2}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)}{\sqrt{5}\,d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,\sec\left(dx+c\right)-3}\,\sqrt{\sec\left(dx+c\right)}}{2\,\sec\left(dx+c\right)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c) + 3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\,\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

maple [C] time = 1.68, size = 142, normalized size = 2.58

$$\frac{i\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{-\frac{2+3\cos(dx+c)}{\cos(dx+c)}}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\sqrt{2}\,\sqrt{\frac{1}{1+c}}}{5d\left(3\left(\cos^2\left(dx+c\right)\right)-\cos\left(dx+c\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*(1/cos(d*x+c))^(1/2)*(-(2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*((2+3*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)^2*EllipticF(I*(-1+cos(
d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*10^(1/2)
/(3*cos(d*x+c)^2-cos(d*x+c)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\,\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3-2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-2*sec(c + d*x) - 3), x)
```

3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/(1+sec
(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = -\frac{\tan(c+dx)\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \sec(c+dx\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\mathrm{ta}}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] time = 26.86, size = 7160, normalized size = 68.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+b\sec\left(c+dx\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 2.09, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

maple [A] time = 0.96, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3), x)
```

3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=362

$$\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3\left(9a^2+32b^2\right)}{}$$

[Out] 3/220*(9*a^2+32*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d-9/44*a*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d+3/11*sec(d*x+c)*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b/d+1/220*a*(18*a^2+49*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^3/d/((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/110*(9*a^4+23*a^2*b^2-32*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)

Rubi [A] time = 0.67, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3865, 4082, 4002, 4007, 3834, 139, 138}

$$\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\left(23a^2b^2+9a^4\right.}{}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]

[Out] (3*(9*a^2 + 32*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*b^2*d) - (9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(44*b^2*d) + (3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + (a*(18*a^2 + 49*b^2)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(110*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - ((9*a^4 + 23*a^2*b^2 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(55*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)(a+b}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx)}{11bd}$$

$$= \frac{3\left(9a^2+32b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx)}{44b^2}$$

$$= \frac{3\left(9a^2+32b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx)}{44b^2}$$

$$= \frac{3\left(9a^2+32b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx)}{44b^2}$$

$$= \frac{3\left(9a^2+32b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx)}{44b^2}$$

$$= \frac{3\left(9a^2+32b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx)}{44b^2}$$

Mathematica [B] time = 26.95, size = 21877, normalized size = 60.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**4, x)
```

3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=305

$$-\frac{\left(6a^2-25b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a\left(a^2-b^2\right)\tan}{}$$

```
[Out] -9/40*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d+3/8*(a+b*sec(d*x+c))^(5/3)*ta
n(d*x+c)/b/d-1/40*(6*a^2-25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))
/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b*se
c(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+3/20*a*(a^2-b^2)*Appell
F1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x
+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*
x+c))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3840, 4002, 4007, 3834, 139, 138}

$$-\frac{\left(6a^2-25b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a\left(a^2-b^2\right)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*b*d) + (3*(a + b*Sec[c +
 d*x])^(5/3)*Tan[c + d*x])/(8*b*d) - ((6*a^2 - 25*b^2)*AppellF1[1/2, 1/2, -
2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[
c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3)) + (3*a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3
, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c
+ d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x
]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + \frac{3\int \sec(c+dx)\left(\frac{5b}{3} - a\sec(c+ \right.}{8}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(}{8bd}$$

Mathematica [B] time = 26.77, size = 18991, normalized size = 62.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**3, x)
```

3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt{2}\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{2\sqrt{2}\,a\tan(c+dx)(a+b}{}$$

```
[Out] 3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+2/5*a*AppellF1(1/2,-2/3,1/2,3/2,b*(
1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(
d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)-2/5*(a^2-b^2
)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b
*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3)/(1+
sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3835, 4007, 3834, 139, 138}

$$-\frac{2\sqrt{2}\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{2\sqrt{2}\,a\tan(c+dx)(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*Sqrt[2]*a*AppellF1[1
/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(
a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*((a +
 b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1/2
, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*S
ec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*(a
+ b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
```

```
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{2}{5}\int\frac{\sec(c+dx)(b+a\sec(c+}{\sqrt[3]{a+b\sec(c+dx)}}$$

$$=\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{(2a)\int\sec(c+dx)(a+b\sec(c}{5b}$$

$$=\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}-\frac{(2a\tan(c+dx))\,\mathrm{Subst}\left(\int\frac{(a+}{\sqrt{1-}}\right.}{5bd\sqrt{1-\sec(c+dx)}\,\sqrt{}}$$

$$=\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}-\frac{\left(2a(a+b\sec(c+dx))^{2/3}\tan(c\right.}{5bd\sqrt{1-\sec(c+d}}$$

$$=\frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{2\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec\right.}{5bd\sqrt{}}$$

Mathematica [B] time = 18.90, size = 2505, normalized size = 9.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(2/3)*((3*a*Sin[c + d*x])/(5*b) + (3*Tan[c + d*x])/5)
)/d - ((-2*b + 3*a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(2/3)*(3*a*(b + a*Cos
[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(2/3) - (3*(b + a*Co
s[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)
])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 -
 b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-
2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2,
1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x
])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(1/3))))/(5*b*d*((3*a*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*
```

```
x])/(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a^2*Sqrt[1 - Cos[c +
 d*x]^2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*a*
(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[
c + d*x] - (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^(5/3)*Sq
rt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*
AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8
/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a
+ 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-
2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a
*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^
(5/3)*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2
 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^
(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2
, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d
*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*S
qrt[b^(-2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x]
)/(1 + a*Sqrt[b^(-2)])]) + (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(
-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c +
 d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -
((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/S
qrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a
 + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[
c + d*x]))*Sin[c + d*x])/(5*b*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(4/3)
) + (2*a*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[
(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*App
ellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a
 + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3,
 -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1
/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x
])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) + ((b + a*Cos[c + d*x
])^(2/3)*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*S
qrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*
(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1
/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[
5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*S
ec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b
*Sqrt[1 - Cos[c + d*x]^2]) - (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c
 + d*x])/(1 - a*Sqrt[b^(-2)])]*(2*a*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b
*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(
-2)])]*Sec[c + d*x]*Tan[c + d*x] - 5*(a^2 - b^2)*((b*AppellF1[5/3, 1/2, 3/2
, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/
(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) -
 (b*AppellF1[5/3, 3/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)
])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/
(5*(-a + 1/Sqrt[b^(-2)]))) + 2*a*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1
/2, 3/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c
+ d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^
(-2)])) - (5*b*AppellF1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1
/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Ta
n[c + d*x])/(16*(-a + 1/Sqrt[b^(-2)])))))/(5*b*Sqrt[1 - Cos[c + d*x]^2]*Sec
[c + d*x]^(1/3))))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+b\sec(dx+c))^{\frac{2}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**2, x)
```

3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec
(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = -\frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \mathrm{se}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 26.40, size = 7142, normalized size = 68.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

```
[Out] int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec\left(c+dx\right))^{\frac{2}{3}}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x), x)
```

3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left((a+b\sec(c+dx))^{2/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (a+b\sec(c+dx))^{2/3}\,dx$$

Mathematica [A] time = 2.06, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

maple [A] time = 0.82, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(2/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3), x)
```

3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/
(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\left((-a-b)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \text{s}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c}}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] time = 27.30, size = 8660, normalized size = 80.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
[Out] Result too large to show
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^2 + a\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

```
[Out] int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{4}{3}}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

3.693 $\int (a+b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left((a+b\sec(c+dx))^{4/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(4/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(4/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (a+b\sec(c+dx))^{4/3}\,dx$$

Mathematica [A] time = 20.45, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(4/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(4/3),x)
[Out] int((a+b*sec(d*x+c))^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(4/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(4/3),x)
[Out] Integral((a + b*sec(c + d*x))**(4/3), x)
```

3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=412

$$\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{1232b^2d}+\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{1232b^2d}+\frac{(36a^4+1}{}$$

```
[Out] 3/1232*a*(18*a^2+97*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/1232*(18
*a^2+121*b^2)*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d-9/77*a*(a+b*sec(d*x+c
))^(8/3)*tan(d*x+c)/b^2/d+3/14*sec(d*x+c)*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)
/b/d+1/1232*(36*a^4+164*a^2*b^2+605*b^4)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec
(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^3/d/
((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/616*a*(18*a^4
+79*a^2*b^2-97*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2
*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c
))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3865, 4082, 4002, 4007, 3834, 139, 138}

$$\frac{\left(164a^2b^2+36a^4+605b^4\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{616\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\,(7}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(18*a^2 + 97*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(1232*b^2*d
) + (3*(18*a^2 + 121*b^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(1232*b^
2*d) - (9*a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(77*b^2*d) + (3*Sec[c
+ d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((36*a^4 + 164*a
^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(616*Sq
rt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) -
(a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c +
 d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1
/3)*Tan[c + d*x])/(308*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{14bd} + \frac{3\int \sec(c+dx)(a+b}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{77b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx)}{14bd}$$

$$= \frac{3\left(18a^2+121b^2\right)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d} - \frac{9a(a+b\sec(c+}{77}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)}{}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)}{}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)}{}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)}{}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)}{}$$

Mathematica [B] time = 27.41, size = 28057, normalized size = 68.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^5 + a\sec(dx+c)^4\right)(b\sec(dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*(b*sec(d*x + c) + a)^(2/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \left(\sec^{4}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4,x)
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/3),x)
[Out] Timed out
```

3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$-\frac{a\left(30a^2-373b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{220\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{3\left(15a^2-64\right.}{}$$

```
[Out] -3/440*(15*a^2-64*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-9/88*a*(a+b*se
c(d*x+c))^(5/3)*tan(d*x+c)/b/d+3/11*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)/b/d-1
/440*a*(30*a^2-373*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/
2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b*sec(d*x+c))
/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/220*(15*a^4-79*a^2*b^2+64*b^4)
*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1
+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3840, 4002, 4007, 3834, 139, 138}

$$-\frac{a\left(30a^2-373b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{220\sqrt{2}\,b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{\left(-79a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*(15*a^2 - 64*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*b*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(88*b*d) + (3*(a + b*Sec[c +
d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - (a*(30*a^2 - 373*b^2)*AppellF1[1/2, 1/
2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]
*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + ((15*a^4 - 79*a^2*b^2 + 64*b^4)*Ap
pellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
+ b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(110*Sqrt[2]*b^2*d
*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)\left(\frac{8b}{3} - a\sec(c+dx\right.}{11b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd} + \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+}{11bd}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+}{88}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+}{88}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+}{88}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+}{88}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+}{88}$$

Mathematica [B] time = 27.34, size = 21890, normalized size = 61.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^4 + a\sec(dx+c)^3\right)(b\sec(dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(b*sec(d*x + c) + a)^(2/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/3),x)
[Out] Timed out
```

3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=299

$$\frac{\left(2a^2+5b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{4\sqrt{2}\,bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(a^2-b^2\right)\tan(c+$$

```
[Out] 3/8*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/8*(a+b*sec(d*x+c))^(5/3)*tan(d*
x+c)/d+1/8*(2*a^2+5*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1
/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d/((a+b*sec(d*x+c))/
(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/4*a*(a^2-b^2)*AppellF1(1/2,1/3,
1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))
^(1/3)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3835, 4002, 4007, 3834, 139, 138}

$$\frac{\left(2a^2+5b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{4\sqrt{2}\,bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(a^2-b^2\right)\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (3*(a + b*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) + ((2*a^2 + 5*b^2)*AppellF1[1/2, 1/2, -2/3, 3/
2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(4*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
+ d*x])/(a + b))^(2/3)) - (a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 -
Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a +
 b))^(1/3)*Tan[c + d*x])/(2*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c
 + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
```

```
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{5}{8}\int \sec(c+dx)(b+a\sec(c+$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c$$

Mathematica [B] time = 27.28, size = 19016, normalized size = 63.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(b*sec(d*x + c) + a)^(2/3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int\left(\sec^{2}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}}{{\cos\left(c+d\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (a+b)*AppellF1(1/2,-5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/
(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\,(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\left((-a-b)(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}dx, x\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 27.77, size = 8668, normalized size = 80.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^2 + a\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \sec(dx+c)(a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{3}}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x), x)
```

3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sec(c+dx))^{5/3},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{5/3}\,dx$$

Mathematica [A] time = 26.49, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

maple [A] time = 0.73, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(5/3), x)
```

3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=313

$$-\frac{a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{\left(18a^2+25b^2\right)\tan(c+d\ldots}{}$$

```
[Out] -9/20*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/8*sec(d*x+c)*(a+b*sec(d*x
+c))^(2/3)*tan(d*x+c)/b/d+1/40*(18*a^2+25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*
(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/
b^3/d/((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/20*a*(9
*a^2+11*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*
x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c))^(1/3
)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3865, 4082, 4007, 3834, 139, 138}

$$-\frac{a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}\,b^3d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}+\frac{\left(18a^2+25b^2\right)\tan(c+d\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*b^2*d) + (3*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((18*a^2 + 25*b^2)*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^3*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (a*(9*a^2 + 11*b^2)*App
ellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^3*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) && !IGtQ[m, 2]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{3\int \frac{\sec(c+dx)\left(a+\frac{5}{3}b\sec(c+dx)-2a\sec^2(c\right.}{\sqrt[3]{a+b\sec(c+dx)}}}{8b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c-}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c-}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c-}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c-}{8bd}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c-}{8bd}$$

Mathematica [B] time = 26.60, size = 19015, normalized size = 60.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
[Out] Result too large to show
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(dx+c\right)}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{4}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(1/3), x)
```

3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=265

$$\frac{\sqrt{2}\left(3a^2+2b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}\,a\tan(c+dx)(a+b}{}$$

```
[Out] 3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-3/5*a*AppellF1(1/2,-2/3,1/2,3/2,b
*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*ta
n(d*x+c)/b^2/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)+1/5*(3*a
^2+2*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c
))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))
^(1/3)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4007, 3834, 139, 138}

$$\frac{\sqrt{2}\left(3a^2+2b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}\,a\tan(c+dx)(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*AppellF1
[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[1/
2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
```

```
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2b}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{5b}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{1}{5}\left(2+\frac{3a^2}{b^2}\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx - \frac{(3a}{\ldots}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(\left(-2-\frac{3a^2}{b^2}\right)\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+\ldots}}\right.}{5d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\mathrm{se}\ldots}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\mathrm{Subst}\ldots}{5b^2d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(\ldots}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} - \frac{3\sqrt{2}\,aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\ldots}{\ldots}\right.}{5b^2d\sqrt{1+\sec(c+d\ldots}}$$

Mathematica [B] time = 26.78, size = 7195, normalized size = 27.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)

[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)),x)

[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/3),x)

[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(1/3), x)

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}\,a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a}}}{bd\sqrt{s}}$$

```
[Out] AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+s
ec(d*x+c))^(1/2)-a*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*
sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*sec(
d*x+c))^(1/3)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3838, 3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}\,a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a}}}{bd\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
+ b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
```

```
x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3838

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Dist[a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b}$$

$$= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}} + \frac{(a\tan(c+dx))\,\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{bd\sqrt{1-\sec(c+d}}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}} + \frac{(a}{}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 19.19, size = 2759, normalized size = 12.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(b + a*Cos[c + d*x])*Tan[c + d*x])/(2*b*d*(a + b*Sec[c + d*x])^(1/3)) -
((b + 3*a*Cos[c + d*x])*(3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]
^2]*Sec[c + d*x]^(2/3) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*S
ec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(
1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*
x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*
AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*(b
+ a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3))))/(2*b
*d*(a + b*Sec[c + d*x])^(1/3)*((3*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/
(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*(b + a*C
os[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[c + d*x]
 - (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2,
5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a
 + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])
/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*
Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-2)])*(b + a*Cos[c + d*x])^
(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 -
a*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])
```

```
*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF
1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b
*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a
+ b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[
b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*Sqrt[b^(-2)])*(b +
 a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Se
c[c + d*x])/(1 + a*Sqrt[b^(-2)])]) - (3*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c
 + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*
Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-
2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[
b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))
*Sin[c + d*x])/(5*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]) + (3
*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^
(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*
AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3
, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a +
1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*
x])^(1/3)*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(7/3)) - (a*(a + b*Sec[c
+ d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(
1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3,
1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c
+ d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec
[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)]
)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(4/3)*Sqrt
[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3)) + (4*(a + b*Sec[c + d*x])*Sqrt[(1
- Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]
*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -
((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/S
qrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a +
 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c
+ d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x
]^2]*Sec[c + d*x]^(1/3)) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b
*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])
/(1 - a*Sqrt[b^(-2)])]*(2*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d
*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec
[c + d*x]*Tan[c + d*x] - 5*a*((b*AppellF1[5/3, 1/2, 3/2, 8/3, -((a + b*Sec[
c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])
]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) - (b*AppellF1[5/3, 3/
2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c +
d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(-a + 1/Sqrt[b^(-
2)]))) + 2*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1/2, 3/2, 11/3, -((a +
b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^
(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^(-2)])) - (5*b*AppellF
1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a +
b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(-a +
1/Sqrt[b^(-2)])))))/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2
]*Sec[c + d*x]^(4/3))))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{2}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(1/3), x)
```

3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(1/3)/(1+sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt[3]{a+bx}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1-\sec(c+dx)), \frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}$$

Mathematica [B] time = 2.07, size = 310, normalized size = 2.95

$$\frac{15(a-b)^2(a+b)\cos(c+dx)\cot^3(c+dx)(\sec(c+dx)+1)(b-b\sec(c+dx)}{b^2d(b-a)\left(3(a-b)(a\cos(c+dx)+b)F_1\left(\frac{5}{3}; \frac{1}{2}, \frac{3}{2}; \frac{8}{3}; \frac{a+b\sec(c+dx)}{a-b}, \frac{a+b\sec(c+dx)}{a+b}\right)+(a+b)\left(10(a-b)\cos(c+dx)F_1\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (15*(a - b)^2*(a + b)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(2/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[5/3, 1/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(10*(a - b)*AppellF1[
2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
 b)]*Cos[c + d*x] + 3*AppellF1[5/3, 3/2, 1/2, 8/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-1/3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-1/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

maple [A] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(1/3),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-1/3), x)
```

3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

```
[Out] AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(2/3)/(1+sec(
d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,(a+bx)^{2/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 1.97, size = 310, normalized size = 2.95

$$\frac{24(a-b)^2(a+b)\cos(c+dx)\cot^3(c+dx)(\sec(c+dx)+1)(b-b\sec(c+}{b^2d(b-a)\left(3(a-b)(a\cos(c+dx)+b)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)+(a+b)\left(8(a-b)\cos(c+dx)F_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (24*(a - b)^2*(a + b)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(1/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[4/3, 1/2, 3/2, 7/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(8*(a - b)*AppellF1[1
/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
b)]*Cos[c + d*x] + 3*AppellF1[4/3, 3/2, 1/2, 7/3, (a + b*Sec[c + d*x])/(a -
 b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)),x)
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-2/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Mathematica [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-2/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

maple [A] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(2/3),x)
[Out] int(1/(a+b*sec(d*x+c))^(2/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(2/3),x)
[Out] int(1/(a + b/cos(c + d*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(-2/3), x)
```

3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] AppellF1(1/2,4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(1/3)/(
1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\,\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = -\frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,(a+bx)^{4/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\,\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}\,dx, x, \sec(c+dx)\right)}{(a+b)d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}$$

Mathematica [B] time = 27.29, size = 10343, normalized size = 94.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
[Out] Result too large to show
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(4/3), x)
```

3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{4/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(4/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-4/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Mathematica [A] time = 27.36, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-4/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(4/3),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(4/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-4/3), x)
```

3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=378

$$-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{3a^2\tan(c+dx}{2bd\left(a^2-b^2\right)(a+}$$

```
[Out] -3/2*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+3/4*(3*
a^2-b^2)*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d-1/4*a*(9*a^2-7*b
^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a
+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1
/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*(9*a^4-10*a^2*b^2-b^4)*AppellF1(1/2,2/
3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b
))^(2/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d
*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4082, 4007, 3834, 139, 138}

$$-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{\left(-10a^2b^2+9a^4-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(2/3)) + (3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b^2*(
a^2 - b^2)*d) - (a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[
c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^(1/3)) + ((9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2
/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[
c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1
+ Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(a^2-\frac{2}{3}ab\sec(c+dx)-\frac{2}{3}\left(3a^2-b^2\right)\sec^2(c+dx\right.}{(a+b\sec(c+dx))^{2/3}}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx}{4b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx}{4b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx}{4b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx}{4b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx}{4b^2\left(a^2-b^2\right)d}$$

Mathematica [B] time = 26.83, size = 21987, normalized size = 58.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^4}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/3),x)
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/3), x)
```

3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=307

$$-\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}+\frac{\left(3a^2-2b^2\right)\tan(c+d\ldots}{\ldots}$$

```
[Out] -3/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+1/2*(3*a^2-2*b^2)*
AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*s
ec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)*
2^(1/2)/(1+sec(d*x+c))^(1/2)-1/2*a*(3*a^2-4*b^2)*AppellF1(1/2,2/3,1/2,3/2,b
*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*ta
n(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2
)
```

Rubi [A] time = 0.38, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3839, 4007, 3834, 139, 138}

$$-\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}+\frac{\left(3a^2-2b^2\right)\tan(c+d\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((3*
a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Se
c[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b^2
*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
 - (a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
d*x])/(Sqrt[2]*b^2*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(-\frac{2ab}{3}-\frac{1}{3}\left(3a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{\left(a\left(3a^2-4b^2\right)\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b^2\left(a^2-b^2\right)} + \frac{(3}{}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(a\left(3a^2-4b^2\right)\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{2b^2\left(a^2-b^2\right)d\sqrt{1-\sec(c+}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{\left(\left(3a^2-2b^2\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+\right.}{2b^2\left(a^2-b^2\right)d\sqrt{1-\sec(c+d}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(3a^2-2b^2\right)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+\right.}{\sqrt{2}\,b^2\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [B] time = 26.70, size = 19126, normalized size = 62.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\sec\left(dx+c\right)^{3}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (dx + c)^{3}}{(b \sec (dx + c) + a)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}(dx + c)}{(a + b \sec (dx + c))^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (c + d\,x)^{3} \left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}(c + dx)}{(a + b \sec (c + dx))^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/3), x)
```

3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=289

$$\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec}}{\sqrt{2}\,bd}$$

```
[Out] 3/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)-1/2*a*AppellF1(1/2,-1/3
,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(1/3)*
tan(d*x+c)/b/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+sec(d*x+
c))^(1/2)+1/2*(a^2-2*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1
/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*tan(d*x+c)/b/(a^2-b^2)/d/
(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3836, 4007, 3834, 139, 138}

$$\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}\,bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec}}{\sqrt{2}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - (a*Appell
F1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b^2)*d*Sqrt[1
 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + ((a^2 - 2*b^2)*App
ellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b
^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
```

```
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{2b}{3}-\frac{1}{3}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{a\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(a^2-\right.}{}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{(a\tan(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(\right.}{2b\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c-}}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(a\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \right.}{2b\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec}}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx}{a+b}\right.}{\sqrt{2}\,b\left(a^2-b^2\right)d\sqrt{1+\sec(c+}}$$

Mathematica [B] time = 27.00, size = 7325, normalized size = 25.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^2}{b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/3), x)
```

3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

```
[Out] AppellF1(1/2,5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*s
ec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(2/3)/(
1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\,(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,(a+bx)^{5/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}\,dx, x, \sec(c+\right.}{(a+b)d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 27.28, size = 10363, normalized size = 94.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
[Out] Result too large to show
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/3), x)
```

3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{5/3}},x\right)$$

```
[Out] Unintegrable(1/(a+b*sec(d*x+c))^(5/3),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-5/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Mathematica [A] time = 23.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(-5/3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(c + d*x))^(5/3),x)
```

```
[Out] int(1/(a + b/cos(c + d*x))^(5/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/3), x)
```

3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};s\right.}{d\left(a^2-b^2\right)}$$

```
[Out] a*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x+
c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)-b*AppellF1(1/2,1/3,1,3
/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x+c)
^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.26, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};s\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3
)) - (b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\sqrt[3]{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c\cdots\right.$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} + \frac{\left(b\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int\cdots\right.}{d}$$

$$= \frac{aF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} - \frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2 s\cdots}{\cdots}\right.}{\cdots}$$

Mathematica [B] time = 21.74, size = 4543, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(5/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -T
an[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Appe
llF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/
(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d*
(Sec[c + d*x]^2)^(2/3)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((9
*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d
*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2
/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 -
b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Se
c[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*Tan[c + d*x]^2*(
(b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan
[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[
```

c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b ^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (12*(a^2 - b^2)*Tan [c + d*x]^2*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d* x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF 1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (- a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2) /(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*Appel lF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(2/3)*(-a^2 + b^2*Se c[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2]*Ta n[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2 *Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2 , 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2* AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2, 2/3, 2, 5/ 2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b ^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) - (b*AppellF1[1 /2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Se c[c + d*x]^2]*(2*(6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Ta n[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] + 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan [c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (A ppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*b^2*((12*b^2*AppellF1 [5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/ 5) + (-a^2 + b^2)*((6*b^2*AppellF1[5/2, 7/6, 2, 7/2, -Tan[c + d*x]^2, (b^2* Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*AppellF1[1/2, 1/6 , 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*Appel lF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^ 2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2 - (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*(-4*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*A ppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 2/3, 2

```
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*(a^2 - b^2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan
[c + d*x]^2*(3*b^2*((12*b^2*AppellF1[5/2, 2/3, 3, 7/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2))
 - (4*AppellF1[5/2, 5/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5) + 2*(-a^2 + b^2)*((6*b^2*AppellF1[
5/2, 5/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c
+ d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - 2*AppellF1[5/2, 8/3, 1, 7/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]))
))/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1,
5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2)
)/((Sec[c + d*x]^2)^(2/3)*(-a^2 + b^2*Sec[c + d*x]^2))))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}\left(dx+c\right)}{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)),x)
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c + dx)}{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x)), x)
```

3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin\right.}{d\left(a^2-b^2\right)}$$

```
[Out] a*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x+
c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)-b*AppellF1(1/2,1/6,1,3
/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x+c)
^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.26, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3
)) - (b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\mathrm{se}\right.$$

$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\mathrm{Subst}}{}$$

$$= \frac{aF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} - \frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\right.}{}$$

Mathematica [B] time = 21.54, size = 4544, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(4/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2,
-Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Ap
pellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2,
(b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d
*(Sec[c + d*x]^2)^(5/6)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((
9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
 b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2
, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2
 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a
^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*
Sec[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*Tan[c + d*x]^2
*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/
3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2
)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2
*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -
```

Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5
/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b
^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (15*(a^2 - b^2)*
Tan[c + d*x]^2*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/
3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*Ap
pellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
+ (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2,
5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*A
ppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)
] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(5/6)*(-a^2 + b^
2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/3, 1,
3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
(b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2
, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c +
d*x]^2) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(
3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*Appe
llF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2,
5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x
]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))
/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d
*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) - (b*
AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2
)]*Sqrt[Sec[c + d*x]^2]*(4*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^
2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[
c + d*x] + 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 -
b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)
/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*b^2*((1
2*b^2*AppellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/
3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^
2*Tan[c + d*x])/5) + (-a^2 + b^2)*((6*b^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(
5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*Ap
pellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
+ 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2
)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2 - (a*AppellF1[1/2, 5/6,
1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*(2*(-6*b^2*Appe
llF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((2*b^2*App

```
ellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*
Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c
+ d*x])/9) + Tan[c + d*x]^2*(-6*b^2*((12*b^2*AppellF1[5/2, 5/6, 3, 7/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]
)/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]) + 5*(a^2 - b^2)*((6*b^
2*AppellF1[5/2, 11/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (11*AppellF1[5/2, 17/6
, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2
*Tan[c + d*x])/5))))/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*Appell
F1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*T
an[c + d*x]^2)^2))/((Sec[c + d*x]^2)^(5/6)*(-a^2 + b^2*Sec[c + d*x]^2))))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

maple [**F**] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x)), x)
```

3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c}}$$

```
[Out] -b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)+a*AppellF1(1/2,-2/3,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x+
c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1
/3))) + (a*AppellF1[1/2, -2/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx = \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{7}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\cos^{\frac{2}{3}}(\right.$$

$$= \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} - \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c+dx)}} + \frac{aF_1\left(\frac{1}{2};-\frac{2}{3},1;\right.}{}$$

Mathematica [B] time = 29.33, size = 7542, normalized size = 43.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\,(a+b\sec(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+}}$$

```
[Out] -b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)+a*AppellF1(1/2,-5/6,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x+
c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\mathrm{si}\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
 - b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2
/3))) + (a*AppellF1[1/2, -5/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
```

```
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\,\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{8}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\sqrt[3]{c\right.$$

$$= \frac{b\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} - \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\,\sqrt[3]{\sec(c\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)} + \frac{aF_1\left(\frac{1}{2};-\frac{5}{6},\right.}{}$$

Mathematica [B] time = 29.26, size = 7588, normalized size = 43.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}\,(a+b\sec(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 37.73, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{7}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

maple [A] time = 1.66, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 49.59, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 2.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

maple [A] time = 1.70, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 32.12, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

maple [A] time = 1.65, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 39.85, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 3.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

maple [A] time = 1.48, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3), x)
```

3.721 $\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 2.67, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 1.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

maple [A] time = 1.99, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a + \frac{b}{\cos (c + d\,x)}} \left(\frac{1}{\cos (c + d\,x)}\right)^{1/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec (c + dx)} \sqrt[3]{\sec (c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3), x)
```

3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 9.39, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

fricas [A] time = 4.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec (dx + c)}}{\sec (dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec (dx + c) + a}}{\sec (dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec (c + dx)}}{\sqrt[3]{\sec (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/3),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(1/3), x)
```

3.723 $\int \frac{\sqrt{a+b \sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)} \, dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 18.33, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

fricas [A] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

maple [A] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec (dx + c)}}{\sec (dx + c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec (dx + c) + a}}{\sec (dx + c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec (c + dx)}}{\sec^{\frac{2}{3}} (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(2/3),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(2/3), x)
```

3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 24.43, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

fricas [A] time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

maple [A] time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sec^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(4/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(4/3), x)
```

3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 35.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

fricas [A] time = 1.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

maple [A] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sec^{\frac{5}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/3), x)
```

3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 41.61, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

fricas [A] time = 1.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)

maple [**A**] time = 1.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec\left(dx+c\right)}}{\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)

[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3),x)

[Out] int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3), x)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/3),x)

[Out] Timed out

3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 39.33, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sec
(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 41.27, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^2+a\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(2/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

maple [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 36.06, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec(dx+c)^2+a\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

maple [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]

[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 42.13, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]

[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]

fricas [A] time = 2.15, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.731 $\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx = \int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 31.84, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 42.65, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

fricas [A] time = 2.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/3),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(1/3), x)
```

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 23.90, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

fricas [A] time = 2.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(2/3),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(2/3), x)
```

3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 28.98, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

fricas [A] time = 3.21, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^{\frac{3}{2}}}{\sec^{\frac{4}{3}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(4/3),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(4/3), x)
```

3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 31.47, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

fricas [A] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)

[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3),x)

[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/3),x)

[Out] Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/3), x)

3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 39.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

fricas [A] time = 2.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/3),x)
[Out] Timed out
```

3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 43.77, size = 0, normalized size = 0.00

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^4 + 2ab\sec(dx+c)^3 + a^2\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

maple [A] time = 1.57, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{7}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 48.44, size = 0, normalized size = 0.00

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

maple [A] time = 1.49, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]

[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 42.23, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]

[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr t(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 49.10, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 2.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(2/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

maple [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.741 $\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx = \int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 36.40, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(1/3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

maple [A] time = 1.32, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(dx+c)\right)(a+b\sec(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a + \frac{b}{\cos (c + d\, x)}\right)^{5/2} \left(\frac{1}{\cos (c + d\, x)}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 50.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

fricas [A] time = 2.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(1/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/3),x)
[Out] Timed out
```

3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 37.71, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

fricas [A] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(2/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

[Out] Timed out

maple [**A**] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)

[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3),x)

[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3), x)

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(2/3),x)

[Out] Timed out

3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 43.84, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

fricas [A] time = 2.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(4/3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(d*x+c))]Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
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 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Evaluation time: 2.78int()
  Error: Bad Argument Value
```

maple [A] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(4/3),x)
[Out] Timed out
```

3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 36.20, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

fricas [A] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(5/3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

[Out] Timed out

maple [**A**] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/3),x)
[Out] Timed out
```

3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)} dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)} dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)} dx$$

Mathematica [A] time = 42.94, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

fricas [A] time = 2.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(7/3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(cos
(d*x+c))]Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
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```

maple [A] time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Timed out
```

3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 30.28, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{7}{3}}}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} \, dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} \, dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]

[Out] Defer[Int][Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} \, dx=\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} \, dx$$

Mathematica [A] time = 36.10, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]

[Out] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]

[Out] Defer[Int][Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]

[Out] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{4}{3}}}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] Timed out
```

maple [A] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**(4/3)/sqrt(a + b*sec(c + d*x)), x)
```

3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 2.23, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 2.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx + c)}{\sqrt{a + b\sec(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)

[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx + c)^{\frac{2}{3}}}{\sqrt{b\sec(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c + dx)}{\sqrt{a + b\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**(2/3)/sqrt(a + b*sec(c + d*x)), x)

3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 1.06, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/sqrt(a + b*sec(c + d*x)), x)
```

3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 3.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

fricas [A] time = 3.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 26.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

fricas [A] time = 2.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 31.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

fricas [A] time = 2.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)^{3}+a\sec(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{4}{3}} \sqrt{a+b\sec\left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\,\sec^{\frac{4}{3}}\left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(4/3)), x)
```

3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 34.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

fricas [A] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)^3+a\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec (dx + c)^{\frac{5}{3}} \sqrt{a + b \sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec (dx + c) + a}\, \sec (dx + c)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos(c+d\,x)}}\, {\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec (c + dx)}\, \sec^{\frac{5}{3}} (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/3)), x)
```

3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 41.79, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

fricas [A] time = 2.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)^4+a\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^4 + a*
sec(d*x + c)^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(dx+c\right)^{\frac{7}{3}} \sqrt{a+b\sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec \left(dx+c\right)+a}\, \sec \left(dx+c\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 34.97, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{7}{3}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 42.04, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 36.22, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{4}{3}}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**(4/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 45.11, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 2.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 37.50, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{1}{3}}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 44.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

fricas [A] time = 2.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(1/3)), x)
```

3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 42.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

fricas [A] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(dx+c\right)^{\frac{2}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}} \sec \left(dx+c\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec \left(c+dx\right)\right)^{\frac{3}{2}} \sec^{\frac{2}{3}} \left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(2/3)), x)
```

3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 49.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

fricas [A] time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec (dx+c)^{\frac{4}{3}} (a+b\sec (dx+c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec (c+dx))^{\frac{3}{2}} \sec^{\frac{4}{3}} (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(4/3)), x)
```

3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 18.83, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

fricas [A] time = 1.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec (dx+c)^{\frac{5}{3}} (a+b\sec (dx+c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2} \left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 51.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

fricas [A] time = 2.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{2}{3}}}{b^2\sec\left(dx+c\right)^5+2\,ab\sec\left(dx+c\right)^4+a^2\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^5 +
2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}}\,(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 41.31, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 1.12, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{7}{3}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{7}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 49.26, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{5}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 40.69, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{4}{3}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{4}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)

[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2),x)

[Out] int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out

3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

[Out] Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]

[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 46.50, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]

[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]

fricas [A] time = 4.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(5/2), x)
```

3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 44.45, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

fricas [A] time = 3.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{1}{3}}(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(5/2), x)
```

3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 51.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

fricas [A] time = 2.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^3\sec(dx+c)^4+3\,ab^2\sec(dx+c)^3+3\,a^2b\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}} (a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2} \left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}} \sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**(1/3)), x)
```

3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 49.97, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

fricas [A] time = 2.89, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b^3\sec(dx+c)^4+3\,ab^2\sec(dx+c)^3+3\,a^2b\sec(dx+c)^2+a^3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

maple [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(dx+c\right)^{\frac{2}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b \sec \left(dx+c\right)+a\right)}^{\frac{5}{2}} \sec \left(dx+c\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 58.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

fricas [A] time = 2.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(dx+c\right)^{\frac{4}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}} \sec \left(dx+c\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 23.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

fricas [A] time = 4.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}} (a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2} \left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 65.96, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

fricas [A] time = 2.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{2}{3}}}{b^3\sec\left(dx+c\right)^6+3\,ab^2\sec\left(dx+c\right)^5+3\,a^2b\sec\left(dx+c\right)^4+a^3\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^6 +
3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec \left(dx+c\right)^{\frac{7}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}} \sec \left(dx+c\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)),x)
[Out] int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=251

$$-\frac{ad\left(a^2(n+1)+3b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{b\left(3a^2(n+2)+b^2(n+1\right.}{}$$

```
[Out] -a*d*(3*b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^
2)*(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+b*(b^2*
(1+n)+3*a^2*(2+n))*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f
*x+e))^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5+2*n)*(d*sec(f*x
+e))^n*tan(f*x+e)/f/(1+n)/(2+n)+b^2*(d*sec(f*x+e))^n*(a+b*sec(f*x+e))*tan(f
*x+e)/f/(2+n)
```

Rubi [A] time = 0.35, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3842, 4047, 3772, 2643, 4046}

$$-\frac{ad\left(a^2(n+1)+3b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{b\left(3a^2(n+2)+b^2(n+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((a*d*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
 Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[
Sin[e + f*x]^2])) + (b*(b^2*(1 + n) + 3*a^2*(2 + n))*Hypergeometric2F1[1/2,
 -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*(2
+ n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 + 2*n)*(d*Sec[e + f*x])^n*Tan[e + f*
x])/(f*(1 + n)*(2 + n)) + (b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[
e + f*x])/(f*(2 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e+fx))}{}$$

$$= \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e+fx))}{}$$

$$= \frac{ab^2(5+2n)(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)(2+n)} + \frac{b^2(d\sec(e+fx))^n(a+b\,\mathrm{s}}{f(2+}$$

$$= \frac{b\left(b^2(1+n)+3a^2(2+n)\right)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e+f}{fn(2+n)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{a\left(a^2+\frac{3b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+f}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.97, size = 231, normalized size = 0.92

$$-\frac{\left(-\tan^2(e+fx)\right)^{3/2}\csc^3(e+fx)(d\sec(e+fx))^n\left(a^3\left(n^3+6n^2+11n+6\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n + 6*n^2 + n^3)*Cos[e + f*x]^3*Hypergeometr
ic2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(3*a^2*(6 + 5*n + n^2)*Cos
[e + f*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] +
b*(1 + n)*(3*a*(3 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sec[e + f*x]^2] + b*(2 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)
/2, Sec[e + f*x]^2])))*(d*Sec[e + f*x])^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*(1
+ n)*(2 + n)*(3 + n)))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{3}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

maple [F] time = 11.29, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{3}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=181

$$-\frac{d\left(a^2(n+1)+b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx)}{fn\sqrt{\sin}}$$

```
[Out] -d*(b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(
d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+2*a*b*hyper
geom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/
(sin(f*x+e)^2)^(1/2)+b^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+n)
```

Rubi [A] time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3788, 3772, 2643, 4046}

$$-\frac{d\left(a^2(n+1)+b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx)}{fn\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos
[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[
e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]
^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^2*(d*S
ec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \frac{(2ab)\int (d\sec(e+fx))^{1+n}\,dx}{d} + \int (d\sec(e+fx))^n\left(a^2+b^2\sec^2(e\right.$$

$$= \frac{b^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)} + \left(a^2+\frac{b^2n}{1+n}\right)\int (d\sec(e+fx))^n$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} +$$

$$= -\frac{\left(a^2+\frac{b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.36, size = 171, normalized size = 0.94

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\sec(e+fx)(d\sec(e+fx))^n\left(a^2\left(n^2+3n+2\right)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e\right.\right.}{fn(n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n + n^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, n/
2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(2*a*(2 + n)*Cos[e + f*x]*Hypergeometri
c2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] + b*(1 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(d*Sec[e + f*x
])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n)*(2 + n))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*sec(f*x + e))^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

maple [F] time = 8.19, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=137

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\right.}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*d*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^(-
1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+b*hypergeom([1/2, -1/2*n],[1-1
/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3787, 3772, 2643}

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\right.}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin
[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = a\int (d\sec(e+fx))^n\,dx + \frac{b\int (d\sec(e+fx))^{1+n}\,dx}{d}$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-n} dx + \frac{\left(b\left(\frac{\cos(e\cdot}{d}\right.\right.}{}$$

$$= -\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.17, size = 107, normalized size = 0.78

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)(d\sec(e+fx))^n\left(a(n+1)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e+fx)\right)+bn\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\right.\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (Csc[e + f*x]*(a*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2
, Sec[e + f*x]^2] + b*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e
+ f*x]^2])*(d*Sec[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)\left(d\sec\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

maple [F] time = 2.60, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{n} \left(a + b \sec{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=192

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-1}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)}{}$$

```
[Out] a*AppellF1(1/2,-1/2+1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*co
s(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/
f-b*AppellF1(1/2,1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(cos(
f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f
```

Rubi [A] time = 0.30, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-1}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, n/2, 1, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])
^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{1-n}(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= -\left(\left(a\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{2-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^n(e+fx)(d\right.$$

$$= -\frac{\left(a\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1-n}{2}}}{-a^2+b^2+a^2x^2}\,dx\right.}{f}$$

$$= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\,s}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 25.85, size = 5280, normalized size = 27.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] Result too large to show
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

maple [F] time = 2.83, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n}{a+\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^n}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=299

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-3}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e\cdots}{}$$

```
[Out] a^2*AppellF1(1/2,-3/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2
)^2/f+b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a
^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2
/(a^2-b^2))*(cos(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/
f
```

Rubi [A] time = 0.44, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2824, 3189, 429}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^nF_1\left(\frac{1}{2};\frac{n-3}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]
^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 -
b^2)^2*f)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{2-n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \left(\frac{b^2\cos^{2-n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3-n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}\right.$$

$$= \left(a^2\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{4-n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - \left(2ab\cos^n(e+fx\right.$$

$$= \frac{\left(a^2\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{3-n}{2}}}{\left(a^2-b^2-a^2x^2\right)^2}\,dx\right.}{f}$$

$$= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3+n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] time = 46.58, size = 13940, normalized size = 46.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
[Out] Result too large to show
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^n}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{\left(b\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(a+b\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{n}}{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\left(a+b\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

3.782 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n,x\right)$$

[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]

[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Mathematica [A] time = 16.82, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]

[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)

maple [A] time = 1.24, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d \sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
[Out] int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec\left(e+fx\right)\right)^{n} \left(a+b \sec\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**(3/2), x)
```

3.783 $\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\sqrt{a+b\sec(e+fx)}\,(d\sec(e+fx))^n,x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
[Out] Defer[Int][(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx = \int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$$

Mathematica [A] time = 0.54, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
[Out] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)+a}\,\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)+a}\,\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

maple [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n \sqrt{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx + e\right) + a}\,\left(d \sec\left(fx + e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a + \frac{b}{\cos\left(e + f\,x\right)}}\,\left(\frac{d}{\cos\left(e + f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec\left(e + fx\right)\right)^{n}\,\sqrt{a + b \sec\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(a + b*sec(e + f*x)), x)
```

3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Defer[Int][(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [A] time = 3.01, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
[Out] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^n}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{n}}{\sqrt{a+b \sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{n}}{\sqrt{b \sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{n}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec\left(e+fx\right)\right)^{n}}{\sqrt{a+b \sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral((d*sec(e + f*x))**n/sqrt(a + b*sec(e + f*x)), x)
```

3.785 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
[Out] Defer[Int][(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
[Out] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

maple [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec \left(f x + e\right)\right)^{n}}{\left(a + b \sec \left(f x + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec \left(f x + e\right)\right)^{n}}{\left(b \sec \left(f x + e\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n}{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec{\left(e + f x \right)}\right)^{n}}{\left(a + b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**(3/2), x)
```

3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sec^n(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
[Out] Defer[Int][Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 2.70, size = 0, normalized size = 0.00

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

maple [A] time = 3.00, size = 0, normalized size = 0.00

$$\int \left(\sec^n\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**n, x)
```

3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((d\sec(e+fx))^n(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

maple [A] time = 2.53, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

```
[Out] int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**m, x)
```

3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=273

$$\frac{\sqrt{2}\left(a^2+b^2(m+1)\right)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}$$

```
[Out] (a+b*sec(f*x+e))^(1+m)*tan(f*x+e)/b/f/(2+m)-a*(a+b)*AppellF1(1/2,-1-m,1/2,3
/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*ta
n(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e))^(1/2)+(a^2
+b^2*(1+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+
e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e))/(a
+b))^m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3840, 4007, 3834, 139, 138}

$$\frac{\sqrt{2}\left(a^2+b^2(m+1)\right)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a +
 b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f
*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 +
 Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2 + b^2*(1 +
 m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]
))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Se
c[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
```

```
x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{\int \sec(e+fx)(b(1+m)-a\sec}{b(2}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{a\int \sec(e+fx)(a+b\sec(e+f}{b^2(2+m)}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+bx}{\sqrt{1-x}}\right.}{b^2f(2+m)\sqrt{1-\sec(e+fx}}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\left(a(-a-b)(a+b\sec(e+fx))^m\right.}{b}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\right.}{}$$

Mathematica [B] time = 26.77, size = 8899, normalized size = 32.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
[Out] Result too large to show
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(fx+e\right)\right) \left(a+b\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{m}}{{\cos\left(e+f\,x\right)}^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x)^3,x)
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m} \sec^{3}\left(e+fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**3, x)
```

3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2}\,(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}$$

```
[Out] (a+b)*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*
(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1+s
ec(f*x+e))^(1/2)-a*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*s
ec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))/(a+
b))^m)/(1+sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3838, 3834, 139, 138}

$$\frac{\sqrt{2}\,(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(
1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[
1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]*(a
+ b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*Sec[e
 + f*x])/(a + b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3838

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Dist[a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{\int \sec(e+fx)(a+b\sec(e+fx))^{1+m}\,dx}{b} - \frac{a\int \sec(e+fx)(a+b\sec(e+
}{b}$$

$$= -\frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{bf\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}} + \frac{(a\tan(e+fx))
}{bf\sqrt{1-}$$

$$= \frac{\left(a(a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-
\right.}{\sqrt{1-x}}\right.}{bf\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\,\mathrm{se}
}{bf\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 23.16, size = 5564, normalized size = 25.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
[Out] Result too large to show
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)+a\right)^m \sec\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^m}{\cos\left(e+f\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^m \sec^2\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**2, x)
```

3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{2}\,\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec
(f*x+e))^m*2^(1/2)*tan(f*x+e)/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e))
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\,\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*
x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{\left((a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}\right.}{\sqrt{1-}}\right.}{f\sqrt{1-\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 14.90, size = 2828, normalized size = 27.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (-6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*(a + b
*Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*
AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]
^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x
)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*AppellF1[3
/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
)])*Tan[(e + f*x)/2]^2)*((6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e +
f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec
[(e + f*x)/2]^2*Sec[e + f*x]^m*Tan[(e + f*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^
2)^2*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*
Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5
/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1
 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (3*(a + b)*AppellF1[1/2, 1 + m,
-m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*C
os[e + f*x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m)/((-1 + Tan[(e + f*x)/2]^2
)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan
[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2,
 Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 +
m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)
/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*a*(a + b)*m*AppellF1[1/2, 1 + m,
 -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*
Cos[e + f*x])^(-1 + m)*Sec[e + f*x]^m*Sin[e + f*x]*Tan[(e + f*x)/2])/((-1 +
 Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)
/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2,
 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (6*(a + b)*m*App
ellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*Sin[e + f*x]*Tan[(e +
 f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/
2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)
*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) -
(6*(a + b)*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(-1/3*((a
```

```
- b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(
e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(a + b) + ((1
+ m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*
x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e +
f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a
- b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1
- m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a +
b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*(a + b)*AppellF1[1/2, 1
+ m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b
+ a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(2*(-((a - b)*m*Appell
F1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(a + b)*(-1/3*((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e +
f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/(a + b) + ((1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2
]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3) + 2*Tan[(e + f*x)/2]^2*(-((a - b)*m*((3*(a - b)*(1 - m)*AppellF1[5/
2, 1 + m, 2 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a +
b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (3*(1 + m)*AppellF1
[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)) + (a + b)*(1 + m)*((-3*(a
- b)*m*AppellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
+ f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (
3*(2 + m)*AppellF1[5/2, 3 + m, -m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
+ f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Ta
n[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1
+ m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])
+ (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a -
b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)^2)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int\sec\left(fx+e\right)\left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(fx+e)+a)^m \sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m/cos(e + f*x),x)
[Out] int((a + b/cos(e + f*x))^m/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(e+fx))^m \sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x), x)
```

3.791 $\int (a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=15

$$\mathrm{Int}\left((a+b\sec(e+fx))^m,x\right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^m,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^m,x]

[Out] Defer[Int][(a + b*Sec[e + f*x])^m, x]

Rubi steps

$$\int (a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 2.08, size = 0, normalized size = 0.00

$$\int (a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^m,x]

[Out] Integrate[(a + b*Sec[e + f*x])^m, x]

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^m, x)

maple [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^m,x)
```

```
[Out] int((a + b/cos(e + f*x))^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m, x)
```

3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\cos(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
[Out] Defer[Int][Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 6.72, size = 0, normalized size = 0.00

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

maple [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx + e\right) + a\right)^{m} \cos\left(fx + e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cos\left(e + f\,x\right)\,\left(a + \frac{b}{\cos\left(e + f\,x\right)}\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b/cos(e + f*x))^m,x)
```

```
[Out] int(cos(e + f*x)*(a + b/cos(e + f*x))^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sec\left(e + fx\right)\right)^{m} \cos\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*cos(e + f*x), x)
```

3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\cos^2(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 5.90, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^m\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

maple [A] time = 1.94, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)+a\right)^{m} \cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m,x)
[Out] int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m} \cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
[Out] Integral((a + b*sec(e + f*x))**m*cos(e + f*x)**2, x)
```

3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{14a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+$$

```
[Out] 14/15*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+14/45*a*cos(d*x+c)^(3/2)*sin(d*x+
c)/d+2/7*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*cos(d*x+c)^(7/2)*sin(d*x+c)/
d+10/21*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{14a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (14*a*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*EllipticF[(c + d*x)/2, 2])/
(21*d) + (10*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (14*a*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(45*d) + (2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) +
 (2*a*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{9}{2}}(c+dx)\,dx + b\int \cos^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}(7a)\int$$
$$= \frac{10b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2b\cos}{}$$
$$= \frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10b\sqrt{\cos(c+dx)}\,\sin(c+}{21d}$$

Mathematica [A] time = 0.39, size = 90, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\,(266a\sin(2(c+dx)) + 35a\sin(4(c+dx)) + 690b\sin(c+dx) + 90b\sin(3(c+dx))) + 1176aE\left(\frac{1}{2}\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (1176*a*EllipticE[(c + d*x)/2, 2] + 600*b*EllipticF[(c + d*x)/2, 2] + Sqrt[
Cos[c + d*x]]*(690*b*Sin[c + d*x] + 266*a*Sin[2*(c + d*x)] + 90*b*Sin[3*(c
+ d*x)] + 35*a*Sin[4*(c + d*x)]))/(1260*d)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4\sec(dx+c) + a\cos(dx+c)^4\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sec(d*x + c) + a*cos(d*x + c)^4)*sqrt(cos(d*x +
c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c) + a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

maple [A] time = 4.02, size = 318, normalized size = 2.36

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + (2240a+720b)\left(\sin^8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a+720*b)*sin(1/2*d*x+1/2*c)^8*c
os(1/2*d*x+1/2*c)+(-2072*a-1080*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
(952*a+840*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a-240*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.31, size = 87, normalized size = 0.64

$$-\frac{2\,a\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,b\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c
+ d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(9/2)*sin(c +
d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(}{}$$

```
[Out] 6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d
+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{7}{2}}(c+dx)\,dx + b\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}(5a)\int \text{co}$$
$$= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b\cos^{\frac{3}{2}}(c+dx)\sin}{5d}$$
$$= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx}{21d}$$

Mathematica [A] time = 0.56, size = 77, normalized size = 0.69

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\,(15a\cos(2(c+dx))+65a+42b\cos(c+dx))+50aF\left(\frac{1}{2}(c+dx)\middle|2\right)+126bE\left(\frac{1}{2}(c+dx)\right|}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (126*b*EllipticE[(c + d*x)/2, 2] + 50*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*a + 42*b*Cos[c + d*x] + 15*a*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^3\sec(dx+c)+a\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(cos(d*x +
c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

maple [A] time = 3.38, size = 290, normalized size = 2.61

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-360a-168b)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a-63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.14, size = 87, normalized size = 0.78

$$-\frac{2\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\,\frac{13}{4};\,\cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,b\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(7/2)*sin(c + d*x
)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2
/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{5}{2}}(c+dx)\,dx + b\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3a)\int$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sqrt{\cos(c+dx)}\sin(c+d}{3d}$$

Mathematica [A] time = 0.27, size = 66, normalized size = 0.76

$$\frac{2\left(\sin(c+dx)\sqrt{\cos(c+dx)}(3a\cos(c+dx)+5b)+9aE\left(\frac{1}{2}(c+dx)\middle|2\right)+5bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(9*a*EllipticE[(c + d*x)/2, 2] + 5*b*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*b + 3*a*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^2\sec(dx+c)+a\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(cos(d*x +
c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 3.64, size = 262, normalized size = 3.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(24a+20b)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*b*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.04, size = 80, normalized size = 0.92

$$\frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*b*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*cos(c + d*x)^(1/2)*sin(c + d
*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))\,dx$$
$$= a\int \cos^{\frac{3}{2}}(c+dx)\,dx + b\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.87

$$\frac{2\left(a\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)+3bE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*b*EllipticE[(c + d*x)/2, 2] + a*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(cos(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 3.51, size = 228, normalized size = 3.74

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-3*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c) + a) \cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.17, size = 53, normalized size = 0.87

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*ellipticE(c/2 + (d*x)/2, 2))
/d + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.798 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4225, 2748, 2641, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\,dx &= \int \frac{b+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= a\int \sqrt{\cos(c+dx)}\,dx+b\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 0.91

$$\frac{2\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
[Out] (2*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/2, 2]))/d
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((b\sec(dx+c)+a)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [A] time = 2.96, size = 152, normalized size = 4.34

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.23, size = 33, normalized size = 0.94

$$\frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\sqrt{\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*b*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + b\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - b\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.17, size = 51, normalized size = 0.89

$$\frac{2\left(aF\left(\frac{1}{2}(c+dx)\middle|2\right) - bE\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
[Out] (2*(-(b*EllipticE[(c + d*x)/2, 2]) + a*EllipticF[(c + d*x)/2, 2] + (b*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [A] time = 3.96, size = 148, normalized size = 2.60

$$-\frac{2\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,a + \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right), \sqrt{2}\right)\sqrt{\frac{1}{2}-}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

[Out] -2*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)

mupad [B] time = 1.25, size = 60, normalized size = 1.05

$$\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(1/2),x)

[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Integral((a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)

3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*
a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{b+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}b\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$
$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.45, size = 65, normalized size = 0.78

$$\frac{\frac{2\sin(c+dx)(3a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)} - 6aE\left(\frac{1}{2}(c+dx)\middle|2\right) + 2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
[Out] (-6*a*EllipticE[(c + d*x)/2, 2] + 2*b*EllipticF[(c + d*x)/2, 2] + (2*(b + 3
*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 7.96, size = 397, normalized size = 4.78

$$2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*b*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*sin(1/2*d*x+
1/2*c)^2-12*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+2*b*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.54, size = 87, normalized size = 1.05

$$\frac{2\,a\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,b\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right)}{3\,d\,\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-3/4, 1/2]
, 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+
2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{b+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3b)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3b)\int$$
$$= -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b}{5d\sqrt{}}$$

Mathematica [A] time = 0.35, size = 95, normalized size = 0.86

$$\frac{10a\sin(c+dx) + 10a\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) + 9b\sin(2(c+dx)) + 6b\tan(c+dx) - 18b\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-18*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*a*Sin[c + d*x] + 9*b*Sin[2*(c + d*x)] + 6
*b*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 9.00, size = 502, normalized size = 4.52

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{6\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\right.}}{3\sqrt{-2\left(\sin^4\right.}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+co
s(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x
+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.70, size = 87, normalized size = 0.78

$$\frac{2\,a\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,b\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +
 d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-5/4, 1/
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(7a^2+9b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{20abF\left(\frac{1}{2}(\right.}{2}$$

```
[Out] 2/15*(7*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(7*a^2+9*b^2)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*
cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(7a^2+9b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{20abF\left(\frac{1}{2}(\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(7*a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos\right.$$

$$= \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\left(10\right.$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(7a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(7a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$

$$= \frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20ab\sqrt{\cos(}}{}$$

Mathematica [A] time = 0.87, size = 113, normalized size = 0.71

$$\frac{84\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(43a^2+36b^2\right)\cos(c+dx)+5a(7a\cos(3(c+dx\right.\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (84*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*a*(156*b
+ 36*b*Cos[2*(c + d*x)] + 7*a*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^4\sec(dx+c)^2+2\,ab\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

maple [B] time = 3.95, size = 398, normalized size = 2.49

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2240a^2+1440ab\right)\left(\sin^8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^2+1440*a*b)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^2-2160*a*b-504*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(952*a^2+1680*a*b+504*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-168*a^2-480*a*b-126*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

mupad [B] time = 1.31, size = 135, normalized size = 0.84

$$-\frac{2\,a^2\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\;\frac{15}{4};\;\cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,b^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4}\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(
c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2\left(5a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{12abE\left(\frac{1}{2}\right.}{5}$$

```
[Out] 12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(5*a^2+7*b^2)*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4264, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2\left(5a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{12abE\left(\frac{1}{2}\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a^2 + 7*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos\right.$$

$$= \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6a\right.$$

$$= \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{5d}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4a}{}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2\left(5a^2+7b\right.}{}$$

Mathematica [A] time = 0.65, size = 98, normalized size = 0.73

$$\frac{10\left(5a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(15a^2\cos(2(c+dx))+65a^2+84ab\cos(c+dx)+7\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2 + 7*b^2)*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(65*a^2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^3\sec(dx+c)^2+2\,ab\cos(dx+c)^3\sec(dx+c)+a^2\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

maple [B] time = 3.90, size = 362, normalized size = 2.68

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(240a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360a^2-336ab\right)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.19, size = 128, normalized size = 0.95

$$\frac{2\left(b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+b^2\sqrt{\cos(c+d\,x)}\ \sin(c+d\,x)\right)}{3\,d}-\frac{2\,a^2\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\ {}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x\right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*(b^2*ellipticF(c/2 + (d*x)/2, 2) + b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))
/(3*d) - (2*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
 cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/2)*
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x
)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2
)*sin(d*x+c)/d+4/3*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2a$$
$$= \frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4ab\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 0.32, size = 79, normalized size = 0.78

$$\frac{6\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20abF\left(\frac{1}{2}(c+dx)\middle|2\right)+2a\sin(c+dx)\sqrt{\cos(c+dx)}\,(3a\cos(c+dx)+10b)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (6*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/2
, 2] + 2*a*Sqrt[Cos[c + d*x]]*(10*b + 3*a*Cos[c + d*x])*Sin[c + d*x])/(15*d
)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

maple [B] time = 3.68, size = 321, normalized size = 3.18

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(24a^2+40ab\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^2+40*a*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-6*a^2-20*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.11, size = 102, normalized size = 1.01

$$\frac{2b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}-\frac{2\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x
)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin
(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=72

$$\frac{2(a^2+3b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2639, 4045, 2641}

$$\frac{2(a^2+3b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos(c\right.$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + (2ab)\int \sqrt{\cos(c+dx)}\,dx - \frac{1}{3}\left(\left(-a^2-3\right.\right.$$
$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{1}{3}\left(-a^2-3b^2\right)\int$$
$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}}{3d}$$

Mathematica [A] time = 0.17, size = 64, normalized size = 0.89

$$\frac{2\left(\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sin(c+dx)\sqrt{\cos(c+dx)}+6abE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
2] + a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)\sec(dx+c)^2+2\,ab\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

maple [B] time = 4.24, size = 283, normalized size = 3.93

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{Ellipt}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*a^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.05, size = 76, normalized size = 1.06

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^2*ellipticF(c/2 + (d*x)/2,
 2))/d + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE(c
/2 + (d*x)/2, 2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.806 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b^2*sin(d*x+c)/d/cos(d*x+c)^(
1/2)
```

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2641, 4046, 2639}

$$\frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2,
 2])/d + (2*b^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \left(2ab\sqrt{\cos\right.$$
$$= \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(\left(a^2-b^2\right)\sqrt{\cos(c+\right.$$
$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a^2-b^2\right)\int \sqrt{\cos(c+dx)}$$
$$= \frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+dx}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.36, size = 62, normalized size = 0.91

$$\frac{2\left(\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2aF\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
[Out] (2*((a^2 - b^2)*EllipticE[(c + d*x)/2, 2] + b*(2*a*EllipticF[(c + d*x)/2, 2
] + (b*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

maple [A] time = 4.14, size = 202, normalized size = 2.97

$$-\frac{2\left(2ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2*(2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.40, size = 81, normalized size = 1.19

$$\frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)
```

3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+4*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\text{co}\right.$$

$$= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+\ldots}}$$

$$= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2ab)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}\left(3a^2+b^2\right)\int \frac{}{\sqrt{\text{c}}}$$

$$= -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(\ldots}{d\sqrt{\cos(\ldots}}$$

Mathematica [A] time = 0.67, size = 73, normalized size = 0.77

$$\frac{2\left(\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6abE\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{b\sin(c+dx)(6a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
 2] + (b*(b + 6*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2 + 2\,ab\sec(dx+c) + a^2}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

maple [B] time = 8.34, size = 514, normalized size = 5.41

$$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin(1/2
*d*x+1/2*c)^2+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-3*a^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.52, size = 108, normalized size = 1.14

$$\frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{4\,a\,b\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+\right.}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*sin(c + d*x)*hypergeom([-3/4
, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c}{5d\cos^{\frac{5}{2}}(c}$$

```
[Out] -2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b^2*sin(d*x+c)/d/c
os(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(5*a^2+3*b^2)*sin
(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2641, 4046, 2639}

$$-\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c}{5d\cos^{\frac{5}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2 + 3*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\cos\right.$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c\right.$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}(2ab)\int \frac{1}{\sqrt{\cos(c}}$$

$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+3b^2\right)\sin(c+d}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin}{3d\cos^{\frac{3}{2}}}$$

Mathematica [A] time = 0.44, size = 124, normalized size = 0.92

$$\frac{-6\left(5a^2+3b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+15a^2\sin(2(c+dx))+20ab\sin(c+dx)+20ab\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*(5*a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 15*a^2*
Sin[2*(c + d*x)] + 9*b^2*Sin[2*(c + d*x)] + 6*b^2*Tan[c + d*x])/(15*d*Cos[c
 + d*x]^(3/2))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

maple [B] time = 10.71, size = 660, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*b^2/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+4*a*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.67, size = 113, normalized size = 0.84

$$\frac{6\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)+30\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c+d\,x\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*a^2*c
os(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20
*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))
/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2), x)
```

3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=160

$$\frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{12ab\sin(c}{5d\sqrt{\cos(c}}$$

```
[Out] -12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*b^2*sin(d*x+c)/d
/cos(d*x+c)^(7/2)+4/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*(7*a^2+5*b^2)*
sin(d*x+c)/d/cos(d*x+c)^(3/2)+12/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{12ab\sin(c}{5d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (12*a*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\cos\right.$$

$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c$$

$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin}{5d\cos}$$

Mathematica [A] time = 0.62, size = 142, normalized size = 0.89

$$\frac{10\left(7a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+35a^2\sin(2(c+dx))+84ab\sin(c+dx)-252ab\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c\right.}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-252*a*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)
*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 84*a*b*Sin[c + d*x] + 252*a
*b*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*Sin[2*(c + d*x)] + 25*b^2*Sin[2*(c
+ d*x)] + 30*b^2*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

maple [B] time = 10.99, size = 689, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*
(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.79, size = 113, normalized size = 0.71

$$\frac{30\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)+70\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\right.}{105\,d\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

```
[Out] (30*b^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*a^2*
cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 8
4*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2
))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{2b\left(15a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

```
[Out] 2/15*a*(7*a^2+27*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(15*a^2+7*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*
a*(7*a^2+27*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a^2*b*cos(d*x+c)^(5/2)
*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+2/21*b
*(15*a^2+7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2b\left(15a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(7*a^2 + 27*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*b*(15*a^2 + 7*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*a^2 + 27*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a^2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*a
^2*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.$$

$$= \frac{2a\left(7a^2+27b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{40a^2b\cos^{\frac{5}{2}}(c+dx)\sin(c+}{63d}$$

$$= \frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a\left(7a^2+27b^2\right)\cos^{\frac{3}{2}}(c+d}{45d}$$

$$= \frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\,\sin(c}{21d}$$

$$= \frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b\left(15a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2}{}$$

Mathematica [A] time = 1.09, size = 137, normalized size = 0.71

$$\frac{84\left(7a^3+27ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+60\left(15a^2b+7b^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7a\left(43a^2+108\right.\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (84*(7*a^3 + 27*a*b^2)*EllipticE[(c + d*x)/2, 2] + 60*(15*a^2*b + 7*b^3)*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*a*(43*a^2 + 108*b^2)*Cos[c
+ d*x] + 5*(234*a^2*b + 84*b^3 + 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c
 + d*x)]))*Sin[c + d*x])/(630*d)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \cos(dx+c)^4 \sec(dx+c)^3 + 3\,ab^2 \cos(dx+c)^4 \sec(dx+c)^2 + 3\,a^2 b \cos(dx+c)^4 \sec(dx+c) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^4*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4)*sqrt(c
os(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3 \cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

maple [B] time = 3.88, size = 470, normalized size = 2.42

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2240a^3+2160a^2b\right)\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 1.34, size = 178, normalized size = 0.92

$$\frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}-\frac{2\,a^3\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right.\right.}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^3*cos(c + d*x)^(1/2)*sin(c
 + d*x))/(3*d) - (2*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11
/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a*b^2*cos(c
+ d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d
*(sin(c + d*x)^2)^(1/2)) - (2*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperge
om([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{2a(5a^2+21b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+$$

```
[Out] 2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*a
^2*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)
)*sin(d*x+c)/d+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(5a^2+21b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a^2*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*a^2*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
```

```
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}$$

$$= \frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+}{35d}$$

$$= \frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+}{35d}$$

$$= \frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a}{}$$

Mathematica [A] time = 0.85, size = 110, normalized size = 0.69

$$\frac{10\left(5a^3+21ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(9a^2b+5b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(15a^2\cos(2(c+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (42*(9*a^2*b + 5*b^3)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^3 + 21*a*b^2)*Ell
ipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(65*a^2 + 210*b^2 + 126*a*b*C
os[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+3\,a^2b\cos\left(dx+c\right)^3\sec\left(dx+c\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^3*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3)*sqrt(c
os(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

maple [B] time = 4.12, size = 421, normalized size = 2.65

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360a^3-504a^2b\right)\left(\sin^6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.22, size = 146, normalized size = 0.92

$$\frac{2\left(b^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\ldots}{9\,d\,\sqrt{\sin\left(c+\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + a*b^2*ellipticF(c/2 + (d*x)/2, 2) + a
*b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*a^3*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(
1/2)) - (6*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=116

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2b\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)}{}$$

```
[Out] 6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c
)^(3/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+8/5*a^2*b*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.23, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4264, 3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2b\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*a^2*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.$$
$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.$$
$$= \frac{8a^2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin}{5d}$$
$$= \frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2b\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a^2\,\mathrm{c}}{}$$
$$= \frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sqrt{\mathrm{c}}}{}$$

Mathematica [A] time = 0.42, size = 84, normalized size = 0.72

$$\frac{2\left(3\left(a^3+5ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sin(c+dx)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+5b}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 + 5*a*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF
[(c + d*x)/2, 2] + a^2*Sqrt[Cos[c + d*x]]*(5*b + a*Cos[c + d*x])*Sin[c + d*
x]))/(5*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\cos(dx+c)^2\sec(dx+c)^3+3ab^2\cos(dx+c)^2\sec(dx+c)^2+3a^2b\cos(dx+c)^2\sec(dx+c)+a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2)*sqrt(c
os(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

maple [B] time = 3.70, size = 376, normalized size = 3.24

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(8a^3+20a^2b\right)\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(8*a^3+20*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-2*a^3-10*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+5*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.18, size = 125, normalized size = 1.08

$$\frac{2b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}-\frac{2\,a^3\cos(c+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a*b^2*ellipticE(c/2 + (d*x)/2, 2
))/d + (2*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*b*cos(c + d*x)^(1/2
)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2,
7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=126

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos}}{}$$

```
[Out] 2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d-2/3*b*(a^2-3*b^
2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3841, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) - (2*b*(a^2 - 3*b^2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\,\sqrt{s\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\,\sqrt{s\right.$$

$$= -\frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))\sin(c}{3d}$$

$$= \frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c}{}$$

$$= \frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2b\left(a^2}{3c}$$

Mathematica [A] time = 0.60, size = 87, normalized size = 0.69

$$\frac{2\left(\frac{\sin(c+dx)\left(a^3\cos(c+dx)+3b^3\right)}{\sqrt{\cos(c+dx)}} + \left(a^3+9ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right) + \left(9a^2b-3b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*((9*a^2*b - 3*b^3)*EllipticE[(c + d*x)/2, 2] + (a^3 + 9*a*b^2)*EllipticF
[(c + d*x)/2, 2] + ((3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d
*x]]))/(3*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\cos(dx+c)\sec(dx+c)^3 + 3\,ab^2\cos(dx+c)\sec(dx+c)^2 + 3\,a^2b\cos(dx+c)\sec(dx+c) + a^3\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x +
c)^2 + 3*a^2*b*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))*sqrt(cos(d*x +
 c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

maple [A] time = 4.56, size = 303, normalized size = 2.40

$$-\frac{2\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+9*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-2*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.24, size = 124, normalized size = 0.98

$$\frac{2a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}+\frac{2\,b^3\,\sin(c+d}{d\,\sqrt{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a^2*b*ellipticE(c/2 + (d*x)/
2, 2))/d + (6*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*cos(c + d*x)^(1
/2)*sin(c + d*x))/(3*d) + (2*b^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, c
os(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.814 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=118

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)(a+b\sec(c}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+16/3*a*b^2*sin(
d*x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^
(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)(a+b\sec(c}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (16*a*b^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)$$
$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)$$
$$= \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(b\left(9a^2+b\right.\right.$$
$$= \frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+}{3d\sqrt{\cos(}}$$
$$= \frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{16ab}{3d\sqrt{}}$$

Mathematica [A] time = 1.29, size = 84, normalized size = 0.71

$$\frac{2\left(3\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)(9a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*((9*a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + (b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2))))/(3*d)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

maple [B] time = 8.46, size = 630, normalized size = 5.34

$$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(6\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d
*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2-18*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(
1/2*d*x+1/2*c)^2-18*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-2*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+36*a*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^4-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*a*b^2+9*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
8*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*b^3*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 2.10, size = 128, normalized size = 1.08

$$\frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)a^3+3\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)a^2\right)}{d}+\frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{6\,a\,b^2\,\sin(c+d}{d\,\sqrt{\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^2*b*ellipticF(c/2 + (d*x)/2, 2)))
/d + (2*b^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a*b^2*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)
^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^3 \sqrt {\cos (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x)), x)
```

3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b}{}$$

```
[Out] -6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(3
/2)+6/5*b*(5*a^2+b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*b*(5*a^2 + b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(a +
b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+d}$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+d}$$

$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 1.09, size = 125, normalized size = 0.84

$$\frac{3\left(5a^2b+b^3\right)\sin(2(c+dx)) + 10a\left(a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6b\left(5a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+d\right.}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(a^
2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a*b^2*Sin[c + d*
x] + 3*(5*a^2*b + b^3)*Sin[2*(c + d*x)] + 2*b^3*Tan[c + d*x])/(5*d*Cos[c +
d*x]^(3/2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \sec(dx+c)^3 + 3\,ab^2 \sec(dx+c)^2 + 3\,a^2b \sec(dx+c) + a^3}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

maple [B] time = 10.75, size = 738, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*b^2
*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3/(8*sin(1/2*d*x+1/2*c)^
6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)+6*a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 2.17, size = 156, normalized size = 1.05

$$\frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos\left(c+d\,x\right)^2\right)}{5\,d\,\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{6\,a^2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos\left(c+\right.\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*sin(c + d*x)*hypergeom([-5/4
, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/
2)) + (6*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*b^2*sin(c + d*x)*hyperge
om([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)^3}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sqrt(cos(c + d*x)), x)
```

3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\left(5a^2+9b^2\right.}{5d\sqrt{\cos(}}$$

```
[Out] -2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*
a*b^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/cos(
d*x+c)^(3/2)+2/7*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/5*a*(
5*a^2+9*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\left(5a^2+9b^2\right.}{5d\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*a*(5*a^2 + 9*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*
(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
 !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+d$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+d$$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx}{7d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+9b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2}{}$$

$$= \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin(c+}{35d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A] time = 0.94, size = 177, normalized size = 0.91

$$\frac{210a^3\sin(c+dx)\cos^2(c+dx)+10b\left(21a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42a\left(5a^2+9b^2\right)\cos^{\frac{5}{2}}(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-42*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*b*
(21*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a*b^2*S
in[c + d*x] + 210*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 378*a*b^2*Cos[c + d*x]^
2*Sin[c + d*x] + 105*a^2*b*Sin[2*(c + d*x)] + 25*b^3*Sin[2*(c + d*x)] + 30*
b^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\sec\left(dx+c\right)+a\right)^3}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

maple [B] time = 12.60, size = 847, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2
+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a^2*b*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*b^2*a/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
```

```
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+2*a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.24, size = 147, normalized size = 0.76

$$\frac{\frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7}+2\,a^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)+\frac{6\,a\,b^2\,\cos(c+d\,x)}{}}{d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*a^
3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
 (6*a*b^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*
x)^2))/5 + 2*a^2*b*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\sec\left(c + dx\right)\right)^3}{\cos^{\frac{3}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/cos(c + d*x)**(3/2), x)
```

3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{2b\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+\right.}{5a^3d}$$

```
[Out] 2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2/3*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d+2*b^4
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*a/(a+b),2^(1/2))/a^4/(a+b)/d+2/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-2/3*
b*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.60, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3853, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2b\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\cos}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*b*(a^2 + 3*b^2
)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^4*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
a^2*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5b}{2}+\frac{3}{2}a\sec(c+dx)+\frac{3}{2}b\sec^2}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+}}{5a}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec}\right.}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{b^4\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(}}{a^4}$$

$$= \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a+b)d} - \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{5ad}$$

$$= \frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2b\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d} + \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{a^4(a+b)d}$$

Mathematica [A] time = 1.80, size = 226, normalized size = 1.49

$$\frac{\frac{2\left(9a^2+5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\left(3a^2+5b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin\right.\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(9*a^2 + 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-5*b + 3*a*Cos[c + d*x])*Sin[c + d*x
] + (6*(3*a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Ellip
ticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Si
n[c + d*x]^2]))/(30*a^2*d)
```

fricas [F] time = 43.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 4.52, size = 668, normalized size = 4.39

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\left(-24a^4+24a^3b\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(24a^4-44a^3b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*a^4+24*
a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^4-44*a^3*b+20*a^2*b^2)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*a^4+16*a^3*b-10*a^2*b^2)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2*b^2-15*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b^4-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
^3-15*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^4/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2*b^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2
^(1/2))/a^3/(a+b)/d+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.39, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2 + 3*b^2)*EllipticF[(c +
d*x)/2, 2])/(3*a^3*d) - (2*b^3*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)+\frac{1}{2}b\sec}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+}}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\sec(c+d}{\sqrt{\sec(c+dx)}}}{3a^3}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{b^3\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a^3} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= -\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{b\int\sqrt{\cos(c+dx)}\,dx}{a^2} + $$

$$= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} - \frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} + 2$$

Mathematica [A] time = 1.93, size = 158, normalized size = 1.41

$$-\frac{\frac{6\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2\sqrt{\sin^2(c+dx)}} - \frac{6b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx\right.}{a+b}}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 4.55, size = 516, normalized size = 4.61

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\left(4a^3-4a^2b\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3+2a^2b\right)\left(\sin^2\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a^3-4*a^2*
b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3+2*a^2*b)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a^2*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+3*b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a+b)
/d
```

Rubi [A] time = 0.24, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4264, 3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticF[(c + d*x)/2, 2])/(a^2*
d) + (2*b^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx \\
&= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}+\frac{\left(b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^2\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a^2}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a}-\frac{(b
\\
&= \frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}+\frac{\int\sqrt{\cos(c+dx)}\,dx}{a}-\frac{b\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2} \\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.34, size = 81, normalized size = 1.08

$$
-\frac{2\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+b\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+aE\left(\sin^{-1}\left(\sqrt{c
}{a^2d\sqrt{\sin^2(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(a^2*d*Sqrt[Sin[c + d*x]^2])
```

fricas [F] time = 83.11, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{b\sec\left(dx+c\right)+a},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

[Out] integral(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)

maple [A] time = 3.68, size = 226, normalized size = 3.01

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\ldots}\right.\right.}{a^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}\ldots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)

[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)

[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)

3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=53

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a+b)/d
```

Rubi [A] time = 0.19, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3848, 2803, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx \\
&= \int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx \\
&= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{a} \\
&= \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.91

$$
\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b))/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 3.86, size = 187, normalized size = 3.53

$$
-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{a\,(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=29

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))/(a+b)/d
```

Rubi [A] time = 0.13, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4264, 3849, 2805}

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx\\ &= \int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx\\ &= \frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 29, normalized size = 1.00

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 3.07, size = 150, normalized size = 5.17

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=77

$$-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b/(a+b)/d+2*sin(d*x+c)/b/d/cos(
d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d) + (2*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx}{b} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= -\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\int\sqrt{\cos(c+dx)}\,dx}{b}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 3.34, size = 195, normalized size = 2.53

$$-\frac{\frac{2\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{6a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -1/2*((6*a*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*b*(2*Ell
ipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
 + b)))/a - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 4.96, size = 353, normalized size = 4.58

$$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/b/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b^2/(a+
b)/d+2/3*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)-2*a*sin(d*x+c)/b^2/d/cos(d*x+c)^(1
/2)
```

Rubi [A] time = 0.54, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
b*d) + (2*a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) - (2*a*Sin[c + d*x])/(b^2*d*Sqr
t[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}b\ldots\right)}{a+b\ldots}}{3b}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\ldots\right.}{\ldots}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\ldots\right.}{3a^2b^2\ldots}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}d\ldots}{b^2}$$

$$= \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d} + \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \ldots$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} + \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d} + \ldots$$

Mathematica [A] time = 4.52, size = 210, normalized size = 1.64

$$\frac{\frac{2(9a^2+2b^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\sin(c+dx)\left((a^2-2b^2)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c\ldots}\right.\right.\right.}{b\sqrt{\sin^2(c+dx)}}}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(9*a^2 + 2*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + (4*(b - 3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)
 + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), Arc
Sin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/(6*b^
2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 10.06, size = 450, normalized size = 3.52

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{b^2\left(a^2-ab\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}-\frac{2a(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^2*a^3/(a^2
-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))-2*a/b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1)+2/b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$\frac{(2a^2-5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(a^2-b^2)}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{(2a^4+16a^2b^2-15b^4)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3a^4d(a^2-b^2)}$$

```
[Out] -b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-15*b^4)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/a^4/(a^2-b^2)/d-b^3*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)
/(a+b)^2/d+1/3*(2*a^2-5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b^
2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 0.73, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3847, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(16a^2b^2+2a^4-15b^4)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d(a^2-b^2)}-\frac{b(4a^2-5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}-\frac{b^3(7a^2-5b^2)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d)) + ((2*
a^4 + 16*a^2*b^2 - 15*b^4)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d)
 - (b^3*(7*a^2 - 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a
- b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a^2+\frac{5b^2}{2}+ab\sec(c+}{\sec^{\frac{3}{2}}(c+dx)(a}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\mathrm{co}\right.}{}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\mathrm{co}\right.}{}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b^3\left(7\right.\right.}{}$$

$$= -\frac{b^3\left(7a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a-b)(a+b)^2d} + \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} +$$

$$= -\frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4+16a^2b^2-15b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4\left(a^2-b^2\right)d} - \frac{b^3\left(7\right.}{}$$

Mathematica [A] time = 2.13, size = 266, normalized size = 1.09

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3b^3}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}+2\right) - \frac{\frac{2\left(5b^3-8a^2b\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^2+2b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\right|\right.}{a+b}}{}}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2 + (3*b^3)/((-a^2 + b^2)*(b + a*Cos[c + d*x])))*Sin
[c + d*x] - ((2*(-8*a^2*b + 5*b^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(4*a^2 - 5*b^2)*(-2*a*b*
EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c
 + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)
))/(12*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 11.37, size = 1063, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/3/a^2/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))-4*(a+b)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*b^3/a^3/(a^2-a*b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1
/2))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\,(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad\,(a^2-b^2)\,\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}-\frac{b\,(4a^2-3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}+\frac{b^2\,(5\cdots}{\cdots}$$

```
[Out] (2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(4*a^2-3*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^
2-b^2)/d+b^2*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+b^2*si
n(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\,(4a^2-3b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}+\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\,(a^2-b^2)}+\frac{b^2\,(5a^2-3b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{\cdots}{ad\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2
 - 3*b^2)*EllipticF[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3*
b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]
))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a^2+\cdots}{\cdots}}{a\left(a^2-b\cdots\right.}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a\left(-a\cdots\right.}{\cdots}}{a^3\left(a^2-\cdots\right.}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(b^2\left(5a^2-3b^2\right)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c\cdots}}{2a^3\left(a^2-b^2\right)}$$

$$= \frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a-b)(a+b)^2d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

$$= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b\left(4a^2-3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+\cdots}\right.}{a^3(a-b)(a\cdots}$$

Mathematica [A] time = 2.08, size = 252, normalized size = 1.37

$$\frac{\frac{4b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a\cos(c+dx)+b)} + \frac{\frac{2\left(2a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(2a^2-3b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\cdots\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x]))
 + ((2*(2*a^2 - b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(-EllipticF[(c + d*x)/2, 2] + (b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b)) + (2*(2*a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2
*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^
2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 10.57, size = 809, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6*b^2/a^2/(a^2-a*b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d/(a^2-b^2)+(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d-b*sin(d*x+c)/(a^2-b^2)/d/(a+
b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4264, 3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2 - b^2)*EllipticF[
(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) - (b*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.}{}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}\right.}{}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)\int \frac{}{\sqrt{\cos(}}}{2\left(a^2\right.}$$

$$= -\frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a-b)(a+b)^2d} - \frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+}$$

$$= \frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2}{a}\right.}{a^2(a-b)(}$$

Mathematica [A] time = 4.21, size = 194, normalized size = 1.16

$$\frac{\frac{4b\sin(c+dx)\sqrt{\cos(c+dx)}}{(b^2-a^2)(a\cos(c+dx)+b)} - \frac{\frac{2\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2\sqrt{\sin^2(c+dx)}} - \frac{10b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\cdots\right)}{a+b}}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))
- (8*EllipticF[(c + d*x)/2, 2] - (10*b*EllipticPi[(2*a)/(a + b), (c + d*x)/
2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elliptic
Pi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c +
 d*x]^2]))/((-a + b)*(a + b)))/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

maple [B] time = 8.36, size = 788, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*b/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2*b^2/a^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
```

```
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)
```

3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/(a^2-b^2)+(a^2+b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2
^(1/2))/a/(a-b)/(a+b)^2/d+a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4264, 3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d)) - (b*EllipticF[(c + d*x)/2, 2]
)/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a*(a - b)*(a + b)^2*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{a^2\,(}$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(a^2+b^2\right)\int \frac{}{\sqrt{\cos(c+dx)}\,(}}{2a\left(a^2-b\right.}$$

$$= \frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a-b)(a+b)^2d} + \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} - \frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx\right.}{a(a-b)(a+b)^2d}$$

Mathematica [A] time = 3.27, size = 229, normalized size = 1.55

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{(a^2-b^2)(a\cos(c+dx)+b)} - \frac{2\left(\frac{\sin(c+dx)\left((a^2-2b^2)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b\sqrt{\sin^2(c+dx)}} - \frac{a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\cdots\right.}{a+b}\cdots\right.}{a(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) -
 (2*(-((a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*b*(2*El
lipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2])))
/(a*(a - b)*(a + b)))/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 8.28, size = 707, normalized size = 4.78

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{(a^2-ab)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} - 2b\left(\frac{a^2}{\cdots}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-2/a*b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
```

```
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec{\left(c + dx \right)}\right)^{2} \cos^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*cos(c + d*x)**(3/2)), x)
```

3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+d$$

```
[Out] a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),
2^(1/2))/(a-b)/b/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))/co
s(d*x+c)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+d$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + EllipticF[(c + d*x)/2, 2]
/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/((a - b)*b*(a + b)^2*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{a^2}$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))} + \frac{\left(a^2-3b^2\right)\int \frac{}{\sqrt{\cos(c+dx}}}{2b\left(a^2-\right.}$$

$$= \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(}$$

$$= \frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d}$$

Mathematica [A] time = 3.83, size = 239, normalized size = 1.55

$$\frac{\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(b^2-a^2\right)(a\cos(c+dx)+b)} + \frac{\frac{2\left(3a^2-4b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{c}\right.\right.\right.}{b\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x])
) + ((2*(3*a^2 - 4*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)
+ 4*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(a + b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elli
pticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c
 + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 6.38, size = 608, normalized size = 3.95

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{b\left(a^2-b^2\right)\left(2a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\right.\right.}}{(a+b)b\sqrt{-2\left(\mathrm{si}\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b)*a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx}}$$

```
[Out] -(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-a*(3*a
^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b
^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+(3*a^2-2*b^2)*sin(d*x+c)/b^2/(a^2-b
^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)) - (a*Ell
ipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])/
(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+d}$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+d}$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+d}$$

$$= -\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b^2(a+b)^2d} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{}{b}$$

$$= -\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2}{a}\right.}{(a-b)b^2(}$$

Mathematica [A] time = 3.57, size = 278, normalized size = 1.27

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a^3\sin(c+dx)}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}+2\tan(c+dx)\right) - \frac{\frac{2\left(9a^3-10ab^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{\left(8a^2b-4b^3\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} +}{}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (-(((2*(9*a^3 - 10*a*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
) + ((8*a^2*b - 4*b^3)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)
/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(3*a^2 - 2*b^2)*(-2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*S
qrt[Cos[c + d*x]]*((a^3*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) +
2*Tan[c + d*x]))/(4*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 10.83, size = 868, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^2/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))+2/b^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1)-2/b*a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)
-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c\right.}{4a^4d\left(a^2-b^2\right)^2}$$

```
[Out] -1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+
128*a^4*b^2-223*a^2*b^4+105*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^5/(a^2-b^2)^2/d-1/4*b^3*(63*a
^4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^5/(a-b)^2/(a+b)^3/d+1/12*(8*a
^4-61*a^2*b^2+35*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/2*b^2
*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b^2*(13*a
^2-7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] time = 1.06, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4264, 3847, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(128a^4b^2-223a^2b^4+8a^6+105b^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^3\left(-86a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
 b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) +
((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^
2 - b^2)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-2a^2+\frac{7b^2}{2}+2ab\,s}{\sec^{\frac{3}{2}}(c+d}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \cdot$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx}{2a\left(a^2-b^2\right)d(a+b\sec(c+d}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx}{2a\left(a^2-b^2\right)d(a+b\sec(c+d}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx}{2a\left(a^2-b^2\right)d(a+b\sec(c+d}$$

$$= -\frac{b^3\left(63a^4-86a^2b^2+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5(a-b)^2(a+b)^3d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos}}{12a^3\left(a^2-b^2\right.}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d} + \frac{\left(8a^6+128a^4b^2-223a^2b^4+105b^6\right.}{12a^5\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 4.35, size = 353, normalized size = 1.02

$$\frac{-\frac{2\left(56a^4b-73a^2b^3+35b^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{16\left(2a^4+14a^2b^2-7b^4\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - \frac{6\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\right.\right.}{(a-b)^2(a+b)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(4*a^6 - 57*a^2*b^4 + 35*b^6 + a*b*(16*a^4 - 83*a^2*
b^2 + 49*b^4)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*
x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(56*a^4*b - 73*a^2*b^3 +
35*b^5)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^4 + 1
4*a^2*b^2 - 7*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/
(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*
a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSi
```

```
n[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[C
os[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
 + b)^2))/(48*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 17.27, size = 2216, normalized size = 6.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+20*b^3/a^4/(a^2-a*b)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))-2/a^5*b^5*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^
2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^
2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/
b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a
^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1
/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b
)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3,x)
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}-\frac{3b\left(8a^4-11a^2\right.}{4}$$

```
[Out] 1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*
a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+15*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/2*b^2*sin(d*x+c)/a/(a^2-b
^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin(d*x+c)
/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3847, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{b^2\left(-38a^2b^2+35a^4+15\right.}{4a^4d(a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2
)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^
4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{2a\left(a^2 \cdots\right.}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+\cdots}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,(a+\cdots}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+\cdots}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,(a+\cdots}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+\cdots}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,(a+\cdots}$$

$$= \frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4(a-b)^2(a+b)^3d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cdots}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{3b\left(8a^4-11a^2b^2+5b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \cdots$$

Mathematica [A] time = 3.27, size = 311, normalized size = 1.10

$$\frac{\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}\left(a\left(13a^2-7b^2\right)\cos(c+dx)+11a^2b-5b^3\right)}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2} + \frac{-\frac{8\left(4a^2b-b^3\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(8a^4-7a^2b^2+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\cdots\right.}{a+b}\cdots}{\cdots}}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*b^2*Sqrt[Cos[c + d*x]]*(11*a^2*b - 5*b^3 + a*(13*a^2 - 7*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((8*a^4 - 7*a^
2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(4*a
^2*b - b^3)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x
]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)
)/(8*a^2*d)
```

fricas [F] time = 136.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

maple [B] time = 15.62, size = 1957, normalized size = 6.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-12*b^2/a^3/(a^2-a*b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2
-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a
^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(
a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2
)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-8/a^4*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+
1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
```

```
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos{\left(c + dx \right)}}}{\left(a + b \sec{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=263

$$\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)}{4ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+\ldots}$$

```
[Out] 3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+1/4*(8*a^4-5*a^2*b^2+3*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/
2))/a^3/(a-b)^2/(a+b)^3/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/c
os(d*x+c)^(1/2)-1/4*b*(7*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(-5a^2b^2+8a^4+3b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{3b\left(-2a^2b^2+5a^4+b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\ldots\right.}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((8
*a^4 - 5*a^2*b^2 + 3*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d
) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(4*a^3*(a - b)^2*(a + b)^3*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]*(a + b*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*Sin[c + d*x])/(4*a*
(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{b\left(7a\right.}{4a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{b\left(7a\right.}{4a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{b\left(7a\right.}{4a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{3b\left(5a^4-2a^2b^2+b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a-b)^2(a+b)^3 d} - \frac{b\,\mathrm{si}}{2\left(a^2-b^2\right)d\sqrt{\cos(c}}$$

$$= \frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+3b^4\right)F\left(\frac{1}{2}(c+dx)\middle|\right.}{4a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 3.15, size = 286, normalized size = 1.09

$$-\frac{\frac{-\frac{2\left(5a^2b+b^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16\left(2a^2+b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6\left(3a^2-b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\right.\right.}{a^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(5*a^2*b + b^3
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2 + b^2)*((
a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]))/(a + b) + (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

maple [B] time = 13.44, size = 1936, normalized size = 7.36

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6/a^2
*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2))-2/a^3*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/
2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+
b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b
)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+6*b^2/a^3*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
```

```
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{3}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x))), x)
```

3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=246

$$-\frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{3\left(a^2+b^2\right)\sin(c+dx)}{4d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}+\frac{}{2d\left(\right.}$$

```
[Out] -1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d-1/4*b*(7*a^2-b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^
2/(a^2-b^2)^2/d+1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)^
2/(a+b)^3/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2
)+3/4*(a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{}{4d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^
2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10*
a^2*b^2 - b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^2*
(a + b)^3*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*Sqrt[Cos
[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)}{4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2(a-b)^2(a+b)^3d} + \frac{a\sin(c+}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} - \frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(3a^4+1\right.}{}$$

Mathematica [A] time = 2.23, size = 272, normalized size = 1.11

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(a\left(5a^2+b^2\right)\cos(c+dx)+3b\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2} - \frac{-\frac{2\left(a^2+5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(5a^2+b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(\ldots}{a^2b\sqrt{\sin^2\ldots}}}{(\ldots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) - ((-2*(a^2 + 5*b^2)*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*b*(2*EllipticF[(c + d*x
)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(5*
a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2
]))/((a - b)^2*(a + b)^2))/(16*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 13.90, size = 1858, normalized size = 7.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+2*b^2/a^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2
+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(
a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-4*b/a^2*(a^2/b/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*c
os(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
```

```
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=253

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{}{4b}$$

```
[Out] 1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-
b^2)^2/d+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a-b)^2/b/(a+b)
^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2)+1
/4*a*(a^2-7*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/
2)
```

Rubi [A] time = 0.72, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{}{2bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2
 - 3*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*S
ec[c + d*x])^2) + (a*(a^2 - 7*b^2)*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{\cdots}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a\left(a^2\right.}{4b\left(a^2-b^2\right)^2 d\sqrt{\right.}}\cdots$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a\left(a^2\right.}{4b\left(a^2-b^2\right)^2 d\sqrt{\right.}}\cdots$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{a\left(a^2\right.}{4b\left(a^2-b^2\right)^2 d\sqrt{\right.}}\cdots$$

$$= \frac{\left(a^4-10a^2b^2-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a(a-b)^2b(a+b)^3d} - \frac{a^2\sin(\cdots}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+\cdots}}$$

$$= \frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-1\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.19, size = 289, normalized size = 1.14

$$\frac{\frac{\frac{6\left(a^3-3ab^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b\left(a^2+2b^2\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2\left(a^2+5b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\cdots\right.\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}\cdots}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a*Sqrt[Cos[c + d*x]]*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((6*(a^3 - 3*a*b^2)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2 + 2*b^2)*(2
*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a + b)))/a + (2*(a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

maple [B] time = 13.65, size = 1760, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2)))+2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
```

```
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=255

$$\frac{\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}-\frac{}{4b^2d}$$

```
[Out] 3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b
/(a^2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)^2/b^2/
(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)
)^2-3/4*a^2*(a^2-3*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {4264, 3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{3\left(-2a^2b^2+a^4+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}-\frac{}{2bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((a
^2 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^
2*(a + b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])^2) - (3*a^2*(a^2 - 3*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b
^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-\right.}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-\right.}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-\right.}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos}}$$

$$= \frac{3\left(a^4-2a^2b^2+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4(a-b)^2b^2(a+b)^3d} - \frac{a^2\sin(c+}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^4-2\right.}{}$$

Mathematica [A] time = 3.16, size = 297, normalized size = 1.16

$$\frac{\frac{\frac{16b\left(a^2-4b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6\left(a^2-3b^2\right)\sin(c+dx)\left(\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos}\right.\right.\right.}{b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a^2*Sqrt[Cos[c + d*x]]*(5*a^2*b - 11*b^3 + 3*a*(a^2 - 3*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(9*a^4 - 19
*a^2*b^2 + 16*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16
*b*(a^2 - 4*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])
*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

maple [B] time = 8.64, size = 1203, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/
2*d*x+1/2*c)^2-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^2-
b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}-\frac{a^2\sin(c+}{2bd\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)(}$$

```
[Out] -1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11
*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/
(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d
*x+c)^(5/2)/(a+b*sec(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x+c)/b^2/(a^2-b
^2)^2/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sin
(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4264, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+15a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{a\left(-38a^2b^2+15a^4+35b^4\right)\Pi\left(\frac{2a}{a+b}\right.}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^
2)^2*d) - (a*(5*a^2 - 11*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)
^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Si
n[c + d*x])/(4*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])
/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*
Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{a^2\left(5a^2\right.}{4b^2\left(a^2-b^2\right)^2d\,c}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4(a-b)^2b^3(a+b)^3d} + \frac{\left(15a^4-29a^2b^2+\right.}{4b^3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} - \frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c+d\right.}{4b^2\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 3.74, size = 334, normalized size = 1.02

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a^3\sin(c+dx)\left(a\left(7a^2-13b^2\right)\cos(c+dx)+9a^2b-15b^3\right)}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+8\tan(c+dx)\right) - \frac{\frac{2\left(45a^5-95a^3b^2+56ab^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b\left(5a^4-1\right.}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (-(((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b) + (8*b*(5*a^4 - 10*a^2*b^2 + 2*b^4)*(2*EllipticF[(c + d*x)/
2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(1
```

```
5*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*
EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt
[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((a^3*(9*a
^2*b - 15*b^3 + a*(7*a^2 - 13*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)
^2*(b + a*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*b^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec \left(dx+c\right)+a\right)^{3} \cos \left(dx+c\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

maple [B] time = 17.37, size = 2014, normalized size = 6.14

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^3/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-2/b*a*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^
2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)
/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),
```

```
2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/
(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/b^3*(-(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2*a/b^2*(a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2
)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{9/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)
[Out] int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] -4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/d+2/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*
x+c))^(1/2)/a/d+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (
2*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2
*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}{}$$

$$= -\frac{4b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+}}{1}$$

Mathematica [C] time = 9.58, size = 340, normalized size = 1.39

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(a\sin(c+dx)(3a\cos(c+dx)+b) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-\left(9a^2-2b^2\right)\tan\left(\frac{1}{2}(c+dx\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(b + 3*a*Cos[c + d*x])*Si
n[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(9*a^3 + 9*a^2*
b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + I*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] - (9*a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))))/(15*a^
2*d)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.45, size = 1724, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(9*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)-7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*cos(d*x+c)*
sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*a^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+9*a^2*b*((a-b)/(a+b))^(1/2)+a*b^
2*((a-b)/(a+b))^(1/2)+9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^3+9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+2*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^3-3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-4*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a^2*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-5*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^2*b-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+9*cos(d*x+c)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a^2*b+2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2-7*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-
2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^2-2*b^3*((a-b)/(a+b))^(1/2)-9*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)/a^2/((a-b)/(a+
b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{}}{}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3857, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{s\right.$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(b\sqrt{\cos(c+dx)}\,\sqrt{se\right.}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+a\right.}{3a\sqrt{\cos(c+}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos}{a-}}\right.}{3a\sqrt{\cos(c+d}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{3ad\sqrt{}}$$

Mathematica [C] time = 8.90, size = 273, normalized size = 1.42

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(a^2\sin(c+dx)+ab\tan\left(\frac{1}{2}(c+dx)\right)+ab\tan(c+dx)-ia(a+b)\sqrt{\sec(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(I*b*(a + b)*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a +
 b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c
 + d*x]] + a^2*Sin[c + d*x] + a*b*Tan[(c + d*x)/2] + b^2*Sec[c + d*x]*Tan[(
c + d*x)/2] + a*b*Tan[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x]))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 1.49, size = 1011, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin
(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-cos(d*x+c)*
sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*b^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b
-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1
/2))/(b+a*cos(d*x+c))/sin(d*x+c)/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(dx + c\right) + a} \cos \left(dx + c\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.839 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3856, 2655, 2653}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}}$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.88, size = 198, normalized size = 2.96

$$\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(\sin(c+dx)\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}-iF\left(i\sinh^{-1}\left(\tan\right.\right.\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(I*Elliptic
E[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*EllipticF[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/(d*Sqrt[(1 +
 Cos[c + d*x])^(-1)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.29, size = 923, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*b+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b+((a-b)/(a+b))^(1/2)*b)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(dx + c\right) + a} \sqrt{\cos \left(dx + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos \left(c + d\,x\right)}\,\sqrt{a + \frac{b}{\cos \left(c + d\,x\right)}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec \left(c + dx\right)} \sqrt{\cos \left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4264, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(\cdots\right.$$

$$= \frac{\left(a\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\cdots}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 28.84, size = 14885, normalized size = 107.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(dx+c) + a}}{\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [C] time = 1.22, size = 275, normalized size = 1.99

$$\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}}\right)a - \mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}}\right)b + 2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(dx+c))}{\sin(dx+c)}\right.}{d\,(-1+\cos(dx+c))\,(b + a\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)^(1/2)*sin(d*x+c)^2*((b+a
*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d*x+c))/((a-b)/(a+b))^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(dx+c) + a}}{\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+d\,x)}}}{\sqrt{\cos(c+d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4264, 3855, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3855

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*
Csc[e + f*x])^(n - 1))/(f*(2*n - 1)), x] + Dist[d^2/(2*n - 1), Int[((d*Csc[
e + f*x])^(n - 2)*Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-a+a\,\mathrm{se}}{\sqrt{\sec(c+dx)}\,\sqrt{}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+}}{\sqrt{\sec(c+dx}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\sqrt{\frac{b}{}}\right.}{2\sqrt{c}}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c}{a+b}}\right.}{2\sqrt{\cos(c+}}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 31.54, size = 33277, normalized size = 140.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [C] time = 1.27, size = 780, normalized size = 3.29

$$\frac{\left(\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\mathrm{EllipticE}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)}, \sqrt{-\frac{a+b}{a-b}}\right)\left(\cos^2(dx+c)\right)\sin(dx+c)\,a-\sqrt{\frac{b+}{(1+c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
[Out] 1/d*(sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a-sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*sin(d*x+c)*cos(d
*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*b+((a-b)/(a+b)
)^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^
(1/2)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{4b\left(41a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+16/35*b*cos(d*
x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d
*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*a^2+3*b^2)*sin(d*x+c)*cos(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^
(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b
))^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+\right.}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(25*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(105*a*d) + (16*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(35*d) + (2*a*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(7*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} - \frac{1}{7}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{16b\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} + \frac{16b}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} + \frac{16b}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} + \frac{16b}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{105ad} + \frac{16b}{}$$

$$= \frac{2\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(41a^2\right.}{}$$

Mathematica [C] time = 11.59, size = 383, normalized size = 1.26

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(a\sin(c+dx)(a\cos(c+dx)+b)\left(15a^2\cos(2(c+dx))+65a^2+48ab\cos(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(a*(b + a*Cos[c + d*x])*(65*
a^2 + 6*b^2 + 48*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x] -
 (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-41*a^3 - 41*a^2*b +
3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
ec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
 I*a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + 2*b*(-41*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec
[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(105*a^2*d*(
b + a*Cos[c + d*x])^2)
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a\cos\left(dx+c\right)^3\right)\sqrt{b\sec\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

maple [B] time = 1.42, size = 2040, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(6*cos(d*x+c)
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^4+25*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-25*a^3*b*((a-b)/(a+b))^(1/2)-8
2*a^2*b^2*((a-b)/(a+b))^(1/2)-3*a*b^3*((a-b)/(a+b))^(1/2)+27*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-82*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+
55*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a*b^3+39*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^4-25*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+25*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)+6*b^4*((a-b)/(a+b))^(1/2)-82*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+51*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)+82*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-82*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-6*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-82*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b+51*cos(d*x+c)*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2+
6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
```

```
1/(1+cos(d*x+c)))^(1/2)*a*b^3+82*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-82*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2-6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)/(b+a*cos(d*x+c))/sin(d*x+
c)/a^2/((a-b)/(a+b))^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=240

$$\frac{2b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a}{}$$

```
[Out] 2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c
)*(a+b*sec(d*x+c))^(1/2)/d+4/5*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/d+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (4*b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} - \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\text{se}}\right.$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}{}$$

$$= \frac{2b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+}}{5}$$

Mathematica [C] time = 9.26, size = 344, normalized size = 1.43

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2\sin(c+dx)(a\cos(c+dx)+b)(a\cos(c+dx)+2b) - \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(2*b
 + a*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2
)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(
c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(5*d*(b +
a*Cos[c + d*x])^2)
```

fricas [F] time = 2.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^2\sec(dx+c) + a\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\,\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.49, size = 1695, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3-4*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+cos(d*x+c)*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*a^3+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-cos(d*x+c)*sin(d*x+c)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a*b^2+cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+3*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-3*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a*b^2+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a*b^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*a^2*b*((a-b)/(a+b))^(1/2)+2
*a*b^2*((a-b)/(a+b))^(1/2)+b^3*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*
x+c)/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{5/2}\left(a+\frac{b}{\cos(c+dx)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=187

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{8b\sqrt{\cos(c+dx)}}{}$$

```
[Out] 2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3864, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{8b\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(4b\sqrt{\cos(c+d\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+\right.}{3\sqrt{\cos(c}$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+}{}}\right.}{3\sqrt{\cos(c+}$$
$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{3d}$$

Mathematica [C] time = 7.44, size = 284, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\left(\frac{1}{2}a\sin(2(c+dx))(a\cos(c+dx)+b)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-i\left(a^2+4ab+3b^2\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((a*(b + a*Cos[c + d*x])*S
in[2*(c + d*x)])/2 + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*(a + b
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*EllipticF
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*d*(b + a*Cos[c + d*x])^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{b\sec(dx+c)+a}\,\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

maple [B] time = 1.47, size = 1209, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+cos(d*x+c)*
sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a^2-4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+3*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*b^2+4*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x
+c)+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)
*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2)-4*b^2*((a-b)
/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*co
s(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \cos (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c + d\,x)^{3/2}\,{\left(a + \frac{b}{\cos (c + d\,x)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.845 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{2b^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {4264, 3868, 3856, 2655, 2653, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2b^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \left(b\sqrt{\cos(c\right.$$

$$= \left(ab\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(b^2\sqrt{\cos\right.$$

$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b+a\cos(c+dx)}\right)}{\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\left(ab\sqrt{\frac{b+a\cos(}{a+b}}\right.}{\sqrt{\cos(c+}}$$

$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+}}$$

Mathematica [C] time = 30.21, size = 25369, normalized size = 121.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
[Out] Result too large to show
```

fricas [F] time = 2.61, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

maple [C] time = 1.32, size = 1365, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b
^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x
+c)*b^2+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^2-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-a*
b*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=249

$$\frac{(2a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{d\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

```
[Out] (2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos
(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+b*si
n(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-b*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A] time = 0.78, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4264, 3866, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(2a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}{d\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (b*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+a^2\,\mathrm{se}\cdots}{\sqrt{\sec(c\cdots}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+\cdots}{\sqrt{\sec(c+d\cdots}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{s}\cdots}}{\sqrt{\sec\cdots}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(\left(2a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c\cdots}}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(\left(2\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(2a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\cdots\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d\cdots}}$$

Mathematica [C] time = 31.93, size = 34674, normalized size = 139.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

maple [C] time = 1.29, size = 1205, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+6*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b-cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*b^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+6*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b-sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)
*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^2+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-b^2*((a-b)/(a+b))^(1/2))*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x
+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sin(c+dx)\sqrt{a+b\sec(c+a}}{4d\sqrt{\cos(c+dx)}}$$

```
[Out] 7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x
+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+1/2*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+5/4*a*sin(d*
x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*a*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A] time = 1.02, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3866, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sin(c+dx)\sqrt{a+b\sec(c+a}}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (5*a*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*d*Cos[c + d*x]^(3/2)) + (5*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{1}{8}\left(5a\sqrt{\right.$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(7ab\sqrt{b\right.}{8\sqrt{}}$$

$$= \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+d}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 32.87, size = 51315, normalized size = 171.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

maple [C] time = 1.18, size = 1742, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c)^3*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*b^2-5*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b+2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2+2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b-4*cos(d*x
+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*b^2+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-5*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b+2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2+2*cos(d*x+c)^2*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
a*b-4*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2+5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+2*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2
+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
b^2-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*b^2*((a-b)/(a+b))^(1/2))*((b+a*c
os(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)/(
(a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=363

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\,\mathrm{s}}}{315ad}$$

```
[Out] 4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/315*(49*a^
2+75*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+38/63*a*b*co
s(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/9*a^2*cos(d*x+c)^(7/2)
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/315*b*(163*a^2+5*b^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/315*(147*a^4+279*a^2*b^2-10*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\,\mathrm{s}}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(163*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49*a^2 + 75*b^2)*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (38*a*b*
Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*C
os[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\,\sqrt{\right.$$

$$= \frac{38ab\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{63d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{}}{}$$

$$= \frac{2\left(49a^2+75b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315d} + \frac{38ab\cos}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{315ad} + \frac{2\left(49\right.}{}$$

$$= \frac{4b\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2\left(147a^4+\right.}{}$$

Mathematica [C] time = 14.58, size = 477, normalized size = 1.31

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{630}\left(133a^2+150b^2\right)\sin(2(c+dx)) + \frac{b\left(747a^2+20b^2\right)\sin(c+dx)}{630a} + \frac{1}{36}a^2\sin(4(c+dx)) + \frac{19}{126}ab\sin(3(c+\right.}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((b*(747*a^2 + 20*b^2)*Sin[c
 + d*x])/(630*a) + ((133*a^2 + 150*b^2)*Sin[2*(c + d*x)])/630 + (19*a*b*Sin
[3*(c + d*x)])/126 + (a^2*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])^2)
 - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec
[c + d*x])^(5/2)*((-I)*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^3 - 1
0*a*b^4 - 10*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ I*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*EllipticF[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*
b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(31
5*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^4\sec(dx+c)^2 + 2\,ab\cos(dx+c)^4\sec(dx+c) + a^2\cos(dx+c)^4\right)\sqrt{b\sec(dx+c)+a}\,\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

maple [B] time = 1.52, size = 2778, normalized size = 7.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/315/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-35*cos(d*x+c
)^6*((a-b)/(a+b))^(1/2)*a^5-14*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5-98*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+147*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5+10*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-147*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5-10*cos(d*x
+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*b^5+147*a^4*b*((a-b)/(a+b))^(1/2)+163*a^3*b^2*((a-b)/(a+b))^
(1/2)+279*a^2*b^3*((a-b)/(a+b))^(1/2)+5*a*b^4*((a-b)/(a+b))^(1/2)-130*cos(d
*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b-170*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*
b^2-82*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+279*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^3*b^2-199*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3-10*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a*b^4-80*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3-272*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2+5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^
4+65*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+147*cos(d*x+c)*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+147
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-279*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+279*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+10*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-261*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+279*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-155*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-10*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
```

```
)))^(1/2)*sin(d*x+c)-147*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-10*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+147*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)-10*b^5*((a-b)/(a+b))^(1/2)+279*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-155*cos(d*x+
c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b^3-10*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+147*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*a^4*b-279*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^2+279*cos(d*x+c)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b
^3+10*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4-261*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b)/(b+a*cos
(d*x+c))/sin(d*x+c)/a^2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^{\frac{9}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c + d\,x)^{9/2} \left(a + \frac{b}{\cos (c + d\,x)}\right)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2b\left(29a^2+3b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E}{21ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+6/7*a*b*cos(d*x+c)^
(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/d+2/21*(5*a^2+9*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)/d+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*c
os(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\right)}{21ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(21*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + (2*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(21*d) + (6*a*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(7*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{6ab\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6ab\,c}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6ab\,c}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6ab\,c}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d} + \frac{6ab\,c}{}$$

$$= \frac{2\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\left(29a^2+\right.}{}$$

Mathematica [C] time = 12.14, size = 419, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{42}\left(23a^2+36b^2\right)\sin(c+dx)+\frac{1}{14}a^2\sin(3(c+dx))+\frac{3}{7}ab\sin(2(c+dx))\right)(a+b\sec(c+dx))^{5/2}}{d(a\cos(c+dx)+b)^2}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((23*a^2 + 36*b^2)*Sin[c +
d*x])/42 + (3*a*b*Sin[2*(c + d*x)])/7 + (a^2*Sin[3*(c + d*x)])/14))/(d*(b +
 a*Cos[c + d*x])^2) + (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(I*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b
^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/
2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(5*a^3 +
 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + b*(29*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]))/(21*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3\right)\sqrt{b\sec\left(dx+c\right)+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c) + a)^{\frac{5}{2}} \cos(dx+c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)

maple [B] time = 1.28, size = 2040, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x)

[Out] -2/21/d*(-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-5*a^3*b*((a-b)/(a+b))^(1/2)-29*a^2*b^2*((a-b)/(a+b))^(1/2)-9*a*b^3*((a-b)/(a+b))^(1/2)+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-29*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+12*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-5*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+5*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*b^4*((a-b)/(a+b))^(1/2)-29*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+27*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+29*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-29*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-29*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b+27*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+29*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3

```
*b-29*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/
((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\cos (dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^{7/2}\,{\left(a+\frac{b}{\cos (c+d\,x)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=239

$$\frac{16b(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*cos(d*x+c)^(3/2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/d+22/15*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec
(d*x+c))^(1/2)/d+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{16b(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (22*a*b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a^2*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{16b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+23b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [C] time = 12.49, size = 391, normalized size = 1.64

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{5}a^2\sin(2(c+dx))+\frac{22}{15}ab\sin(c+dx)\right)(a+b\sec(c+dx))^{5/2}}{d(a\cos(c+dx)+b)^2} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((22*a*b*Sin[c + d*x])/15 +
(a^2*Sin[2*(c + d*x)])/5))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3/
2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I)
*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] + I*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^3)*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2 + 23*b^2)*(b + a*Cos
[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[c
 + d*x])^3*Sec[c + d*x]^(5/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^2\sec(dx+c)^2 + 2\,ab\cos(dx+c)^2\sec(dx+c) + a^2\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\,\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.34, size = 1921, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/15/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(9*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)-23*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-17*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+23*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*cos(d*x+
c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*a^3+23*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3+9*a^2*b*((a-b)/(a+b))^(1/2)+
11*a*b^2*((a-b)/(a+b))^(1/2)+9*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^3+9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-23*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*b^3-3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-14*cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a^2*b-34*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+5*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-15
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)*sin(d*x+c)*b^3+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-23*cos(d*x+c)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*a*b^2-17*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+23*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+
23*b^3*((a-b)/(a+b))^(1/2)-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+23*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+9*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-15*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
```

```
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^3*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^3\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2
)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+14/3*a*b*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4264, 3841, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^3\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+d\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+d\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(7ab\sqrt{\cos(c+\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(a\left(a^2+2b^2\right)\sqrt{\right.}{3\sqrt{\cos}}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{se}}}{3d}$$

$$= \frac{2a\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi}{d\sqrt{\cos(c+dx)}\sqrt{}}$$

Mathematica [C] time = 32.89, size = 36372, normalized size = 138.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

maple [C] time = 1.26, size = 1651, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*cos(d*x+c
)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^2-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*b^3+7*cos(d*x+c)*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*
b-7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+6*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)-7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+7*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a^3+8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a*b^2-a^2*b*((a-b)/(a+b))^(1/2)-7*a*b^2*((a-b)/(a+b)
)^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+
c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.852 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=263

$$\frac{b\left(4a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\,\mathrm{s}}{}$$

```
[Out] b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+
b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(2*a^2-b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4264, 3842, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{b\left(4a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\left(4a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right.}{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b\left(4a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(\right.}{d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Mathematica [C] time = 32.17, size = 44191, normalized size = 168.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
[Out] Result too large to show
```

fricas [F] time = 2.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{b\sec\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

maple [C] time = 1.27, size = 1947, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+6*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^2*b-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a*b^2+10*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a*b^2+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3-2*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2*b-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2+((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*
b^3-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^2+10*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+2*cos(d*x+c)*
sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*a^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-cos(d*x+c)*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
```

```
),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*b^3+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a^3+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b+cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-cos(
d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-b^3*((a
-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^(1/2)/((a-b)/(a+b)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \sqrt{\cos (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos (c + d\,x)}\, \left(a + \frac{b}{\cos (c + d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=314

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+}{2a}$$

```
[Out] 1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+
b*sec(d*x+c))^(1/2)+1/2*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^
(3/2)+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-9/4*a*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + (9*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+d}}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{1}{8}\left(9ab\sqrt{\right.}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(a\left(8a^2+\right.\right.}{}$$

$$= \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a\left(8a^2+11b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2\right.}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec}}$$

Mathematica [C] time = 32.33, size = 52888, normalized size = 168.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

maple [C] time = 1.30, size = 1972, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+8*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^3*b^3-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2*b+9*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^3*a*b^2+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^3-6*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^2*b+
2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)*cos(d*x+c)^3*a*b^2-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*b^3+30*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+8*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b
^3-9*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2+8*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*a^3-6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+2*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+
c)^2*a*b^2-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^3+9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-9*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^2*a^2*b+9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+2*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*b^3-11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-2*b^3*((a-b)/(a+
b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^
(3/2)/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=369

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{b(59a^2+16b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{(33a^2+16b^2)}{}$$

```
[Out] 1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b
*sec(d*x+c))^(1/2)+1/3*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(
5/2)+13/12*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/24*(3
3*a^2+16*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/24*(33
*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(
1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{b(59a^2+16b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{(33a^2+16b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (13*a*b*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((33*a^2 +
16*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+d\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+\cdots}{\cdots}$$

$$= \frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\,\sin(c+d\cdots}{3d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{b\left(59a^2+16b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2\cdots\right.}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec\cdots}}$$

Mathematica [C] time = 32.64, size = 61979, normalized size = 167.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

maple [C] time = 1.17, size = 2285, normalized size = 6.19

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/24/d*(59*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b+33*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
4*a^3-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3-26*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^
2*b-16*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2-33*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^
2*b+16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-26*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^2*b+44*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)*cos(d*x+c)^3*a*b^2-33*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-18*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)^3*a^3+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3+33*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^3-18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-33*co
s(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^
2-18*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^4*a^3-30*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a
^3+33*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3-30*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3-16*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3+
44*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^4*a*b^2-120*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*
a*b^2-120*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2-33*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^4*a^2*b+16*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
```

```
/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2-26*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2
*b+8*b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)/cos(d*x+c)^(5/2)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
[Out] Timed out
```

3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{8b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{15a^2d}-\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+8b\right.}{}$$

```
[Out] -2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(
1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d-8/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/a^2/d+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3863, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{15a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
 - (8*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad} - \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{5a}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{5ad}$$

$$= -\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+8b^2\right)\sqrt{\cos(c+dx)}\,E\left(\right.}{15a^3d\sqrt{}}$$

Mathematica [C] time = 9.68, size = 340, normalized size = 1.37

$$\frac{2a\sin(c+dx)(a\cos(c+dx)+b)(3a\cos(c+dx)-4b) + \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(\left(9a^2+8b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*b + 3*a*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(9*a^2 + 2*a*
b + 8*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^
2 + 8*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]
))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.29, size = 1726, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-9*a^2*b*
((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d*x+c)*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a
^3-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a*b^2+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+2*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b-8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-8*b^3*((a-b)/(a+b)
)^(1/2)+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a^3/((a-b
)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx}{}$$

```
[Out] 2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/a/d-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1
/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3863, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cdots}{\sqrt{\text{se}\cdots}}}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{3a^2}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(\left(a^2+2b^2\right)\sqrt{b+a\cos(c+dx)}\right)\int \cdots}{3a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec\cdots}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{\cdots}{\sqrt{\frac{b}{a+b}+\cdots}}}{3a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c\cdots}}$$

$$= \frac{2\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{4b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a\cdots}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.64, size = 265, normalized size = 1.36

$$\frac{2\sqrt{\cos(c+dx)}\left(a^2\sin(c+dx)-2ab\tan\left(\frac{1}{2}(c+dx)\right)+ab\tan(c+dx)-ia(a-2b)\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*((-2*I)*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a - 2*b)*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] + a^2*Sin
[c + d*x] - 2*a*b*Tan[(c + d*x)/2] - 2*b^2*Sec[c + d*x]*Tan[(c + d*x)/2] +
a*b*Tan[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.47, size = 1014, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin
(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*cos(d
*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*b^2+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a*b-((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)+2*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a*b-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-a*b*((a-b)/(a+b))^(1/2)+2*b^2*
((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)/a^2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4264, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int}{a\sqrt{b+a\cos(c+d}}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int\sqrt{\frac{b}{a+}}}{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.03, size = 216, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\sqrt{\frac{1}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)-ia\sqrt{\frac{a\cos(c+dx}{(a+b)(\cos(c+}}\right.}{ad\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(I*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
```

```
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*a*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2
]))/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

maple [B] time = 1.42, size = 732, normalized size = 5.15

$$2\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\left(\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)\text{EllipticF}\left(\frac{(-1+\cos(d}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d
*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*b+((a-b)/(a+b))^(1/2)*b)/(b+a*co
s(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {4264, 3858, 2663, 2661}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 0.89, size = 102, normalized size = 1.52

$$-\frac{2i\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\,F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] ((-2*I)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[(
1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)\sec(dx+c)+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)*sec(d*
x + c) + a*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

maple [A] time = 1.34, size = 163, normalized size = 2.43

$$\frac{2\left(\sin^2(dx+c)\right)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)\left(\sqrt{\cos}\ (dx+c)\right)\sqrt{\frac{b+a\cos}{\cos(d}}}{d\,(-1+\cos(dx+c))\,(b+a\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*sin(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(-1+c
os(d*x+c))/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(dx + c) + a}\,\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(c + d\,x)}\,\sqrt{a + \frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec(c + dx)}\,\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)
^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4264, 3859, 2807, 2805}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 29.05, size = 14986, normalized size = 220.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

maple [C] time = 1.30, size = 206, normalized size = 3.03

$$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\sqrt{b}}{d\,(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{a-b}{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
[Out] 2/d*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))-2*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))
```

```
*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/(1/(
1+cos(d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2
)/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {4264, 3860, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cdots}{\sqrt{\cdots}}}{2b}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{2b}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos(c+dx)}\cdots}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+\cdots}{bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\cdots}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+\cdots}}$$

Mathematica [C] time = 30.99, size = 21698, normalized size = 88.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

maple [C] time = 1.46, size = 986, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(-sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a+sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*sin(d*x+c)*cos
(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a+2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*cos(d*x+c)*sin(d*x+c)*b-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+2*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)/((
a-b)/(a+b))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{5/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}+\frac{3a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{4b^2d\sqrt{\frac{a\cos(c}{a}}}$$

```
[Out] -1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*
x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^
(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)+1/2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(3/2)-3/4*a*s
in(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+3/4*a*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/
(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3860, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}+\frac{3a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{4b^2d\sqrt{\frac{a\cos(c}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*b*d*Cos[c + d*x]^(3/2)) - (3*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{s}}{}}{4}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}}{2bd\cos^{\frac{3}{2}}(c}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a}}$$

Mathematica [C] time = 32.31, size = 51323, normalized size = 164.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

maple [C] time = 1.34, size = 1745, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*(6*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*cos(d*
x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*b^2-3*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+c)^3*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b-6*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^2+6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a*b+4*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b-6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^2+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-2*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+
3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b
^2-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)/((a-
b)/(a+b))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec\left(dx + c\right) + a} \cos\left(dx + c\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+dx)^{7/2}\sqrt{a+\frac{b}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Timed out
```

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)}-\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{a\cos(c}{a}}}{5a^4d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-8/5*
b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^4
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*cos(d*x+c)^(3/2)
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/5*b*(3*a^2-8*b^2)*sin(
d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d+2/5*(3*a^4+8
*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)}-\frac{2b\left(3a^2-8b^2\right)\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b
^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(a^2 - 6*b
^2)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+3b^2+\frac{1}{2}ab\sec(}{\sec^{\frac{5}{2}}(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin}{5a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{5a^3\left(a^2-b^2\right)d}$$

$$= -\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{\cos(c+}}{5a^4\left(a^2-\right.}$$

Mathematica [C] time = 12.75, size = 419, normalized size = 1.16

$$\frac{(a\cos(c+dx)+b)\left(a\sec^{\frac{3}{2}}(c+dx)\left(6b\left(b^2-a^2\right)\sin(c+dx)(a\cos(c+dx)+b)+a\left(a^2-b^2\right)\sin(2(c+dx))(a\cos(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(a*Sec[c + d*x]^(3/2)*(10*b^4*Sin[c + d*x] + 6*b*(-a^
2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + a*(a^2 - b^2)*(b + a*Cos[c + d
*x])*Sin[2*(c + d*x)]) + 2*(a^2 + 4*b^2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
(3/2)*(I*(3*a^3 + 3*a^2*b - 4*a*b^2 - 4*b^3)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] - I*a*(3*a^2 - a*b - 4*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(5*a^4*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}{b^2\sec(dx+c)^2+2ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)

maple [B] time = 1.43, size = 1851, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)

[Out] -2/5/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-16*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-3*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4+3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b+8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*a^3*b*((a-b)/(a+b))^(1/2)-8*a*b^3*((a-b)/(a+b))^(1/2)-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+16*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-3*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*b^4*((a-b)/(a+b))^(1/2)-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-12*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-4*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b-12*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)

```
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^
2*b^2-16*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+8*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2)/(b+a*
cos(d*x+c))/sin(d*x+c)/a^4/(a+b)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx + c\right)^{\frac{5}{2}}}{\left(b \sec\left(dx + c\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2+8b^2\right)\sqrt{\frac{a\,\mathrm{co}\ldots}{}}}{3a^3d\sqrt{\cos(c+\ldots}}$$

```
[Out] 2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*
(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/3*b*(5*a^2-8*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/
d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2+8b^2\right)\sqrt{\frac{a\,\mathrm{co}\ldots}{}}}{3a^3d\sqrt{\cos(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+2b^2+\frac{1}{2}ab}{\sec^{\frac{3}{2}}(c+}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\left(a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{3a^3\left(a^2-b^2\right)d}$$

Mathematica [C] time = 9.32, size = 382, normalized size = 1.32

$$-\frac{2(a\cos(c+dx)+b)\left(a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(3b^3-\left(a^2-b^2\right)(a\cos(c+dx)+b)\right)-\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a*(3*b^3 - (a^2 - b^2)*(b + a*Cos[c + d*x]))*Sec[
c + d*x]^(3/2)*Sin[c + d*x] - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*b*
(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] - I*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(-5*a^2 + 8*b^2)*(b + a*Cos[c
+ d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2 - b^2)*d
*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.54, size = 1307, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3/d*(5*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^3-cos(d*x+c)
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3-6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2
+5*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-6*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)^3*((a-b)/(a+
b))^(1/2)*a^2*b+4*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b+4*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-4*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^2*b+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3+a^2*b*((a-b)/(a+
b))^(1/2)-4*a*b^2*((a-b)/(a+b))^(1/2)-8*b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a^3/
(a+b)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-4*b*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))
^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3847, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Si
n[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots}{a\left(a^2-b^2\right)\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\left(-\frac{a^2}{2}+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(\cdots}}{a^2\left(a^2-b\cdots\right.}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right)\right)\sqrt{b+a\,c\cdots}}{a^2\left(a^2-b^2\right)\sqrt{\cos(c+dx\cdots}}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right)\right)\sqrt{\frac{b+a\cos(\cdots}{a+b\cdots}}}{a^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\cdots}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2\cdots}{a+\cdots}\right.}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c\cdots}{a+b}}}$$

Mathematica [C] time = 9.66, size = 330, normalized size = 1.54

$$\frac{2(a\cos(c+dx)+b)\left(ab^2\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(\left(a^2-2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a\cos(c+dx)+b)-ia\left(a^2-a\cdots\right.\right.}{\cdots}\right)}{a^2d\left(a^2-b\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(a*b^2*Sin[c + d*x] + ((Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(3/2)*(I*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(
Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a^2*(a^2
 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.49, size = 997, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*
x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*x+c)
*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a*b+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2
+a*b*((a-b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*
x+c)/((a-b)/(a+b))^(1/2)/(a+b)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\,c}{}}}{ad\sqrt{\cos}}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(
a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {4264, 3843, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\,c}{}}}{ad\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{b+a\cos(c+d}}{a\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int}{a\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{a\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [C] time = 7.80, size = 245, normalized size = 1.22

$$\frac{2\sqrt{\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(b(b-a)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)} - ia(a+b)\sqrt{\sec(c+dx}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(I*b*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] - I*a*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + b*(-a + b)*S
qrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(3/2))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)\sec(dx+c)^2+2\,ab\cos(dx+c)\sec(dx+c)+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)*sec(
d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 1.41, size = 502, normalized size = 2.51

$$2\left(-\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-\sqrt{\frac{b+a\cos}{(1+\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
[Out] 2/d*(-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*
x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/
2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/a/(a+b)/
((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {4264, 3844, 21, 3856, 2655, 2653}

$$\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{a^2}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\,s\right.}{\left(a^2-b\right.}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\,s\right.}{\left(a^2-\right.}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{}{\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [C] time = 7.85, size = 260, normalized size = 2.06

$$\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left((a-b)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1}}\right.}{d\left(a^2-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(3
/2)*((-I)*(b + a*Cos[c + d*x])*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + I*(b + a*Cos[c + d*x])*EllipticF[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + (a - b)
*Sqrt[Sec[c + d*x]]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
*Sin[c + d*x]))/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*(a + b*Sec[c + d*x])^(3/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2*se
c(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 1.26, size = 491, normalized size = 3.90

$$-\frac{2\left(\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)\sin(dx+c)\cos(dx+c)-\sqrt{\frac{b+a}{(1+\cos}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*cos(d*x+c)-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)+EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos
(d*x+c)*((a-b)/(a+b))^(1/2)-((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/
(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos}{}}}{bd\sqrt{\cos}}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c
),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3845, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos}{}}}{bd\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(
a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+\right.}{b\left(a^2-b^2\right)\sqrt{\mathrm{co}}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+d}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{b\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [C] time = 32.27, size = 47811, normalized size = 232.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

maple [C] time = 1.35, size = 1134, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+2*cos(d*x+c)*sin(d*x+c)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*b-2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a+2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+2*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+a*((a-b)
/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*co
s(d*x+c))/sin(d*x+c)/b/(a+b)/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(dx+c\right)+a\right)^{\frac{3}{2}} \cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} dx$

Optimal. Leaf size=345

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\sqrt{\cos(c+d}}{b^2d\,(}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)+(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)
/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(3*a^2-b^2)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)-(3*a^2-b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^
2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3845, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\sqrt{\cos(c+d}}{b^2d\,(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+}}{b^2\left(a^2-b\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+}}{b^2\left(a^2-b\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+}}{b^2\left(a^2-b\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+}}{b^2\left(a^2-b\right.}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

Mathematica [C] time = 32.56, size = 51610, normalized size = 149.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

maple [C] time = 1.42, size = 1492, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+6*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*a*b+3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2
-6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*a^2-4*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+6*cos(d*x+c)*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a^2+6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+3*cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2-cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-6*cos(d*x+c)*sin(d*x+c)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^
2-4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)-cos(d*x
+c)*((a-b)/(a+b))^(1/2)*b^2+a*b*((a-b)/(a+b))^(1/2)+b^2*((a-b)/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c
)^(1/2)/((a-b)/(a+b))^(1/2)/(a+b)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{7/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{4b^2(5a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{a\cos(c+dx}{a+b}}}{3a^4d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(
d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)/a^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^
(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*co
s(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {4264, 3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4b^2(5a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{2(-13a^2b^2+a^4+8b^4)\sin(c+d}{3a^3d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

```
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a^2}{2}+3b^2+\frac{3}{2}a\cdots}{\sec^{\frac{3}{2}}(c+\cdots}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2\left(a^4+16a^2b^2-16b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{\cdots}}{3a^{4}\cdots}$$

Mathematica [C] time = 14.71, size = 527, normalized size = 1.35

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2\sin(c+dx)}{3a^3}+\frac{2b^4\sin(c+dx)}{3a^3\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{8\left(2b^5\sin(c+dx)-3a^2b^3\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+d\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*Sin[c + d*x])/(3*a^3) + (2*b^4*Sin[c + d*x])/(3
*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (8*(-3*a^2*b^3*Sin[c + d*x] + 2*
```

```
b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c +
d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-4*
I)*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*EllipticE[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^5 - 8*a^4*b + 7*a^3
*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] - 4*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*(b + a*Cos[c + d*x])*
(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b
*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

maple [B] time = 1.52, size = 3604, normalized size = 9.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6-a^4*b^2*((a-b
)/(a+b))^(1/2)+7*a^3*b^3*((a-b)/(a+b))^(1/2)+20*a^2*b^4*((a-b)/(a+b))^(1/2)
-8*a*b^5*((a-b)/(a+b))^(1/2)-16*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^5+((a-b)
/(a+b))^(1/2)*cos(d*x+c)^4*a^5*b-((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4*b^2-(
(a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*b^3-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*
a^5*b-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^4*b^2+6*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^3*a^3*b^3+6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b^4+7*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^2*a^5*b-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b^2-34*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*b^3+8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^
2*b^4+24*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^5-2*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*a^5*b+14*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b^2+22*((a-b)/(a+b))^(1/2
)*cos(d*x+c)*a^3*b^3-34*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b^4-16*sin(d*x+c
)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
```

b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^6+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-12*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+28*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^6+((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^6-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^6-16*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-16*b^6*((a-b)/(a+b))^(1/2)-16*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^5+28*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^3+28*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^4+9*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+16*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-12*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-16*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-8*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5*b+28*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b^3-16*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^5+10*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+25*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-28*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b

```
)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a^5*b-8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b^2)*((b+a*cos(d*x+c))/cos(d*
x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/a^4/(a-b)/(a+b)^
2/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8\right.}{3a^3d\left(a^2-b\right.}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*
x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*
(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3847, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8\right.}{3a^3d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{3a}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= -\frac{2b\left(9a^2-8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{\cos(}}{3a^3\left(a^2\right.}$$

Mathematica [C] time = 14.59, size = 507, normalized size = 1.60

$$\frac{(a\cos(c+dx)+b)^3\left(-\frac{2\left(5b^4\sin(c+dx)-9a^2b^2\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} - \frac{2b^3\sin(c+dx)}{3a^2\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} - \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((-2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) - (2*(-9*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c +
d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^5 + 3*a^4*b - 15*a^3
*b^2 - 15*a^2*b^3 + 8*a*b^4 + 8*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] + I*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^4 - 15*a^2*b
^2 + 8*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^3\sec\left(dx+c\right)^3+3\,ab^2\sec\left(dx+c\right)^2+3\,a^2b\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)

maple [B] time = 1.50, size = 3101, normalized size = 9.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] -2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^5+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^5-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5-3*a^3*b^2*((a-b)/(a+b))^(1/2)-11*a^2*b^3*((a-b)/(a+b))^(1/2)+4*a*b^4*((a-b)/(a+b))^(1/2)+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-12*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+18*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+18*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-15*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-15*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)

```
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b^4-9*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+6*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*c
os(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*b^5*((a-b)/(a+b))^(1/2)-3
*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^3*b^2+14*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*cos(d*x+c
)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^4+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4*b-15*cos(d*x+c)*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a
^3*b^2-15*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3+8*cos(d*x+c)*sin(d*x+c)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4-12*co
s(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^4*b)/sin(d*x+c)/(b+a*cos(d*x+c))^2/a^3/((a-b)/(a+b))^
(1/2)/(a+b)^2/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{2b(5a^2-b^2)\sin(c+dx)}{3ad(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)}{3d(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2-2b}{3a^2d(a^2-b^2}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-2/3*b
*(5*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3*a^2-
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2
/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3843, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b(5a^2-b^2)\sin(c+dx)}{3ad(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)}{3d(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2-2b}{3a^2d(a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sin[c + d*x])/(3*a*(a^
2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(3a^2-b^2\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [C] time = 9.24, size = 398, normalized size = 1.32

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2b\sin(c+dx)\left(\left(2ab^2-6a^3\right)\cos(c+dx)-5a^2b+b^3\right)}{a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} - \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(2b\left(b^2-3a^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c +
d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (2*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^3 + 6*a^2*b
 + a*b^2 - 2*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ 2*b*(-3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
 + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*x]^(3/2))))/(3*d*Cos[c + d*x]^(5/2)
*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)\sec(dx+c)^3+3\,ab^2\cos(dx+c)\sec(dx+c)^2+3\,a^2b\cos(dx+c)\sec(dx+c)+a^3\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)*sec(
d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)*sec
(d*x + c) + a^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

maple [B] time = 1.53, size = 2062, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*(-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4+3*cos(d*
x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*a^4+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2-3*sin(d*x+c)*c
os(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a^3*b-2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2+6*sin(d*x+c)*cos(d*x+c)
^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b-2*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+5*a^2*b^2*((a-b)/(a+b))^(1/2)-a*b^
3*((a-b)/(a+b))^(1/2)-6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a*b^3+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+2*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*b^4*((a-b)/(a+b))^(1/
2)+3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-2*cos(d*x+c)*sin(d*x+c)*EllipticF((
```

```
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+6*cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3*b+6*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*cos(d*x+c)*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b^3)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\frac{a}{}}}{3ad\left(a^2-b^2\right.}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+4/3*(a
^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/
3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(
d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3844, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\frac{a}{}}}{3ad\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^{\cdot}\right.}{3\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^{\cdot}\right.}{3\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^{\cdot}\right.}{3\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^{\cdot}\right.}{3\left(a^2-b^2\right)^2 d\sqrt{c}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2+b^2\right)\sqrt{c}}{}$$

Mathematica [C] time = 11.70, size = 447, normalized size = 1.59

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2b\sin(c+dx)}{3\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{2\left(3a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
d*x])^2) + (2*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b
 + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (
2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*
x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^
2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(
b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^2
 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^2\sec(dx+c)^3+3ab^2\cos(dx+c)^2\sec(dx+c)^2+3a^2b\cos(dx+c)^2\sec(dx+c)+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^2*se
c(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)
^2*sec(d*x + c) + a^3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

maple [B] time = 1.31, size = 1804, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a^3+EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*a*b^2-3*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*a^3+((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+3*cos(d*x+c)*sin(d*x+c)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^3+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b+cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^2+cos(d*x+c)*s
in(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*b^3-3*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*cos(d*x+c)*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+cos
(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^2+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+((a-b)/(a+b))^(1
```

```
/2)*cos(d*x+c)^2*a^2*b+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-3*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*b+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-cos(d*x+c)*((a
-b)/(a+b))^(1/2)*b^3+2*a^2*b*((a-b)/(a+b))^(1/2)-a*b^2*((a-b)/(a+b))^(1/2)+
b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/
2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{a}{}}}{3d\left(a^2-b^2\right.}$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+2
/3*a*(a^2-5*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/(a^2
-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4264, 3845, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{a}{}}}{3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c +
 d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a\right.}{3b\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a\right.}{3b\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a\right.}{3b\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a\right.}{3b\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{3\left(a^2-\right.}$$

Mathematica [C] time = 9.52, size = 311, normalized size = 1.12

$$\frac{2(a\cos(c+dx)+b)^2\left(\frac{a\sin(c+dx)\left(a^2-4ab\cos(c+dx)-5b^2\right)}{a\cos(c+dx)+b} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-i\left(a^2+4ab+3b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\,F\left(i\,\mathrm{s}\right.\right.}{}\right)}{3d\left(a^2-b^2\right)^2\cos^{\frac{5}{2}}(c+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^2*((a*(a^2 - 5*b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d*
x])/(b + a*Cos[c + d*x]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*
(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*Ell
ipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/Sqrt[Sec[c + d*x]]))/(3*(a^2 - b^2)^2*d*Cos[
c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^3\sec(dx+c)^3+3\,ab^2\cos(dx+c)^3\sec(dx+c)^2+3\,a^2b\cos(dx+c)^3\sec(dx+c)+a^3\,\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)

maple [B] time = 1.51, size = 1333, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] -2/3/d*(cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2-3*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b+4*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*b^2+4*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-((a-b)/(a+b))^(1/2)*a^2-a*b*((a-b)/(a+b))^(1/2)+4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a}{3bd\left(a^2-\right.}$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)-2
/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec
(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1
/2)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*s
ec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^
(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A] time = 1.20, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {4264, 3845, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a}{3bd\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a
^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+dx}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi}{b^2d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 33.47, size = 92128, normalized size = 248.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)

maple [C] time = 1.38, size = 3844, normalized size = 10.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)

[Out] 2/3/d*(6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4+6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^2+4*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^3*b-9*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-4*a^3*b*((a-b)/(a+b))^(1/2)-a^2*b^2*((a-b)/(a+b))^(1/2)+7*a*b^3*((a-b)/(a+b))^(1/2)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3-3*a^4*((a-b)/(a+b))^(1/2)*cos(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4-6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^4+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4-3*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/

```
2))*a^3*b*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+10*cos(d*x+c)*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*a^3*b-5*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^2-12*cos(d*x+c)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3-3*co
s(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3*b+7*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+7*cos(d*x+c)*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b^3-12*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b+12*sin(d*x+c)*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*a*b^3-3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^3+7*sin(d*x+c)*co
s(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2-6*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b+6*sin(d*x+
c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3+6*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*a^2*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x
+c))^2/sin(d*x+c)/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

3.875 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=266

$$-\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a\left(a^2(1-n)-3b^2n\right)\sin(e+$$

[Out] -b*(b^2*(1-n)+3*a^2*(2-n))*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a*(a^2*(1-n)-3*b^2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5-2*n)*(d*cos(f*x+e))^n*tan(f*x+e)/f/(n^2-3*n+2)+b^2*(d*cos(f*x+e))^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(2-n)

Rubi [A] time = 0.46, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4264, 3842, 4047, 3772, 2643, 4046}

$$-\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a\left(a^2(1-n)-3b^2n\right)\sin(e+$$

Antiderivative was successfully verified.

[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]

[Out] -((b*(b^2*(1 - n) + 3*a^2*(2 - n))*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f*x]^2])) - (a*(a^2*(1 - n) - 3*b^2*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 - 2*n)*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n)*(2 - n)) + (b^2*(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 - n))

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+b\sec(e$$

$$= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{\left((d\cos(e+f\right.}{}$$

$$= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{\left((d\cos(e+f\right.}{}$$

$$= \frac{ab^2(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{b^2(d\cos(e+fx))^n(a+}{f(}$$

$$= -\frac{b\left(b^2(1-n)+3a^2(2-n)\right)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e\right.}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{b\left(b^2(1-n)+3a^2(2-n)\right)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e\right.}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.75, size = 222, normalized size = 0.83

$$-\frac{\sqrt{\sin^2(e+fx)}\,\csc(e+fx)\sec^2(e+fx)(d\cos(e+fx))^n\left(\frac{1}{2}a(n-2)\cos(e+fx)\left(2a(n-1)\cos(e+fx)\left(an\,co\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b^3*n*(-1 + n^2)*Hypergeometric2F1[1/2,
 (-2 + n)/2, n/2, Cos[e + f*x]^2] + (a*(-2 + n)*Cos[e + f*x]*(6*b^2*n*(1 +
n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + 2*a*(-1
+ n)*Cos[e + f*x]*(3*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
 + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
```

```
Cos[e + f*x]^2])))/2)*Sec[e + f*x]^2*Sqrt[Sin[e + f*x]^2])/(f*(-2 + n)*(-1
+ n)*n*(1 + n)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*cos(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

maple [F] time = 11.27, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(d\,\cos\left(e+f\,x\right)\right)}^n\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=186

$$-\frac{\left(a^2(1-n)-b^2n\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin(e+fx)(d\cdots}{}$$

```
[Out] -2*a*b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(
f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-(a^2*(1-n)-b^2*n)*cos(f*x+e)*(d*cos(f*x+e))
^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+
1)/(sin(f*x+e)^2)^(1/2)+b^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

Rubi [A] time = 0.22, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3772, 2643, 4046}

$$-\frac{\left(a^2(1-n)-b^2n\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin(e+fx)(d\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a*b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - ((a^2*(1 - n) - b^2*n)*Co
s[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (b
^2*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+b\sec(e+$$

$$= \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}\left(a^2+b^2\sec^2(e\right.$$

$$= \frac{b^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)}{d}$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{b^2(}{}$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{\left(a^2\right.}{}$$

Mathematica [A] time = 0.43, size = 161, normalized size = 0.87

$$-\frac{d\sqrt{\sin^2(e+fx)}\csc(e+fx)(d\cos(e+fx))^{n-1}\left(a(n-1)\cos(e+fx)\left(an\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+f\right.\right.\right.}{f(n-1)n(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(d*Cos[e + f*x])^(-1 + n)*Csc[e + f*x]*(b^2*n*(1 + n)*Hypergeometric2F
1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + a*(-1 + n)*Cos[e + f*x]*(2*
b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[
e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]))*Sqr
t[Sin[e + f*x]^2])/(f*(-1 + n)*n*(1 + n)))
```

fricas [F] time = 2.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

maple [F] time = 8.81, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^{n}\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
```

```
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos\right.}{fn\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x+
e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4225, 16, 2748, 2643}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos\right.}{fn\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\cos(e+fx))^n(b+a\cos(e+fx))\sec(e+fx)\,dx$$
$$= d\int (d\cos(e+fx))^{-1+n}(b+a\cos(e+fx))\,dx$$
$$= a\int (d\cos(e+fx))^n\,dx + (bd)\int (d\cos(e+fx))^{-1+n}\,dx$$
$$= -\frac{b(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d\,c}{}$$

Mathematica [A] time = 0.13, size = 106, normalized size = 0.80

$$-\frac{\sqrt{\sin^2(e+fx)}\,\csc(e+fx)(d\cos(e+fx))^n\left(an\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)+b(n+1)\,{}_2F_1\left(\frac{1}{2},\right.\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b*(1 + n)*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

maple [F] time = 3.04, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)),x)
[Out] int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \cos{\left(e + f x \right)}\right)^{n} \left(a + b \sec{\left(e + f x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-$$

```
[Out] a*AppellF1(1/2,-1/2-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*co
s(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2)/
f-b*AppellF1(1/2,-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(d*c
os(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*n))
```

Rubi [A] time = 0.37, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4264, 3869, 2823, 3189, 429}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -n/2, 1, 3/2, Sin[e + f*x]^
2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2
 - b^2)*f*(Cos[e + f*x]^2)^(n/2))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
```

```
x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{a+b\sec(e+fx)}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{1+n}(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= -\left(\left(a\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^{-n}(e+fx)(d\right.$$

$$= -\frac{\left(a\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1+n}{2}}}{-a^2+b^2+a^2x^2}\,dx, x,\right.}{f}$$

$$= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1-n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 25.73, size = 5216, normalized size = 26.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] Result too large to show
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

maple [F] time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\cos\left(e+f\,x\right)\right)^{n}}{a+\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
[Out] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=309

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2}{}$$

```
[Out] a^2*AppellF1(1/2,-3/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*
cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2
)^2/f+b^2*AppellF1(1/2,-1/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-
b^2))*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a
^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2
/(a^2-b^2))*(d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/f/((cos(f*x+e)^2)^(1/2*
n))
```

Rubi [A] time = 0.51, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4264, 3869, 2824, 3189, 429}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*
x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x]^
2)^(n/2))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx &= \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{(a+b\sec(e+fx))^2}\,dx\\ &= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx\\ &= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \left(\frac{b^2\cos^{2+n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}-\frac{2ab\cos^{3+n}(e+\text{}}{\left(b^2-a^2\cos^2(e+\text{}\right.}\right.\\ &= \left(a^2\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{4+n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx-\left(2ab\cos^{-n}(e\right.\\ &= \frac{\left(a^2\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{3+n}{2}}}{\left(a^2-b^2-a^2x^2\right)^2}\right.}{f}\\ &= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3-n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos}{\left(a^2-b^2\right)^2f}\end{aligned}$$

Mathematica [B] time = 45.58, size = 14144, normalized size = 45.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^n}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

maple [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(a+b\sec\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,\cos\left(e+f\,x\right)\right)^{n}}{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{\left(a+b\sec\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(exppn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```